DISCRETE COSINE TRANSFORM DOMAIN RESTORATION OF DEFOCUSED IMAGES

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ABSTRACT

In DCT-based compressions such as JPEG, it is a common practice to use the same quantization matrix for both encoding and decoding. However, this need not be the case, and the flexibility of designing different matrices for encoding and decoding allows us to perform image restoration in the DCT domain. This is especially useful when we have severe limitations on the computational power, for instance, in on-camera image manipulation for programmable digital cameras. Here, we provide an algorithm that would compensate partially for a defocus error in image acquisition, and experimental results show that the restored image is closer to the in-focus image than is the defocused image.

Key words: Image reconstruction-restoration, transforms, aberration compensation, focus, noise in imaging systems.

1. INTRODUCTION

The JPEG algorithm is a very popular lossy image compression standard for still-frame, continuous-tone images [1]. In this algorithm, the image is first divided into 8×8 non-overlapping blocks, and each block is subjected to a discrete cosine transform (DCT). The coefficients are then quantized according to the quantization matrix Q_e. This is done by rounding off the quotients when the DCT coefficients are divided entrywise by Q_e. They are then entropy-coded before transmission. Upon receiving, the decoder reverses the process for the entropy coding, dequantizes the coefficients by multiplying entrywise with the matrix Q_d, and performs the inverse DCT. The compression is lossy because of the quantization process.

It is customary and convenient to use the same quantization matrix for both encoding and decoding, i.e. $Q_e = Q_d$. The JPEG committee actually recommends the matrix as shown in figure 1 for both Q_e and Q_d, which takes into account some of the human visual system properties, although the use of it is strictly voluntary. In the literature, one could find various attempts to vary the dequantization matrix Q_d slightly to achieve better images

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Figure 1: The recommended JPEG Quantization Matrix

(see, for example, [2], [3], and [4]). In this paper, we will first show that using $Q_e = Q_d$ is not necessarily optimal in the sense of reducing noise, and in fact, in some circumstances the flexibility of using a Q_e different from Q_d allows us to restore images in the DCT domain. In particular, we provide an algorithm for image restoration when it has been taken out-of-focus. We assume here that the amount of focusing error is known *a priori*, or estimated beforehand by some external means. We will show that after compression and decompression, the image is closer (in the mean-square sense) to the original in-focus image than is the uncompressed defocused image.

Before we proceed, however, it is imperative for us to explain why and when this algorithm would be useful. The motivation behind the scheme is that as the sensors in digital cameras move from being CCD-based to CMOS-based, it is possible to allow certain computational power on the camera to devote to image restoration. For example, instead of mechanically moving the lens to adjust focusing, we can use the computational capacity to manipulate the image concurrently, thereby reducing the need for an exact positioning of the lens. However, the cost of on-camera computation is much more expensive than if done offline, and a full-frame fast Fourier transform of the image is not desirable because its complexity goes up too rapidly with the size of the image. Yet, as most cameras have built-in JPEG compression and decompression algorithms in place, it would be very cost-effective if we can take advantage of the blockwise DCT that is readily available to us as part of the JPEG standard. Our goal, therefore, is to make use of the blockwise DCT frequency components in the design of Q_e and Q_d to approximate the desirable image restoration had we been able to perform the full-frame FFT.

2. NOISE TRADEOFF

Consider one 8×8 block of the image. Let its DCT domain representation be $X(u, v)$, where u and v are the spatial frequencies in the horizontal and vertical directions, both ranging from 0 to 7. Using $X_q(u, v)$ to denote the quantized coefficients and $X_n(u, v)$ for the quantization noise, we see that they are related by:

$$\frac{X(u,v)}{Q_e(u,v)} = X_q(u,v) + X_n(u,v). \tag{1}$$

Since we only transmit $X_q(u,v)$, for decoding, we have:

$$\hat{X}(u,v) = Q_d(u,v)X_q(u,v), \tag{2}$$

where we have used $\hat{X}$ to denote the estimate of X. Now to compare the original and the decompressed image, we normally would have to calculate the mean-square error (MSE) in the space domain. However, because of the unitary nature of the 2-D DCT II, we could employ the Parseval's theorem and perform the calculation in the DCT domain [4]. Therefore,

