MGF Approach to the Capacity Analysis of Generalized Two-Ray Fading Models

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***Abstract*—We propose a class of Generalized Two-Ray (GTR) fading channels that consists of two line of sight (LOS) components with random phase and a diffuse component. Observing that the GTR fading model can be expressed in terms of the underlying Rician distribution, we derive a closed-form expression for the moment generating function (MGF) of the signal-to-noise ratio (SNR) of this model. We then employ this approach to compute the ergodic capacity with receiver side information. The impact of the underlying phase difference between the LOS components on the average SNR of the signal received is also illustrated.**

***Index Terms*—Envelope statistics, fading channels, hyper-Rayleigh fading, moment generating function, multipath propagation, Rician fading, small-scale fading, Two Ray.**

I. INTRODUCTION

We consider a class of fading channels where the fading amplitude is built from two line of sight (LOS) components and multiple non-LOS (NLOS) components. The arriving LOS components can be regarded as individual multipath waves with constant amplitude and random phase, whereas the multiple NLOS components can be grouped into an aggregate diffuse component [1]. We will denote this general class of fading channels as Generalized Two-Ray (GTR) fading models and specify the phase distribution between the LOS components when used in the analysis.

When uniformly distributed phases for the LOS components are assumed, the resultant GTR fading model (GTR-U) reduces to the Two Wave with Diffuse Power (TWDP) model proposed by Durgin, Rappaport and de Wolf as a generalization of the Rayleigh and Rician fading models [2]. This model was shown to closely match field measurements in indoor scenarios [3]. By varying the power of the LOS and NLOS components, the TWDP fading model encompasses the Rayleigh and Rician models along with the LOS case with no diffuse components (i.e., a two-ray model). Another fading behavior that TWDP fading can model is when the fading is more severe than Rayleigh fading [4]. This regime, termed *hyper-Rayleigh* fading, has been observed in wireless sensor networks deployed in cavity structures such as an aircraft or a bus [5], or in vehicle-to-vehicle communication links [6]. Other distributions, such as the κ-μ extreme distribution, have been proposed to model *hyper-Rayleigh* fading behavior [7].

Although this fading model can indeed suit a variety of propagation conditions, its complicated statistical characterization has been its main drawback. The original pdf in [2] is given in integral form, which has hindered the wireless system performance analysis using this model. To circumvent this issue, an *approximate* closed-form pdf was proposed in [2] to facilitate obtaining analytical results for this channel. This approximate TWDP fading pdf has been widely used to characterize the performance of wireless communication systems in TWDP fading, in terms of the bit error rate (BER) in single-antenna and multi-antenna reception using various modulation schemes [8–11], as well as in relay networks [12, 13]. Other performance metrics such as the secrecy capacity associated with physical layer security have also been investigated [14]. In [15], alternative *exact* expressions for the TWDP fading cdf and pdf were given in terms of infinite series of Laguerre and Legendre polynomials.

In [16], it was observed that the pdf of the GTR-U fading model conditioned on the phase difference α between the LOS components resulted in the Rician pdf. This implied that *any performance metric* that is a linear functional of the envelope statistics of the GTR-U fading model can be expressed as a finite integral of the corresponding metric for Rician fading. As a key result, the Moment Generating Function (MGF) of the GTR-U fading model was obtained as well as statistics such as the Amount of Fading and the level crossing rate (LCR).

Inspired by the connection between the Rician and GTR-U fading unveiled above, in this work we show that the statistical properties of the phase difference between the two LOS components α have an impact on the fading experienced by the signal. Allowing this phase difference α to be arbitrarily distributed, we analyze a more general fading propagation condition: the GTR fading model with arbitrary phase. We will show that this additional degree of freedom models a much larger range of fading behavior, and hence can be useful to characterize *hyper-Rayleigh* fading in more severe scenarios than the ones considered in [4–6].

Interestingly, we also obtain a closed-form expression for the MGF of the GTR fading model when the phase difference is distributed according to the von Mises (or circular normal) distribution [17, 18], which includes the uniform distribution as a particular case. The analysis in this new general scenario is of similar complexity to the conventional GTR-U fading case. With the MGF in closed-form, we can easily evaluate the ergodic capacity [19, 20] in GTR-U fading and study its behavior in asymptotic regimes.

