Discrete Mathematics and Algorithms Qualifying Exam

September 2012

1. Imagine n cars, each of which travels at a different maximum speed. Initially, the cars are queued in uniform random order at the start of a semi-infinite, one lane highway. Each car drives at the minimum of its maximum speed and the speed at which the car in front of it is driving. The cars will form "clumps". What is the expected number of clumps? Prove your answer.

2. A connected d-regular graph on n nodes is one in which all vertices have degree d. Prove that the diameter of a connected d-regular graph is $O(n/d)$.

3. Given an undirected graph $G = (V, E)$ with nonnegative edge costs satisfying the metric inequality and whose vertices are partitioned into two sets, *Required* and *Steiner*, find a minimum cost tree in G that contains all the Required vertices and any subset of the Steiner vertices. Finding an optimal solution to this problem is NP-hard. Find and prove a factor 2 approximation algorithm for this problem.