

Color Appearance: The Effects of Illumination and Spatial Pattern *

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Abstract

The color we perceive at each point in an image depends on information spread across the three spatial arrays of cone photoreceptors. I describe experiments aimed at clarifying how information is integrated across the spatial arrays to yield a color experience. We have found that changes of color appearance due to changes of the ambient illumination and the pattern's spatial frequency can be described using a simple set of optical and neural transformations. Each transformation can be thought of having two parts. First, the transformation converts the color representation into a new coordinate frame that is independent of the image contents. Second, the transformation scales the neural responses in the new coordinate frame by a gain factor that depends on the image contents.

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Introduction

The impressive successes in color science and technology derive mainly from the observation that the human eye is a very poor instrument for discriminating lights of different spectral power distributions. Figure 1 illustrates one way in which color technology takes advantage of our poor spectral resolution. The two spectral power distributions shown in the figure have the same color appearance. One signal is a smooth version of the spectral power distribution emitted from the disk of the sun. The second signal is the spectral power distribution emitted by a typical CRT monitor whose phosphors are set to create the same color appearance. It is impractical to replicate the physical signal, so color CRT technology relies on our poor spectral resolution to arrange appearance matches. Pairs of lights that are physically different but perceptually identical are called *metamers*.

The central theory of color science, *trichromacy*, explains our inability to discriminate these spectral power distributions by assuming that we have only three types of cone photoreceptors [1]. A pair of lights will be indiscriminable, according to theory, when the two lights cause equal rates of photopigment absorption in the three classes of cone photoreceptors. Human cone photopigments and photocurrent spectral sensitivities have only been measured recently [2, 3, 4, 5]; but, by 1931 scientists and engineers at the International Commission on Illumination meetings agreed to standardize on a set of spectral functions believed to be within a linear transformation of the true photopigment spectral sensitivities. These functions can be used to predict the observed metamers, and they form the basis of modern color standards [6, 7, 8, 9, 10].

Color standards have been very successful aids in writing contracts for color products (e.g. textiles and paints) and in color signalling (e.g. broadcast television). It is surprising to realize that current standards are successful because they specify differences we do not see; trichromacy establishes equivalence classes of lights that

Disk of the Sun

CRT Metamer to Disk of the Sun

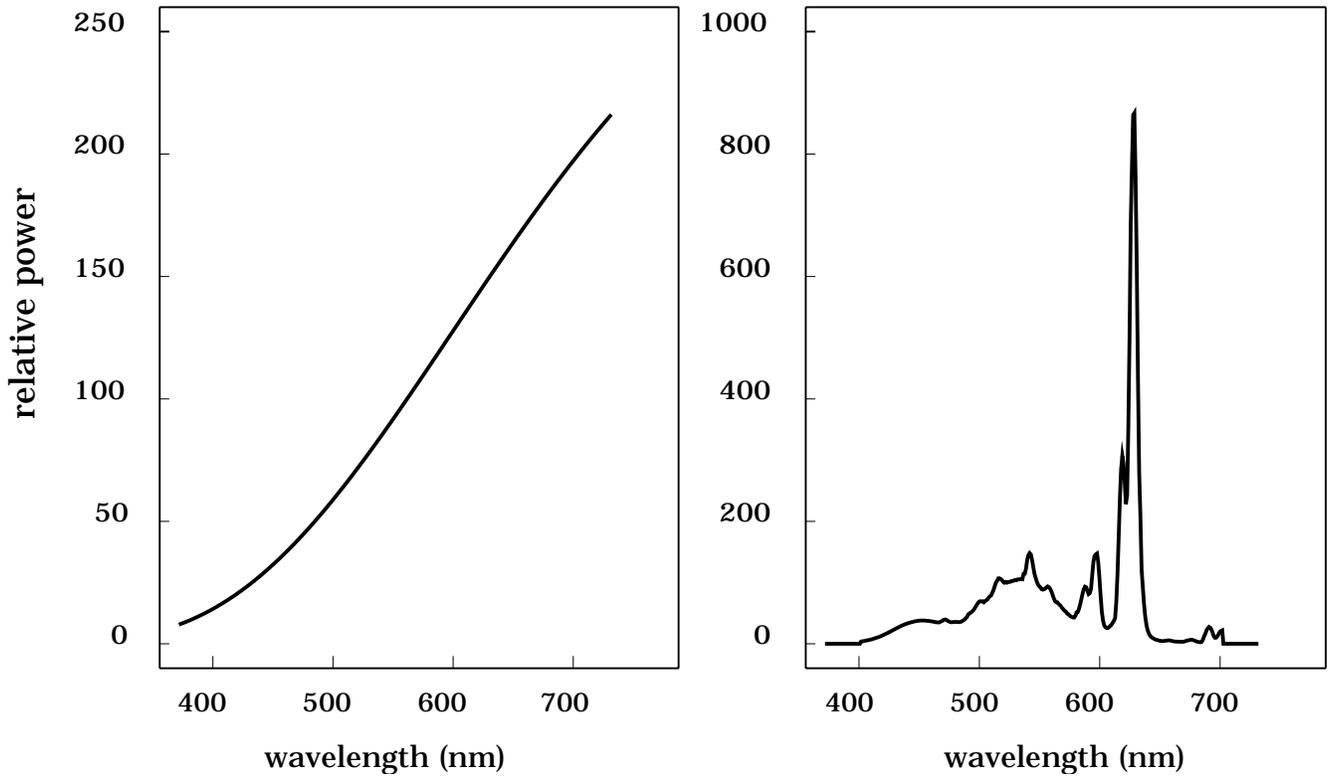


Figure 1: When these two spectral power distributions are presented as uniform fields, they have equivalent effects on the three classes of cone photoreceptors. Consequently, they are indistinguishable to the normal color observer. (Note the difference in vertical scales.)

cannot be discriminated from one another when the lights are viewed in the same context.