The rest of the paper is organized as follows: in Section II, the family of GTR fading models is introduced as a natural generalization of the TWDP fading model. In Section III, the connection between Rician and GTR fading is unveiled which allows us to derive closed-form expressions for the MGF of the GTR fading models. We use the MGF-based approach to performance analysis in Section IV by analyzing the ergodic capacity with receiver side information in the GTR fading channel. The implications for system design enabled by our analysis are presented in Section V. The main conclusions are outlined in Section VI.

II. Generalized Two-Ray Fading Models

A. A Brief Description of the GTR Fading Models

As presented in [21], the complex baseband received signal $s(t)$ in narrowband multipath fading is:

$$s(t) = \Re\left\{u(t)\sum_{n}\alpha_n e^{j\phi_n}\right\}, \tag{1}$$

where $u(t)$ is the transmitted signal in baseband, α_n and ϕ_n represent the amplitude and phase of the n-th multipath component and $\Re\{.\}$ denotes the real part.

The GTR fading model described in [2, eq. 7] consists of two specular components and a diffuse component, as

$$V_r = V_1\exp(j\phi_1) + V_2\exp(j\phi_2) + X + jY, \tag{2}$$

where V_r is the received signal, components 1 and 2 are specular components. V_1 and V_2 are constant and in the diffuse component $X, Y \sim \mathcal{N}(0,\sigma^2)$. ϕ_1 and ϕ_2 are the phases of the LOS components with the phase difference $\alpha = \phi_1 - \phi_2$ being an RV arising from distribution $f_\alpha(.)$.

The model is conveniently expressed in terms of the parameters K and Δ, defined as

$$K = \frac{V_1^2 + V_2^2}{2\sigma^2}, \tag{3}$$

$$\Delta = \frac{2V_1V_2}{V_1^2 + V_2^2}. \tag{4}$$

Similar to the Rician parameter, here K represents the ratio of the power of the specular components to the diffuse power; Δ is related with the ratio of the peak specular power to the average specular power and serves as the comparison of the power levels of the two specular components. We observe that $\Delta = 1$ only when the two specular components are of equal amplitude, and $\Delta = 0$ when either LOS component has zero power.

In GTR fading with uniform phase (GTR-U), $\phi_1, \phi_2 \overset{\text{i.i.d}}{\sim} \mathcal{U}(0, 2\pi)$ and the model reduces to the TWDP fading model. Phase difference $\alpha \sim \mathcal{U}(0, 2\pi)$ [16]. Special cases of the GTR-U fading model are detailed in [2], encompassing the *One Wave*, *Two Wave*, Rayleigh and Rician fading models. In [4] it is shown that when $K > 0$ and $\Delta \approx 1$ the channel exhibits worse fading than Rayleigh, referred to as hyper-Rayleigh behavior. As K increases, the fading becomes more severe and with the extreme condition of $K \to \infty$, the most severe two-wave fading model emerges

B. GTR Fading Model with Arbitrary Phase

In the previous analysis, uniformly and independently distributed phase difference α between the two LOS components was considered. While this consideration for the LOS components has been verified through field measurements [2], in [4] it was observed that the uniform phase assumption for α does not hold in some practical scenarios.

The hyper-Rayleigh behavior exhibited by the GTR-U fading model when the two LOS components have equal power (i.e., $\Delta = 1$) has an intuitive explanation. When α is uniformly distributed, there is a finite probability that α takes values close to π, i.e. the LOS components are out of phase and are cancelled. This is especially important in the simple Two-Ray (or Two Wave) model, in which the diffuse part is absent; therefore, even in the presence of two very strong LOS components the actual fading behavior is more severe compared to other NLOS models like Rayleigh. In fact, limiting the range of valid phases for α caused a worse fading condition than the Two-Ray model [4]. Based on this observation, we note that in the limit case where the phase difference is deterministic $\alpha \equiv \pi$ we would have total cancellation. Hence, using a distribution for α that concentrates the probability close to π would reduce the SNR of the received signal for the same transmit power as compared to the Two-Ray model. Our motivation for considering arbitrary distribution for α is to approximate received signal fading behavior (for constant transmit power) that ranges from complete cancellation to the best single ray case.

Although a different distribution for the GTR fading model arises for any particular choice of $f_\alpha(\alpha)$, we will focus on some specific distributions that can help us model harsh propagation conditions.

