Lecture-11 2/10/2015

* The most important thing is Home Work for mid term.

Basic Graph Theory. (Perfect for easy questions)

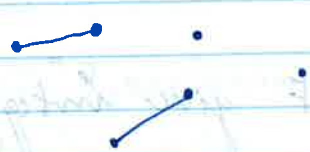
- Trees.
- Matchings (Perfect!)
- Independent Sets. / Vertex Covers.
  Complement operation.
- Edge Cover: Ques (2011 midterm).
- Diameter
- Hamiltonian cycles.
- Eulerian Tour.

Ques: Min edge cover C minimum Edge cover.

$|C| + |M| \le n$ $\quad |C| \le |C_1|$

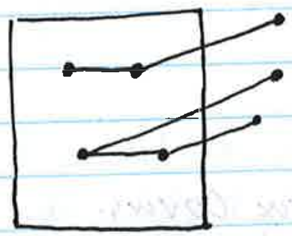
maximal matching.

Proof.



Proof:

Take the matching



$C \leq |M| + n - 2|m|$

by cover property (min edge)

Guarantee there will be a cover time.

covertime constraints.

→ $C(G) \leq 2n(n-1)$

→ $R_{dia} = \max_{u,v} R_{u,v}$

→ $mR \leq C(G) \leq 2e^3 m \log(n) R + n$
$= O(mR\log(n))$

$C_{uv} = 2m R_{uv}$

Markov Inequality: $\Pr[x > a] \leq \frac{E[x]}{a}$

Max Flow / min cut.

→ Max min

→ Create graph ⟷ F.F give integer solns. on integer capacities

→ Encode constraints.

→ How to compute output.

REDUCTIONS

There is going to be reduction on the mid term.
make a new problem & prove it is NP complete.
Do it by a black box.
"Know all the N-P complete problems".

NP hard problems → VERTEX COVER

→ Integer Programming
→ Independent Set.
→ TSP → metric, → hamiltonian cycle.
→ SAT / 3 SAT
→ Max cut

(We are not going to go beyond NP).

~~APPROXIMATION ALGO~~

CUTS

- s-t min cut → F.F (know the proof).
- $^nC_2$ global min cut → (extended global min-cut Karger algo) $\leq \binom{n}{2}$ possibilities
- max cut → Randomized ratio = 1/2 → Deterministic + Greedy

→ $O(2^n)$ realisation for one st.
$(n-1)$ values for s.t

→ have an handle on OPT

APPROXIMATION ALGORITHMS.

1. Max-cut (number of edges) ← OPT

2. TSP
   - 2- approximation. (MST + doubling + shortcutting)
     handle on OPT
   - 1.5 approximation (MST + min matching + shortcutting)

3. Vertex Cover, 2- approx (Relaxation)

OPT: universal notation for objective function's maximum or minimum value depending on the problem.