As important as it is, trichromacy has two important limitations that restrict its applicability to many current problems. First, the equivalences specified by trichromacy do not generalize when the lights are seen in different contexts, e.g. when we compare printed hardcopy compared with a CRT display. It has long been known that when we compare stimuli across contexts, the image properties across a significant spatial extent of the visual field can influence color appearance; Nearly all of the visual illusions that we use to engage students' attention in introductory psychology and color science courses are designed to illustrate this point [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 9]

Second, trichromacy has only been applied studied extensively using large uniform visual fields (two and ten degrees of visual angle). The design of color television signals through the National Television Standards Committee [22] did provide some important qualitative understanding of the relationship between color appearance and pattern; but, there has been little scientific study of how color appearance depends on the local pattern within an image. Improving our understanding the relationship between pattern and color is essential in order to predict and control the color appearance of image data.

Phenomenology

To understand the challenge of calculating color appearance from the spatial array of photoreceptor responses, consider some classic color illusions. One can alter the appearance of a fixed target easily by presenting the object against backgrounds with different colors. When the background changes the appearance of the object changes.

Some of the nicest examples of this phenomenon are contained in the classic collections of color paper collages by Joseph Albers [23]

The basic principle for creating many of these illusions was described by Helson [24, 25]. The illusions are easy to create using simple computer drawing programs. Place a neutral and a saturated background side by side. Select a drawing color similar to the saturated background, but less saturated. Draw a line across the two backgrounds. As the pen moves across the backgrounds, the color appearance of the line changes. It would be interesting to create a program that altered the CRT values of the pen so that the appearance remained constant as the pen traversed different backgrounds.

A second way in which color appearance changes with the spatial context is illustrated in Figure 2. The bars of the two squarewaves in the figure are identical. But, as the spatial frequency of the squarewave increases, the color appearance of the squarewave bars desaturates. For patterns above eight or ten cycles per degree, the bars of any spectral power distribution appear to be intensity modulations with the same appearance as the background spectral power distribution.

The loss of color at high spatial frequencies must be due, in part, to the axial chromatic aberration of the eye [26, 27] Marimont and I have calculated the modulation transfer function of a model eye (5.0mm pupil) at different wavelengths assuming an optimum focus at 580nm [28]. We used Hopkins [29] formulae and measurements of axial chromatic aberration in the human eye [30, 29]. Our results, which probably represent an upper bound on the human sensitivity, are plotted in Figure 3. The height of the surface shows the theoretical contrast modulation for different spatial frequency modulations of different monochromatic lights. Beyond ten cycles per degree there is almost no contrast outside of a small wavelength region, near the peaks of the long- and middle-wavelength photoreceptors. Our ability to

Pattern-Color Interactions

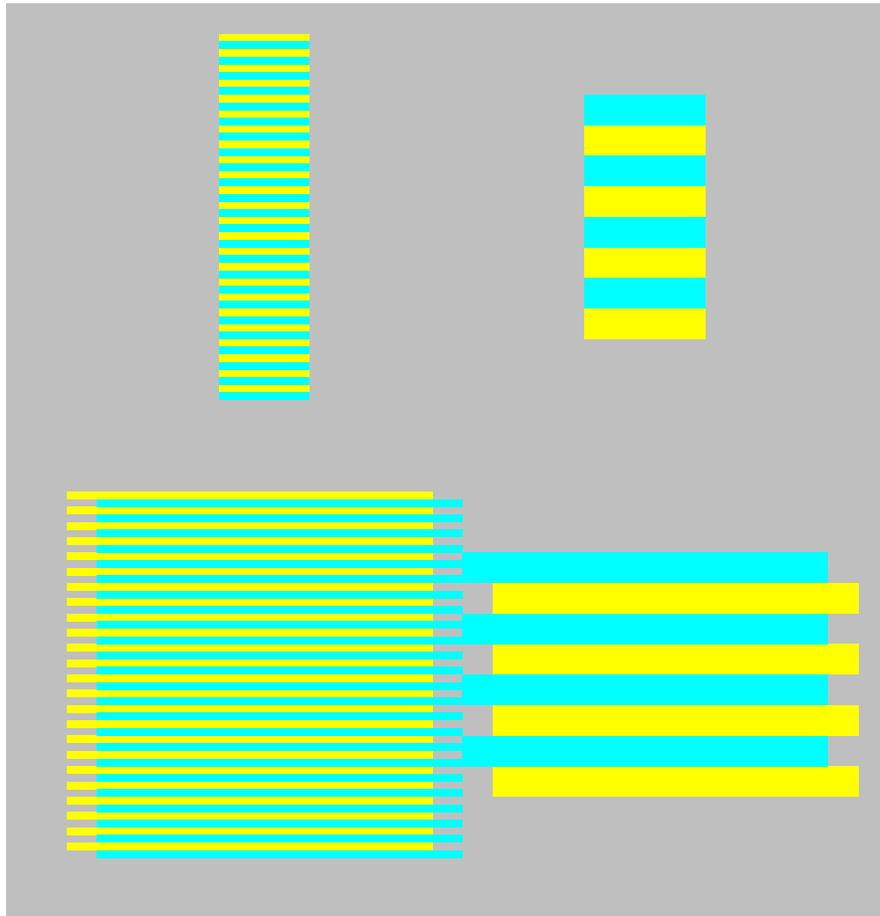
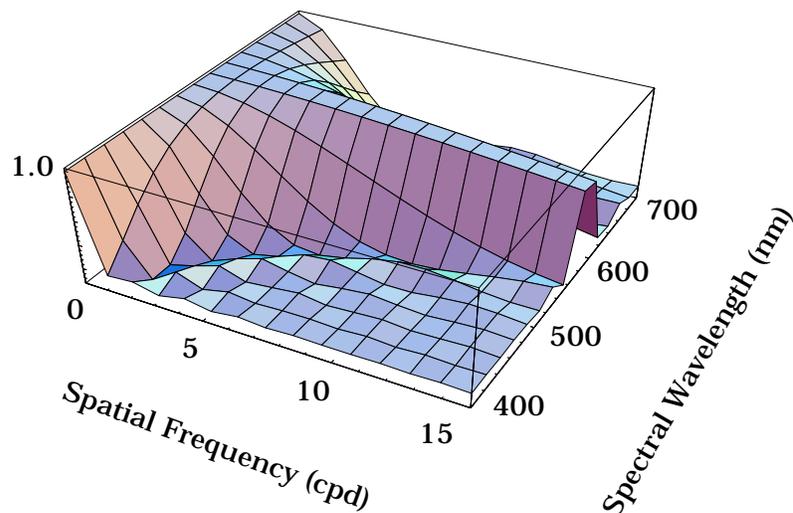


