# Distributed Graph Coloring 

Jane Bae

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## How to color a graph?



Given a graph $\boldsymbol{G}=(\mathrm{V}, \mathrm{E})$ with $|\mathrm{V}|=\mathrm{n}$, and maximal degree $\Delta$, there is always a $\Delta+1$ coloring.

## Naïve Algorithm

- Color the graph with n colors
- Reduce the colors down to $\Delta+1$
- Requires $n-\Delta-1$ iterations.

Depth: $\mathrm{O}(\Delta \log \Delta)$

Total Depth: O(n- $\Delta-1)$

## Distributed Algorithm

- Color the graph with n colors
- Reduce to $5 \Delta^{2}$ logn colors with Linial's Algorithm

Depth: $\log ^{*} n+0(1)$

- Reduce the colors down to $\Delta+1$ in parallel
- The number of iterations required becomes $O(\Delta \log \Delta)$ with Kuhn-Wattenhofer reduction
Depth: $\mathrm{O}(\Delta \log \Delta)$
Total Depth: $O(\Delta \log \Delta)+\log ^{*} n$
Communication Cost: $\mathrm{O}(\Delta)$ per processor