$$\begin{aligned} \text{MSE} &= \sum_{u=0}^{7}\sum_{v=0}^{7}\left(X(u,v) - \hat{X}(u,v)\right)^2 \\ &= \sum_{u=0}^{7}\sum_{v=0}^{7}((Q_e - Q_d)X_q + Q_e X_n)^2 \end{aligned} \tag{3}$$

where it is understood that Q_e, Q_d, X_q, and X_n all have arguments (u,v). We can see that when X_n is small and X_q large, it is reasonable to set $Q_e = Q_d$ to generate small MSE. However, typically for high frequencies we have larger X_n and smaller X_q, and changing Q_e at those frequencies would provide a tradeoff between the sources of noise and may reduce the overall MSE. With this in mind, we now look at a specific case where the original image is in fact corrupted by an out-of-focus optical transfer function (OTF), and we modify the quantization matrix Q_e to perform partial restoration.

3. DEFOCUSING

In imaging of a diffraction-limited system with incoherent light, the object and the image are related by [5]:

$$\mathcal{G}_i(f_x, f_y) = \mathcal{H}(f_x, f_y)\mathcal{G}_g(f_x, f_y) \tag{4}$$

where $\mathcal{G}_i$ is the normalized frequency spectrum of the image intensity I_i, $\mathcal{G}_g$ is the normalized frequency spectrum of the object intensity I_g, and $\mathcal{H}$ is called the optical transfer function (OTF). If the ideal focusing plane is at z_i from the lens, but we focus on z_a, and radius in the pupil is r, we define, as in [5]:

$$W_m = \frac{1}{2}\left(\frac{1}{z_a} - \frac{1}{z_i}\right) r^2 \tag{5}$$

which is an indication of the severity of the focusing error when normalized by wavelength λ. The path length error is therefore

$$W(x,y) = W_m \frac{x^2 + y^2}{r^2} \tag{6}$$

and

$$\mathcal{H}(f_x, f_y) = \frac{\int\int_{\mathcal{A}(f_x,f_y)} \exp\left(jk[W(x+x_0, y+y_0) - W(x-x_0, y-y_0)]\right) dxdy}{\int\int_{\mathcal{A}(0,0)} dxdy} \tag{7}$$

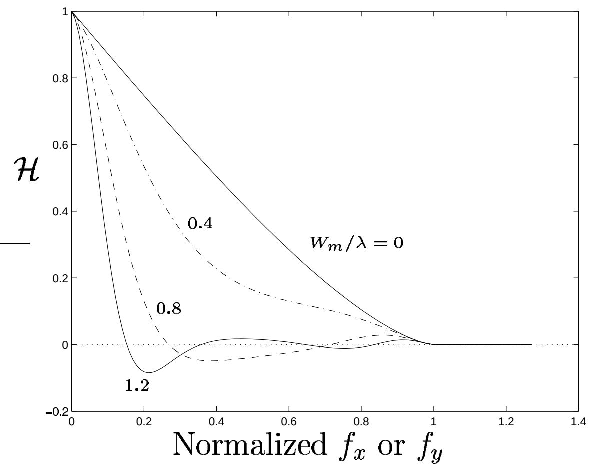

Figure 2: Cross-section view of the OTF

with $\mathcal{A}(f_x, f_y)$ being the area of overlap of the pupil displaced by $\pm(x_0, y_0)$ and

$$x_0 = \frac{\lambda z_i f_x}{2}, \quad y_0 = \frac{\lambda z_i f_y}{2}. \tag{8}$$

Figure 2 shows a cross-section of the OTF for a circular pupil with various amounts of defocus. We see that even for in-focus imagery, we have a cutoff because of the finite size of the pupil, and we also see attenuation at frequencies before cutoff. For restoration of a defocused image, our goal is to modify it to resemble the in-focus image rather than to reproduce the object itself.