Figure 2: (Color Plate). The bars in the squarewaves are physically the same, measured pointwise. But, color appearance desaturates as we increase the spatial frequency of a squarewave. The effect of spatial frequency can be increased by placing the color print further away from your eyes or simply tilting the page away from you. To eliminate the effect, inspect the prints up close. The size of the effect can also vary depending on the illuminant. The effect depends in large part on chromatic aberration, prior to absorption in the photoreceptors.

Axial Chromatic Aberration of a Model Eye



Assumptions

Only spherical aberration
Pupil diameter = 5.0mm
Accommodation for 580nm
Theory due to Hopkins (1955)
Measurements from Wald and Griffin (1948)
and Bedford and Wyszecki(1957)

Figure 3: The height of the surface shows the optical contrast reduction of spatial frequency patterns composed of monochromatic light. The calculation is based on the assumption that the optics has only spherical aberration, the pupil radius is 5.0 mm (monitor light levels), and the eye is accommodated to focus at 580nm. Very little contrast is transmitted to the retina from the short wavelength phosphor beyond 5 cycles per degree. Since the long-wavelength phosphor in most modern monitors have their energy concentrated near 630nm, very little contrast remains in the signal from this phosphor past 10 cycles per degree.

discriminate colors and contrasts diminishes rapidly with spatial frequency; but we retain our ability to discriminate the position of fine patterns. This observation is the basis for current image compression methods.

Experimental Studies

Introduction

Researchers have used a variety of phenomena and methods to measure appearance [31, 32, 33, 34, 35, 36]. Many of these methods require asking people to make subjective evaluations of appearance such as rating the brightness or similarity of very different lights. In my lab we have studied color appearance by having observers set complete color matches between lights presented in different visual contexts.

Observers do not evaluate the hue or saturation of lights; they only judge whether two lights appear to have the same color. The method is called *asymmetric color-matching*.

Our experimental results consist of vector pairs of photopigment absorption rates, one vector from each visual context, that appear visually equivalent. We study the transformations that map the photopigment absorption vector in one context into the corresponding vector from the second context. The logic of asymmetric color-matching is similar to the basic color-matching experiment: the matches establish visual equivalence classes between lights. But, the method extends color-matching since we establish appearance equivalence across visual contexts [37, 38, 39, 40, 41, 42].

(Chapter 5 in Reference [10] has a nice discussion of the asymmetric color-matching methodology.)

We have found that some important appearance phenomena are described by transformations that are simple compared to formulations found in some modern

theories of color appearance [43, 44] Conceptually, the transformations first change the coordinate frame of the color representation and then scale the sensitivity of each of the new coordinates. Most important, it is possible to understand these transformations in terms of the optical and neural transformations in the visual pathway. I will expand on these ideas after describing our experiments.

Illuminant Variation

Brainard and I studied the effect of changing the ambient illuminant (see Figure ??). We presented subjects a collection of colored squares on a CRT monitor. The squares were simulations of a random sample of Lambertian surfaces, illuminated by a diffuse daylight. We trained subjects to remember one particular surface. After training, we changed the spectral properties of the simulated illuminant slowly, much like a cloud passing in front of the sun. After adapting to the new simulated illumination, subjects adjusted the appearance of a simulated surface until it matched the color appearance of the original [45, 46, 47, 48].

