CME 305: Discrete Mathematics and Algorithms

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Midterm – 02/16/10

Problem 1. Show that a graph has a unique minimum spanning tree if, for every cut of the graph, the edge with the smallest cost across that cut is unique. Show that the converse is not true by giving a counterexample.

Problem 2. The *edge connectivity* $\lambda(G)$ of an undirected graph $G(V, E)$ is defined as the cardinality of a minimum set of edges $S \subseteq E$, whose removal disconnects G. Give a polynomial-time algorithm for computing $\lambda(G)$.

Problem 3. Recall the job scheduling problem. We have m machines and n jobs such that any machine j takes time t_i to process job i. Let A_j be the set of jobs assigned to machine j. In that case, $T_j = \sum_{i \in A_j} t_i$ will be the *load* of machine j. Our goal is to find an assignment of jobs to machines that would minimize $\max_j T_j$. Denote this minimum value OPT.

Consider a slightly modified greedy approach to the one showed in class. First, we sort the jobs so that $t_1 \geq t_2 \geq \cdots \geq t_n$. Then, we assign them iteratively to the machines, every time to the machine with the smallest load.

Show that the approximation factor of this algorithm is at most 3/2.

Hint: Note that if $n > m$ then $t_{m+1} \leq OPT/2$. Why is this true?

Extra Credit: Prove that the approximation factor of the algorithm is actually equal to 4/3.