

CME 305: Discrete Mathematics and Algorithms

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Midterm Winter 2015

1. (5 points) Prove that every tree on n nodes has a vertex cover of size at most $\lceil \frac{n-1}{2} \rceil$
2. (10 points) The long-bar-bell graph on n nodes consists of two cliques of size $n/3$ with a chain of length $n/3$ connecting the two cliques. Prove that the long-bar-bell graph has covering time $\Theta(n^3)$ (upper and lower bound needed)
3. (10 points) Prove that every tree has at most one perfect matching.
4. (15 points) The SETCOVER problem is as follows: Given a set E of elements and a collection S_1, \dots, S_n of subsets of E , is there a collection of at most k of these sets whose union equals E ? Prove that SETCOVER is NP-complete.
5. (10 points) Prove that a graph with minimum degree $n/2$ must have effective resistance $O(1)$.