Our experiments identify pairs of lights that appear the same in different visual contexts. We fit various simple models to describe the relationships between the matching lights. We begin by assuming that the lights can be represented in terms of the three rates of photopigment absorptions. We then evaluate the quality of fit from three transformations from the photopigment absorptions under one illuminant to the photopigment absorptions under a second illuminant. The three transformations are nested in their complexity: an affine mapping, a general linear transformation, and finally a diagonal linear transformation that only scaled the photopigment absorptions. If we plot covariance cross-sections of repeated matches in photopigment coordinates, we find a strong correlation (see top of Figure 4). Therefore, we evaluated the quality of the fit of these three models by transforming the data nonlinearly into a coordinate

Covariance Cross-Sections of Repeated Matches

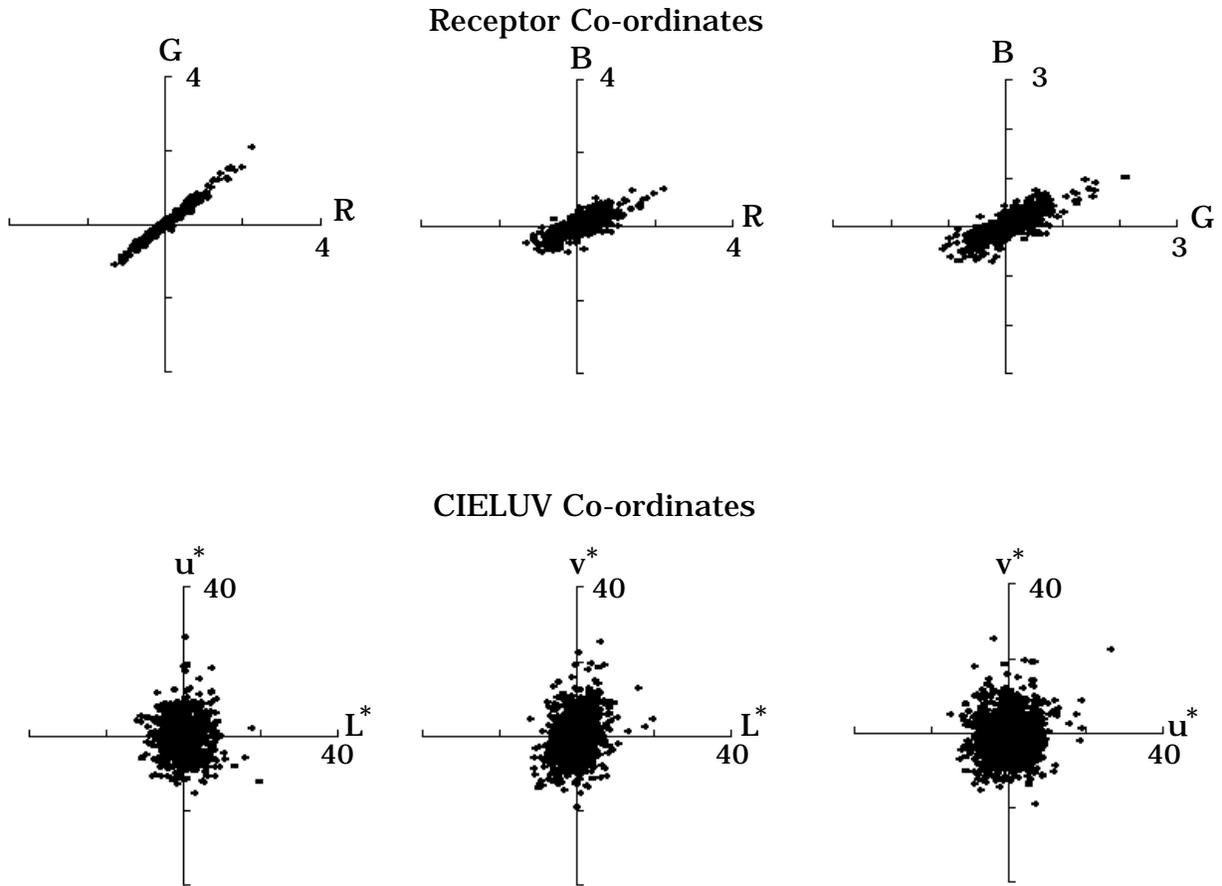


Figure 4: The covariance of the color matches is highly elliptical when plotted in photoreceptor coordinates. To evaluate the model errors, we transformed the data to the CIELUV representation where the covariance clouds are much more nearly spherical. The individual plots show cross-sections of the covariance in different planes.

frame (CIELUV) that reduces the correlation. Cross-sections of the covariance plots in the transformed space are shown on the bottom of Figure 4. We evaluated the three transformations by minimizing the root mean squared error of the transformation fits in the CIELUV space.

Figure 5 compares the performance of the models. The bar on the left indicates the relative size of the effects by showing the error if we make no correction at all. This bar shows us that the size of the effect far exceeds the tolerances usually demanded in imaging applications (below 5 CIELUV units). The next bar shows the subjects' variability on repeated matches, with no illuminant change. The next three bars show the error for the affine, linear and diagonal transformations. The diagonal model is barely worse than the affine model and predicts the subjects behavior about as well as the subjects' own matches predict behavior. The diagonal matrix has a simple interpretation for the visual organization: the matrix entries represent the relative receptor signal gain. Under our measurement conditions, using a CRT of moderate luminance levels, the appearance changes can be explained by regulation of the photoreceptor signal gain.

