1/8/2015.

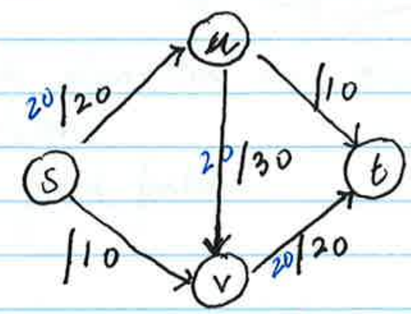
GLOBAL MIN CUT (Karger's algorithm).

This class:

s-t min cut, max-flow

→ focus on directed weighted graphs.

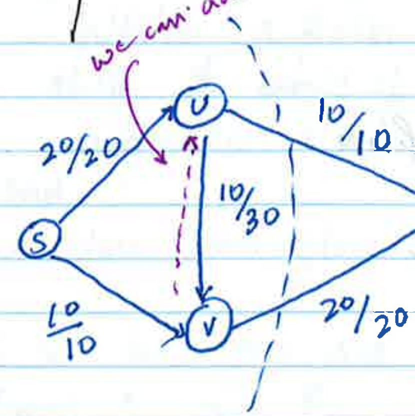
for undirected graphs, will follow by adding and (add both directions, with same capacities)

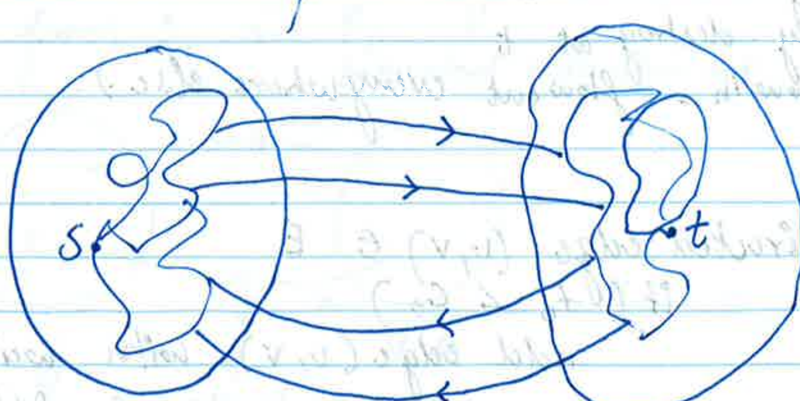


Greedy doesn't work.

min cut problem

we can add edge.





Key: The max flow from s to t is upper bound by any cuts separating s from t
$\downarrow$
cutsize

→ FORD FULKERSON ALGORITHM

finds a flow whose value equals some cut size (and is thus optima)
↳ that cut is also the minimum cut.

Directed cut?

Tight duality?

Linear program.

Define residual Graph: R_f

Let f be a legal flow

(flows are non negative
- → only leave s
- → only created at s
- → only destroy at t
- → flow in = flow out everywhere else.)

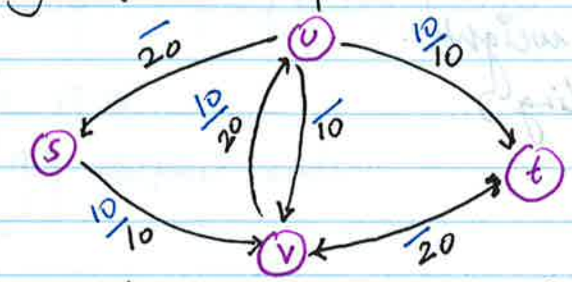
→ For directed edge $(u,v) \in E$
if ($f_e < c_e$)
add edge (u,v) with weight $c_e - f_e$ "forward"

we are shipping

if $f_e > 0$, add (v,u) with weight f_e "backward"

→ This is a new graph.

→ Greedy algorithm for residual algorithm.

→ 

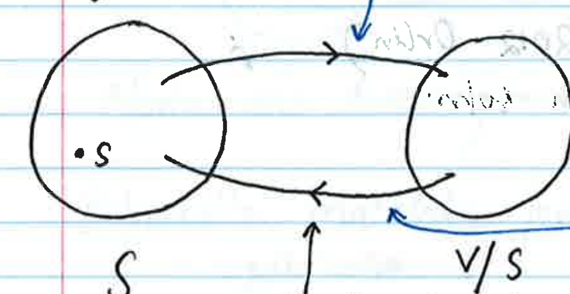
Greedy algorithm on residual graph
cons
find path

→ FORD FULKERSON (1956)
While ∃ path P in R_f from s to t
ship 1 unit of flow from s to t
· update f and R_f $\xrightarrow{\text{corrected}}$ updated R_f only.
end.

↳ How does this give optimality guarantee.

Let $S \subseteq V$ be the set of nodes that are reachable from s in R_f.

saturated with flow in f.



claim 0 flow. (if ≠0, then an edge going the opposite way).

⇒ flow upon termination is equal to cutsize of S.

→ This is the instance when termination already happened.

→ only rational weights.

(Kleinberg Tardos reading).

RUN TIME ANALYSIS :

$O(m)$ time to create R_f

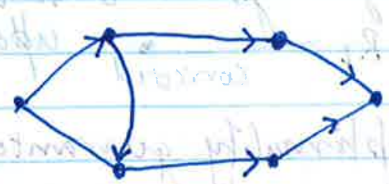
$O(n)$ time to find path in R_f.

Repeat c times where c is max-flow.

→ no. of edges. $m = |E|$ size of edgeset.

$O(mc)$ - - - → (1956) FF

BFS, DFS ?.



In 1972, Edmunds Karp. $O(nm^2)$ (1972 - E Karp shortest path)

$O(nm)$ (2012 - Orlin)

Min-cut max problem soln.

→ Comparison to Global min cut.

Run it $n-1$ time

⇒ global min-cut $O(n^2 m)$

Karger $(O(n^2 \log^3(n)))$.

→ In H.W

$\binom{n}{2}$ distinct cuts.

there are only $(n-1)$ distinct cuts. induction, two smaller. , using max-flow

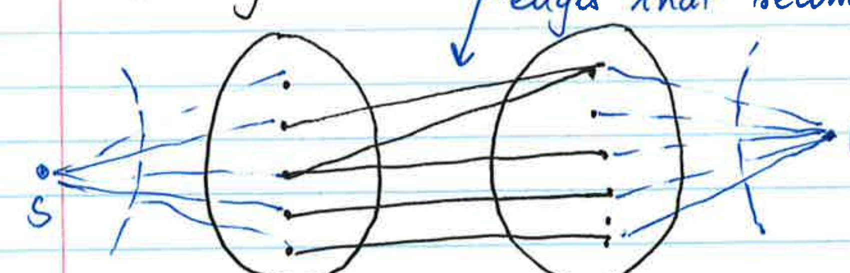
→ Global min-cuts ${}^{n}C_2$.

BIPARTITE GRAPH (undirected): G is one in which nodes are split in two sets A, B.

→ all edges go between A & B

→ No edges on inside A, B.

edges that become saturated are optimal soln.



Nodes are heterogeneous.

Bipartite matching problem. (Max flow to solve).

~~jobs~~ person → jobs.