4. DCT-DOMAIN RESTORATION

Since the OTF multiplies the Fourier transform of the images, we expect that the middle to high frequencies (where the OTF of $W_m/\lambda = 0$ and $W_m/\lambda > 0$ differ most) are more severely suppressed and ideally we would like to boost those frequencies. However, in the DCT domain the blockwise frequencies do not bear a simple relationship with the frequency components in the Fourier domain. Indeed, blocking destroys the space-invariance of the system, and so we cannot relate the object and the image by a transfer function as in (4). Therefore, it is not possible to specify analytically the amount of pre-emphasis required for each of the DCT frequency components from the OTF expressed in the Fourier domain. Instead, we collect a set $\mathcal{C}_{\text{in}}$ of in-focus images and another set $\mathcal{C}_{\text{out}}$ of the corresponding out-of-focus images. Let $X_{\text{in}}(u, v)$ be the vector containing the (u, v) DCT coefficients in all the blocks of the in-focus images, and $X_{\text{out}}(u, v)$ be the corresponding vector for the out-of-focus images, and we seek a multiplicative factor such that

$$X_{\text{out}}(u, v)a(u, v) \approx X_{\text{in}}(u, v). \tag{9}$$

-1	-1	-1
-1	8	-1
-1	-1	-1

Figure 3: Laplacian convolution kernel

The best $a(u, v)$, in the mean-square sense, is given by:

$$a(u, v) = \frac{\langle X_{\text{in}}(u, v), X_{\text{out}}(u, v)\rangle}{\langle X_{\text{out}}(u, v), X_{\text{out}}(u, v)\rangle} \tag{10}$$

with $\langle \cdot, \cdot \rangle$ denoting inner product. This method is related to the variance-matching method in [6] designed for text sharpening in scanned images, but is superior in terms of mean-square error reduction. In the appendix we provide an examination of when the variance-matching method converges to the minimum mean-square error (MMSE) method used here. Unfortunately we have no guarantee that $a > 0$, and indeed it may not be if its calculation is dominated by noise. Therefore, we clip a to be ≥ 0.001. Note that we do not have to multiply the DCT coefficients by $a(u, v)$ before quantization, but could simply change the encoding quantization matrix to:

$$Q_e(u, v) = \frac{Q(u, v)}{a(u, v)} \tag{11}$$

where Q could be the quantization matrix in figure 1.

5. REGULARIZATION

Because of the ill-posed nature of the image restoration problem, an MMSE solution is known to be highly sensitive to noise, especially at high frequencies [7]. It is very important to incorporate some regularization constraint to compromise between fidelity of the data and consistency with *a priori* knowledge of the image. Here we propose a method of regularization similar to [4] that can be incorporated in the dequantization matrix Q_d.

Assume we have B blocks altogether. Let $l(j, k)$ be a highpass filter, such as that of a Laplacian kernel as shown in figure 3 [8]. Let L be the 2-dimensional Discrete Fourier Transform (DFT) of l with size 8×8, and so

$$G_i = \sum_{u=0}^{7} \sum_{v=0}^{7} \left(X_{\text{in}}(u, v) L(u, v)\right)^2 \tag{12}$$

would measure the amount of high-frequency content in the block i. Therefore, we want

$$\sum_{u=0}^{7} \sum_{v=0}^{7} \left(\hat{X}_{\text{out}}(u, v) L(u, v)\right)^2 \leq G = \frac{1}{B} \sum_{i=1}^{B} G_i. \tag{13}$$

This serves as our regularization constraint. At the same time, we also want to constrain the MSE of the reconstructed image compared to the in-focus image. Therefore we have:

$$\begin{aligned}\sum_{u=0}^{7}\sum_{v=0}^{7}\left(\frac{\hat{X}_{\text{out}}(u,v)}{Q(u,v)} - X_{q,\text{out}}\right)^2 &\leq R \\ &= \frac{1}{B}\sum_{i=1}^{B}\left(\sum_{u=0}^{7}\sum_{v=0}^{7}X_{n,\text{in}}^2(u,v)\right).\end{aligned} \tag{14}$$

The expression on the left is our objective function, and our goal is to minimize it subject to the constraints mentioned above. Having set up the optimization problem, we could now solve it via Lagrangian multiplier. Therefore we have:

$$J(\nu) = \sum_{u=0}^{7}\sum_{v=0}^{7}\left(\frac{\hat{X}_{\text{out}}(u,v)}{Q(u,v)} - X_{q,\text{out}}\right)^2 + \nu\sum_{u=0}^{7}\sum_{v=0}^{7}\left(\hat{X}_{\text{out}}(u,v)L(u,v)\right)^2 \tag{15}$$

where we have used ν to represent the Lagrangian to avoid confusion with the wavelength λ. As discussed in [9] and [10], ν is given by (R/G). Now, setting

$$\frac{\partial J(\nu)}{\partial \hat{X}_{\text{out}}} = 0 \tag{16}$$

we obtain, after some simplification,

$$\hat{X}_{\text{out}}(u,v) = \frac{Q(u,v)G}{G + Q^2(u,v)L^2(u,v)R}X_{q,\text{out}}. \tag{17}$$