This sort of experiment had been tried several times in the past [39, 40, 41]. At the time of these experiments, the photopigment spectral curves were not known, so the investigators used a clever mathematical method to evaluate whether the color transformation is diagonal in photopigment coordinates. After finding that the matching transformation is essentially linear, they asked whether the linear transformations could be diagonalized by purely real eigenvectors [49, 50, 51]. The eigenvectors of the estimated linear transformation are complex, leading them to reject the diagonality hypothesis. The argument does not treat the estimation error properly. The best eigenvector solutions are complex for our data, too. But, at Chilchilnisky's suggestion we also looked for and found purely real solutions within the measurement

Transformation Comparisons

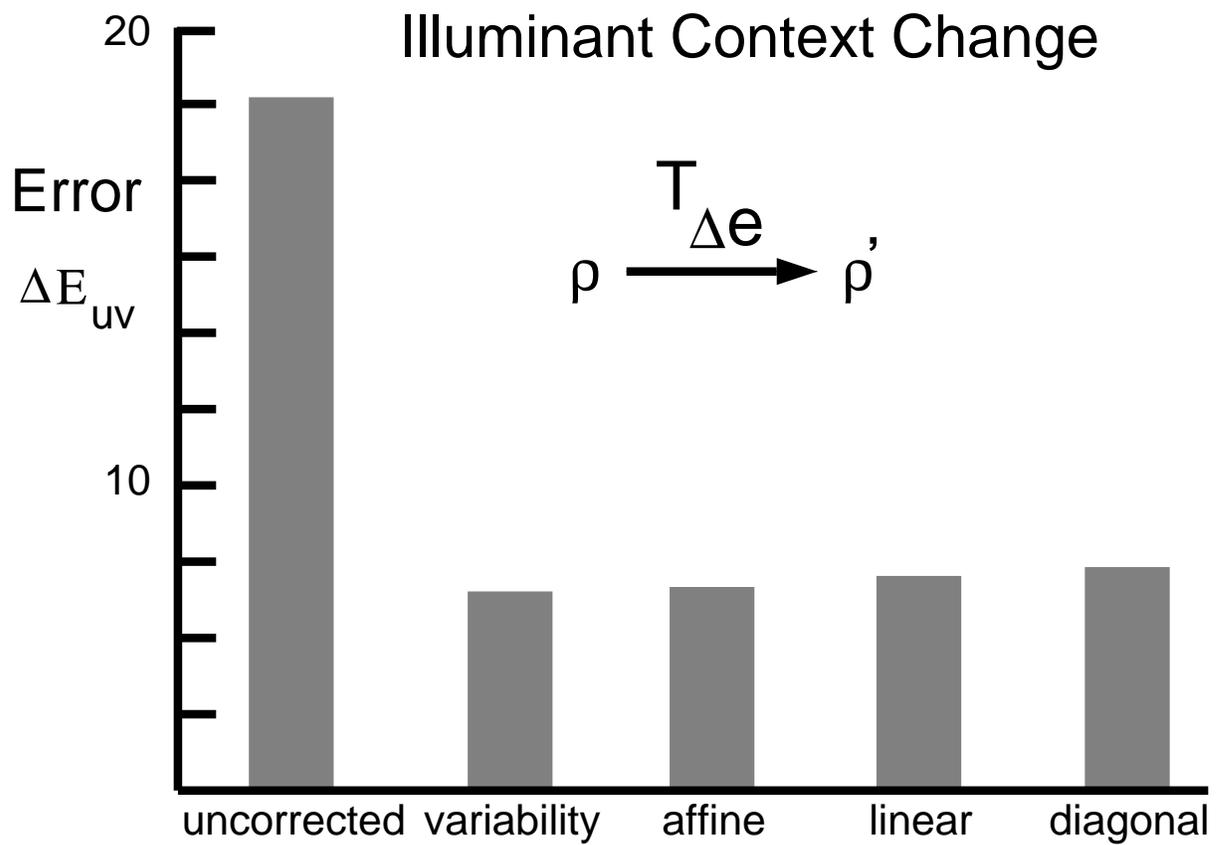


Figure 5: The bar on the left shows the size of the error (CIELUV *DeltaE* units) when no correction at all is applied to the data. The next bar shows the observer's variance to repeated matches. The next three bars show the best affine, linear and diagonal models subject to minimization of the CIELUV error.

noise [48].

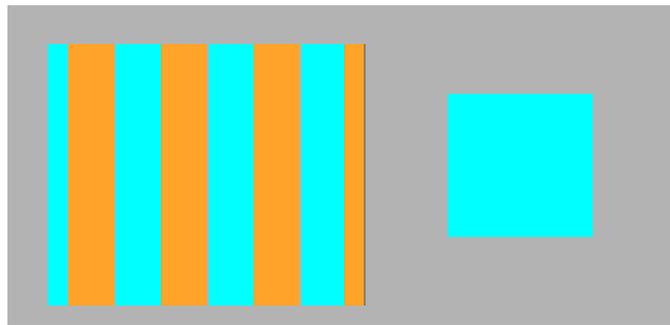
How do image properties determine the photoreceptor gain, i.e., how do the diagonal entries of the transformation vary as we change the illuminant? For the range of simulated illuminants on our CRT, we find that the diagonal entries are a linear function of the illuminant change. This is a new observation and represents a remarkable simplification of the theory.

Spatial Pattern Variation

Poirson and I used similar logic to study the influence of the stimulus spatial frequency stimuli on color appearance [52, 53]. Subjects viewed squarewave patterns, modulated symmetrically about a neutral background. They set the appearance of a two degree uniform patch, presented alternately with the squarewave, to match the color appearance of one of the bars in the pattern. We measured the appearance mapping between the color representation of the squarewave bars and the color representation of the matching box (see Figure 6).

