Lecture-7 1/27/2015

→ OVERVIEW OF CUTS
→ Reductions.
→ NP-hard problems

Global min cut
- $\binom{n}{2}$ different cuts that realize the global (HW2)
- 1 value.

$\binom{n}{2}$ different cycles.

s-t min cut.
- for fixed s,t there could exponentially many s-t min cuts. HW2.
- at most (over all s,t) n-1 different values. (HW1)

MAX CUTS (very hard)

→ $\frac{1}{2}$ approximation (randomized)

→ $\frac{1}{2}$ approx. (deterministic)

→ 0.878 approx (SDP relaxation 1995).

→ generalize to submodular functions.

Linear time, Polynomial time?

k or less? questi.

exponential time

Easy	Not so easy.
MST	Degree constrained-MST
2-SAT	3-SAT
Eulerian circuits	Hamiltonian circuits.
Min cut	Max cut.
	⋮

Live in NP.

How are they all in the same problem class?

DECISION PROBLEM.
→ Is a problem that has a yes/no answer.

→ All problems in NP decision problems.

NP: The set of decision problems whose "Yes" answer can be verified in poly time. (nothing has been said about the "no" answers).

* Problems that have poly time solns. are in P.
$P \subseteq NP$.

* Tautology: Problem that is not NP.

REDUCTIONS:

- Independent set (bunch of nodes with no edges): Is there an induced subgraph with k nodes or more with no edges between nodes?

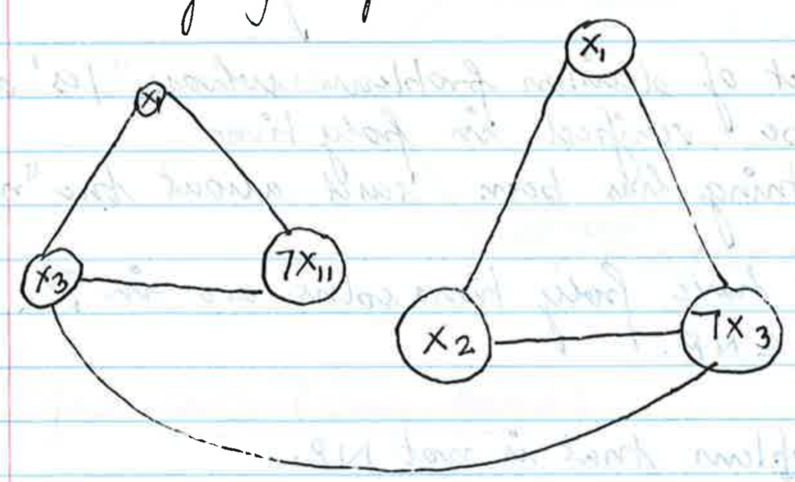
- 3-SAT : Is there a satisfying assignment to a boolean of 3-SAT form.

eg. $(\underbrace{x_1 \vee x_3 \vee \neg x_{11}}_{\text{gadget 1}}) \wedge (\underbrace{x_2 \vee x_1 \vee \neg x_3}_{\text{gadget 2}}) \wedge \ldots$

→ 3-Sat can be reduced to $\le_p$ Independent-set.

→ Given an instance of 3-SAT, and a Blackbox for Independent set, solve the instance of 3-SAT in polynomial time addition work and polynomial # of calls to the Black Box.

- We create a gadget for each clause.



call Independent set on this graph. return "yes" iff $\exists$ independent set of size k.

Show

- $A \le_p B$ $B \le_p C$ $\Rightarrow$ $A \le_p C$.

(COOK 1971)   ✓ property np hardness

If $X \in NP$ then $X \le_p SAT$

(KARP)

Set $S = 21$ problems ($\hookrightarrow \in NP$), if $x \in S$ then $SAT \le_p X$

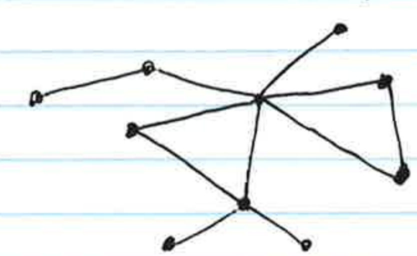
NP Hard Problem X : If $\forall$ $y \in NP$ $y \le_p X$

NP-complete X : If $X \in NP$ and X is NP-hard.

- SAT is NP-complete.

VERTEX COVER PROBLEM

- Subset of nodes such that all edges are incident on some node of the cover.



decision version problem → VC : Is there a vertex cover of size k or smaller?

Claim / Theorem : An independent set S of size k exists iff a vertex cover of size n-k exists.

Proof: If $VC_k \Rightarrow$ ✓ exercise.

you are left with just nodes.

If we have independent set $S \Rightarrow V-S$ is a cover.

exercise.

VC $\leq_p$ independent set.

Independent set $\leq_p$ VC.