Compared with equation (2), it is clear that we should set Q_d as:

$$Q_d(u,v) = \frac{Q(u,v)G}{G + Q^2(u,v)L^2(u,v)R}. \tag{18}$$

Note that, as $G \to \infty$, i.e. as we relax the regularization constraint, we obtain $Q_d = Q$. Finally, we need to round off Q_d to contain integers in the range of 0 to 255, since the entries of the quantization matrix are represented as 8-bit unsigned integers in the JPEG algorithm [1].

6. SIMULATION RESULTS

We implement the algorithm on a 256×256 "bridge" image, and the resulting images are shown in figure 4. To obtain a quantitative comparison among the images, we use the signal-to-noise (SNR) ratio as the metric. SNR is defined as [11]:

$$\text{SNR}(x,\hat{x}) = 10\log_{10}\left(\frac{\sum_j\sum_k(x(j,k))^2}{\sum_j\sum_k(x(j,k)-\hat{x}(j,k))^2}\right) \tag{19}$$

where (j, k) range over the whole image. Now for the figures, (a) is the ideal in-focus image that we would like to obtain with an aberration-free lens. (b) is the image we get with a defocus parameter $W_m/\lambda = 0.4$. We also add some Gaussian noise to the defocused image, with SNR = 30dB. We compress and decompress (b) with the standard quantization matrix as recommended by the JPEG standard (see figure 1) to arrive at (c). Based on this quantization table, we make the changes according to our algorithm outlined above for Q_e and Q_d and we obtain the image in (d). We also did a set of experiments when no noise is present, and the SNR of these two sets of experiments are shown in table 1. We note that our method generally gives a higher SNR than even the uncompressed defocused image.

	Signal-to-noise ratio	
Conditions	Without noise	With noise
Before compression	25.1dB	24.3dB
Standard $Q_e = Q_d$	23.4dB	23.2dB
Customize Q_e and Q_d	25.9dB	25.1dB

Table 1: SNR of Various Images

7. CONCLUSIONS

In this paper we have presented an algorithm that would take advantage of the flexibility in designing the quantization matrices in JPEG to perform partial restoration of the images. Experimental results show that the restored images, although quantized, could have a smaller distortion in terms of mean-square error compared to the unquantized images before processing.

8. APPENDIX

In the variance-matching method [6], the multiplicative factor $a'(u, v)$ is chosen such that:

$$a'(u,v) = \sqrt{\frac{\mathrm{Var}(X_{\mathrm{in}})}{\mathrm{Var}(X_{\mathrm{out}})}} = \sqrt{\frac{\frac{1}{n}\sum X_{\mathrm{in}}^2 - (\frac{1}{n}\sum X_{\mathrm{in}})^2}{\frac{1}{n}\sum X_{\mathrm{out}}^2 - (\frac{1}{n}\sum X_{\mathrm{out}})^2}} \tag{20}$$

and we want to compare it against our calculation of a in (10). By the Cauchy-Schwarz inequality, we know that:

$$\left(\sum X_{\mathrm{in}} X_{\mathrm{out}}\right)^2 \leq \sum X_{\mathrm{in}}^2 \sum X_{\mathrm{out}}^2 \tag{21}$$

with equality if and only if X_{out} equals X_{in} or a scaled version of it. Now taking square root on both sides and rearranging terms in (21), we obtain:

$$\frac{\sum X_{\mathrm{in}} X_{\mathrm{out}}}{\sum X_{\mathrm{out}}^2} \leq \sqrt{\frac{\frac{1}{n}\sum X_{\mathrm{in}}^2}{\frac{1}{n}\sum X_{\mathrm{out}}^2}}. \tag{22}$$