We studied color matches between stimuli up to eight cycles per degree: that is the range where people have good color and contrast vision. The experimental matches are remarkably regular. An example of one set of matches, for a single spatial frequency and single color direction are shown in Figure 7 from Poirson's dissertation. The axes plot the contrast of the photopigment absorption, uncorrected for chromatic aberration. The contrast of the squarewave and the matching box are linearly related for both positive and negative contrast values, passing smoothly through the origin. This linear relationship is typical of all of the measurements we have made. This data set also illustrates that photoreceptor matches do not predict these asymmetric appearance matches. Notice that **increasing** the squarewave contrast to the nominal middle-wavelength photopigment (uncorrected for chromatic aberration) caused the

Asymmetric Color-Matching (spatial)



The observer adjusts the box to match the appearance of the bars in the squarewave

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix} \xrightarrow{T_f} \begin{pmatrix} r \\ g \\ b \end{pmatrix}'$$

(squarewave) (box)

Figure 6: We studied the color appearance equivalences between bars in squarewave gratings and uniform patches. Subjects adjusted the uniform patch to appear the same as the bars in various squarewave gratings.

subject to **decrease** the middle-wavelength contrast in the matching box.

If we plot the data for all spatial frequencies and all color directions in the photopigment coordinate frame, we find very little order. It is possible to find a color coordinate frame in which the data from all of the colored squarewaves plot on a single, unified graph, shown in Figure 9. Poirson and I have shown that the color coordinates in Figure 9 have the unique property that for each spatial frequency (grouped in the columns), the data from all the color directions plot along a single line. The slopes of the lines define the relative spatial sensitivity of the coordinates in the new frame.

The coordinate frame in Figure 9 is a linear transformation of the photopigment absorption curves. Since the spectral sensitivities of the new coordinates have both positive and negative values, the new coordinate frame is called an *opponent-color representation*. The photopigment spectral responsivities and the opponent-colors responsivities are plotted next to one another in Figure 8. These opponent-colors responsivities, derived from first principles based on asymmetric spatial color matches, are remarkably similar to curves derived by Hurvich and Jameson [54, 31, 55] in unique hue cancellation experiments. In those experiments opponent-colors representations are derived from observers judgments about the color appearance of test lights; observers are asked: Is the light red or green?. Observers in our experiments never comment on the color appearance of the test lights directly, they merely set matches. The convergence of these two independent methods is encouraging and should permit us to relate our results to appearance measurements made using hue cancellation methods [32, 34, 36].

Spatial Color Matches

Color line (+r,-g,-b),1cpd, obs: JL

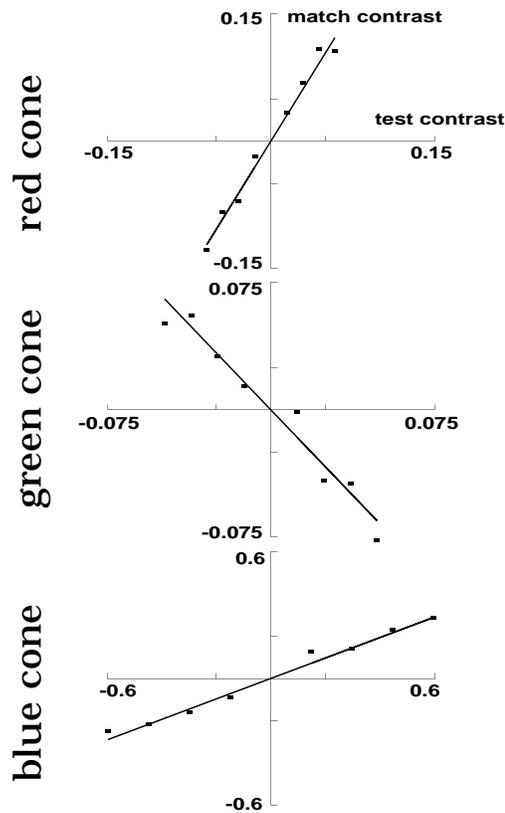


Figure 7: These three graphs show the matching contrast between the bars in a one cycle per degree squarewave and a uniform patch. The squarewave contrast is plotted on the horizontal axis and the contrast of the uniform patch is plotted on the vertical axis. The three graphs show the data with respect to the photopigment absorptions of the three cone classes, uncorrected for chromatic aberration.

