MATH 145: UNDERGRADUATE ALGEBRAIC GEOMETRY, SPRING 2024

Instructor: Zhiyu Zhang, zyuzhang@stanford.edu.
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Course webpage. Canvas (see also https://stanford.edu/~zyuzhang/MATH145.html).
Textbook. Algebraic Curves (Fulton). Available for free at course webpage.
Time And Place. MWF 09:30 AM - 10:20 AM, GESB150.
Office Hours. Zhiyu: 382-C, Tuesday 12:30-2:30 pm and by appointment. Jiahao: 381-B, Thursday 10:30 am-12:30 pm, 3-4:30 pm.
Grade. 70% homework, 30% take home final projects.

Homework and final projects. Homework will be assigned on Tuesdays and will be due in Canvas on the following Tuesday 11:59 pm. The lowest homework grade will be dropped, and a missed homework will count as a 0. It is encouraged for students to discuss the exercises with each other. It is important to do the homework and practice ideas you learn from the textbook and each class. The final project requires you to write a short note on certain questions and theorems related to algebraic geometry. You may choose the topic and discuss it with the instructor (hopefully set up the topic by 7th week of quarter).

Course description. Solving equations with coefficients is a fundamental topic in mathematics. We may use linear algebra to solve linear equations and find applications in the real world. In general, polynomial equations are hard to solve (and there may be infinitely many solutions). Instead, in algebraic geometry we study invariants and geometry of solutions to polynomial equations. These invariants (e.g. dimensions, coordinate rings, tangent spaces) have geometric meanings and could also be understood using (commutative) algebra. Geometric intuition (from $\mathbb{C}$) could be used to predict behaviors of algebraic invariants (not just over $\mathbb{C}$) and vice versa. We assume that you are comfortable with the use of (commutative) algebra and the use of (topological) spaces. From a purely logical point of view, we will never need any results depending on calculus.

In this course, we study algebraic geometry over an algebraically closed base field e.g. $\mathbb{C}$. Geometric intuition will be emphasized. In the first half you will learn the language and dictionary of algebraic geometry, including affine varieties and maps between them, Hilbert basis theorem and Nullstellensatz. In the second half, you will learn more about algebraic curves. A key topic is Bézout's theorem and basic intersection theory. You may also learn about resolution of singularity and the Riemann-Roch theorem. Moreover, we will find many enjoyable applications of alegbraic geometry. We will try to learn about how to invent and develop algebraic geometry by yourself.

Access and Accommodations. Stanford is committed to providing equal educational opportunities for disabled students. Disabled students are a valued and essential part of the Stanford community. We welcome you to our class. If you experience disability, please register with the Office of Accessible Education (OAE). Professional staff will evaluate your needs, support appropriate and reasonable accommodations, and prepare an Academic Accommodation Letter for faculty. To get started, or to re-initiate services, please visit oae.stanford.edu. If you already have an Academic Accommodation Letter, please use this form (https://forms.gle/jbuEJW798a5oNbnEA) to upload it and detail the specific accommodations you will need in this course. Letters are preferred by the end of week 2, and at least two weeks in advance of any exam, so we may partner with you and OAE to identify any barriers to access and inclusion that might be encountered in your experience of this course. New accommodation letters, or revised letters, are welcome throughout the quarter; please note that there may be constraints in fulfilling last-minute requests.