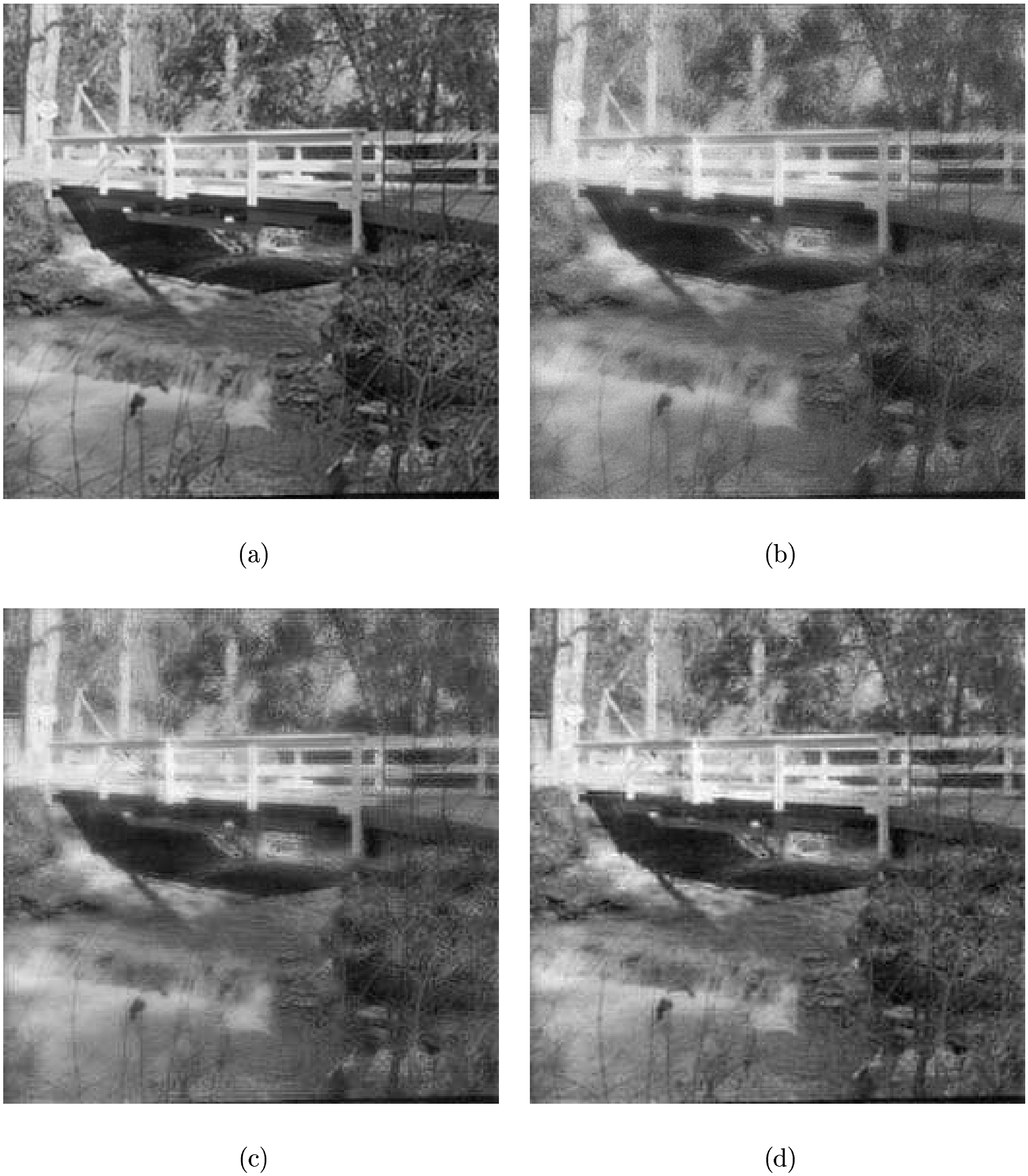

Figure 4: (a) In-focus image (b) Defocused image (c) Image compressed and decompressed with normal Q (d) Image compressed and decompressed with improved Q_e and Q_d

Except for the DC frequency, X_{in} and X_{out} are reasonably approximated to possess zero-mean Laplacian distributions [12], so we have $\frac{1}{n}\sum X_{\text{in}} \approx 0$ and $\frac{1}{n}\sum X_{\text{out}} \approx 0$, and (22) now implies $a \leq a'$. Since X_{out} is generally not simply a scaled version of X_{in}, the variance-matching algorithm tends to over-compensate the defocus by selecting $a' > a$.

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