Spectral Coordinate Frames

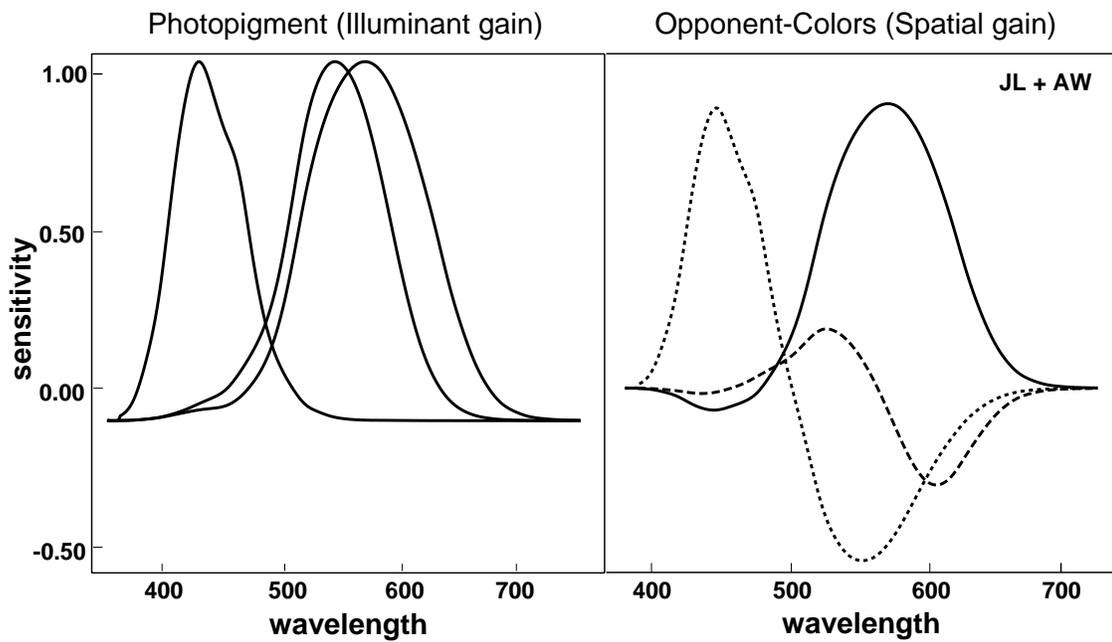


Figure 8: The spectral sensitivities of the photopigment coordinates (left) and the spectral sensitivity of the mechanisms after transformation into the opponent-colors coordinates (right).

Color-matches for Different Spatial Frequencies

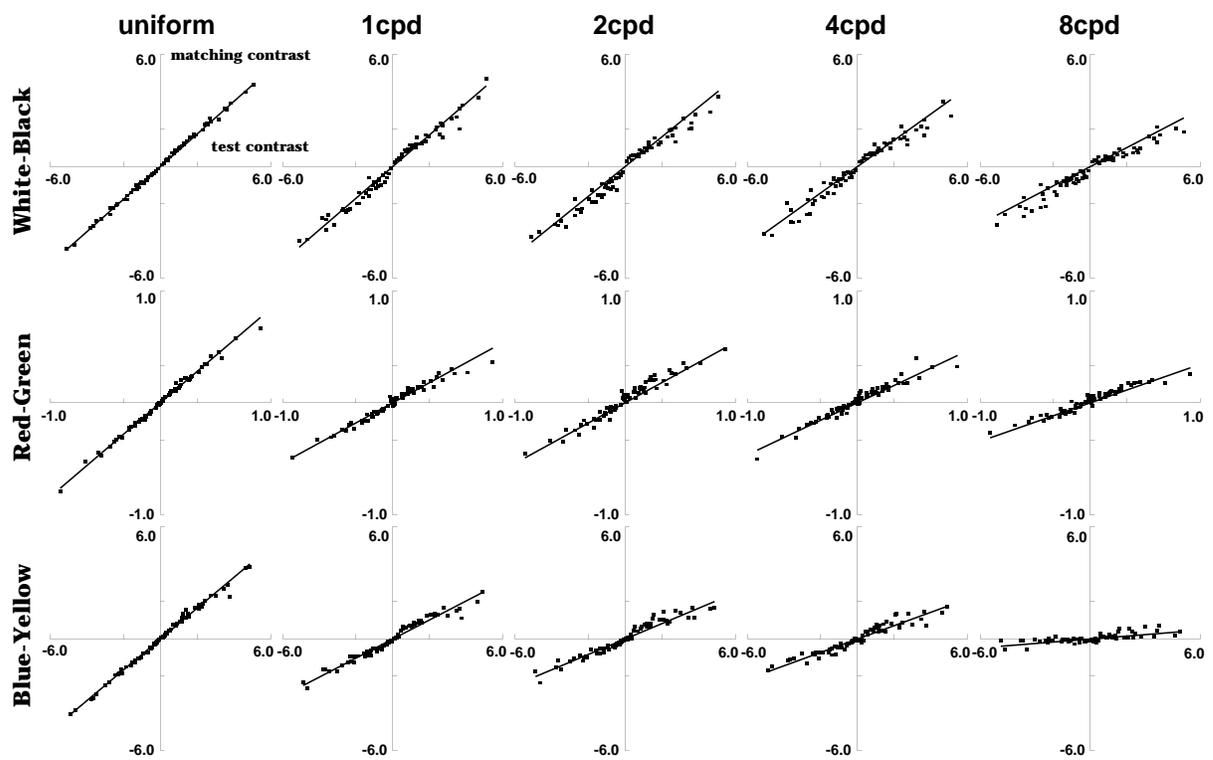


Figure 9: There is a unique coordinate frame such that for a given squarewave frequency matches for all the different colors fall as close as possible to a line. The slope of the line defines the spatial sensitivity of that coordinate.

Theory and Discussion

From our measurements and much related work, we can begin to see how to unify the different components of color appearance models. To simplify the mathematical presentation of the theory, I have sketched the main color appearance components (Figure 10) as a matrix tableau. The simple matrix formulation illustrated in the Figure only applies when the input stimulus has a sinusoidal spatial profile.

We begin by specifying the stimulus in the wavelength domain. When we assume the input signal is a spatial sinusoid, we can represent it using only its spectral power distribution. We describe the spectral power distribution of the input by the vector on the right of the matrix tableau. The first visual transformation is *axial chromatic aberration*. Because the optics of the eye are approximately a shift-invariant linear system near the fovea, when the input stimulus is sinusoidal we can represent axial chromatic aberration as a diagonal transformation applied to the spectral components of the spatial sinusoid. In this sense, we can think of axial chromatic aberration as a regulation of the gain of the spectral components of the spatial sinusoid. The amount of axial aberration (i.e. gain) for any target depends on factors such as the pupil aperture and the accommodation of the lens (see, e.g. Figure 3). Poirson and I estimated the spatial sensitivity loss in all three color mechanisms after correcting for chromatic aberration. We find that more than half the spatial resolution loss is due to chromatic aberration.

Photoreceptor transduction is the second stage; transduction is represented by the $3 \times N$ matrix whose rows are the photopigment responsivities. Transduction changes the representation of the signal from the wavelength domain of physics to the biological domain of neural signals. We can think of this transformation as a shift of coordinate frames. Gain regulation occurs in this new coordinate frame, too, as represented by the next diagonal transformation. The photoreceptor gain depends on factors such as the

illumination change, and probably on the distribution of nearby portions of the image.

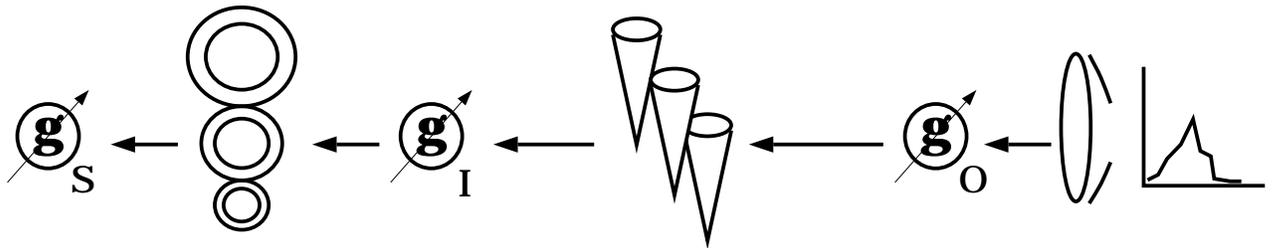
The third stage repeats the basic process; there is a transformation into a new representation, again followed by sensitivity regulation. In this case the coordinate frame is an opponent-colors representation. The sensitivity regulation in this coordinate frame depends upon the spatial structure in the image. High frequency spatial patterns are passed very poorly by two of the opponent-color mechanisms.

The computation described in Figure 10 yields three numbers that specify the color appearance of the sinusoidal input stimulus. Two sinusoids, with different spatial frequencies, seen in different contexts, match in appearance only if the three numbers for each stimulus agree.

There are several features of our observations on color appearance to date that strike me as remarkable. First, we have been able to segregate those components of the model that depend on changes in the ambient illumination from those parts that depend on the pattern of the stimulus. Second, we have kept the image-dependent parts of the computation segregated from the image-independent parts. The coordinate transformations are image-independent, representing a fundamental neural transformation. The gain factors in the diagonal matrices are image-dependent and are related to image properties such as the local mean excitation or local spatial frequency.

Finally, I am struck by the implication of this model for the neural representation of appearance judgments. The computation in Figure 10 is a series of common optical and neural calculations. Each step in the process performs the same basic action: the data are transformed into a useful coordinate frame, and then a scale factor is applied to each coordinate. It would be lovely to discover that neural appearance mechanisms use common computational elements and processes repeatedly to achieve a variety of objectives. A coordinate frame transformation followed by image-dependent gain control is a simple but powerful computational framework. The framework provides a

Gain Regulation in a Series of Color Representations



OPPONENT

RECEPTOR

OPTICS

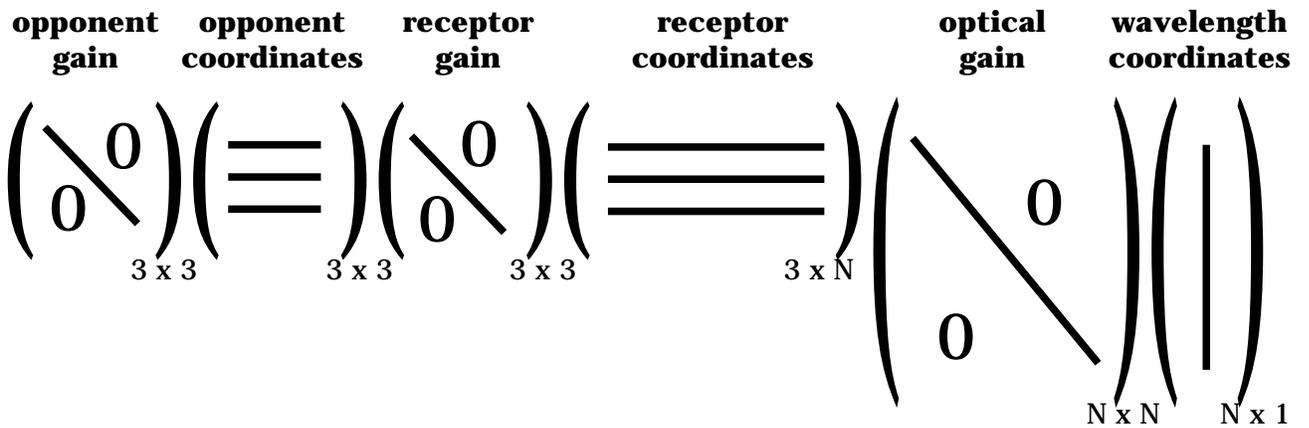


Figure 10: A matrix tableau overview of the three stages of color appearance we have analyzed: spectral gain regulation due to chromatic aberration, photoreceptor gain regulation, and opponent-colors gain regulation.

useful account the color appearance of simple patterns. How broadly do these principles extend within the neural representation of appearance?

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