Following this reasoning, we first study GTR fading with truncated phase (GTR-T), where $\alpha \sim \mathcal{U}(\pi(1-p), \pi(1+p))$, and $p \in (0, 1]$. When $p = 1$, the GTR-T fading model reduces to the conventional GTR-U fading model, whereas as $p \to 0$ we observe that the probability of the two LOS components to cancel each other is increased, causing a fading worse than hyper-Rayleigh fading. Hence, the GTR-T fading distribution can model fading from extremely favorable propagation conditions when $\Delta = 0$ and $K \to \infty$ to very severe fading, close to complete cancellation, as $p \to 0$ with $\Delta = 1$. This is illustrated in Fig. 1, where the cdf of the GTR-T fading model is represented and compared with Rician, Rayleigh and Two-Ray fading models. The transmitted signal envelope amplitude is normalized to $\sqrt{\overline{\gamma}}$.

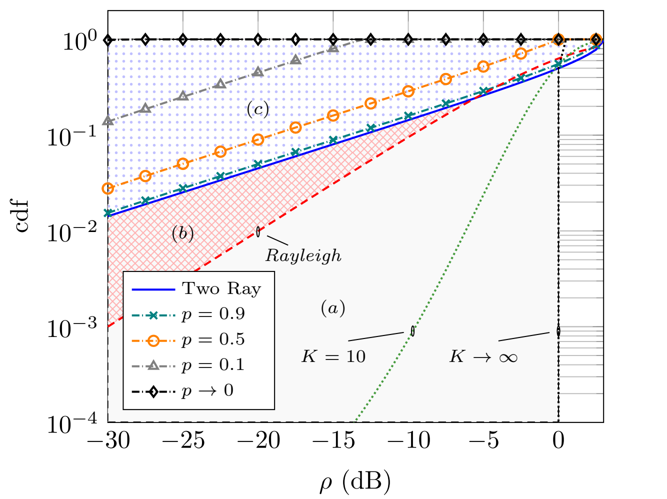

Fig. 1. The cdf of the GTR-T fading model vs. the normalized envelope amplitude, for different values of the phase truncation parameter p. Parameter values for GTR-T fading are $\Delta = 1$ and $K \to \infty$ as in the Two Ray model (obtained for $p = 1$). Shaded regions correspond to (a) Rician fading, (b) Hyper-Rayleigh fading, (c) Hyper Two-Ray fading.

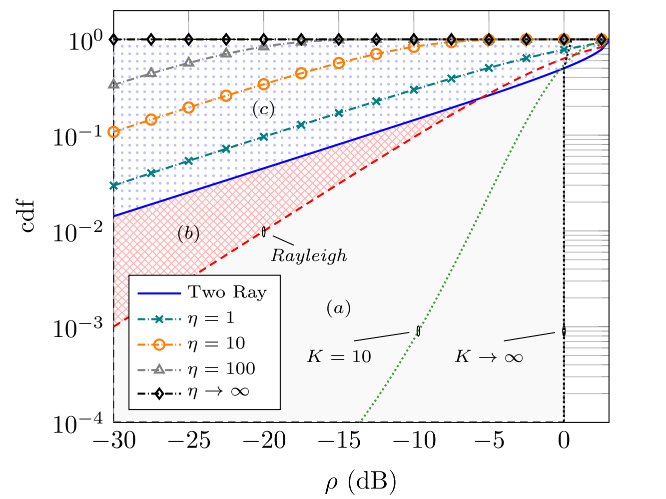

Fig. 2. The cdf of the GTR-V fading model vs. the normalized envelope amplitude, for different values of the phase scale parameter η. Parameter values for GTR-V fading are $\Delta = 1$ and $K \to \infty$ as in the Two-Ray model (obtained for $\eta = 0$). Shaded regions correspond to (a) Rician fading, (b) Hyper-Rayleigh fading, (c) Hyper Two-Ray fading.

The GTR-T fading model provides for a simple way to characterize a variety of fading behavior. However, it may be argued that a truncated model for the phase α might not occur in practice. For this reason, we now present another alternative for the family of GTR fading models.

Assume that α is distributed according to the von Mises (VM) distribution [17] with pdf given by

$$f_\alpha^{\text{VM}}(\alpha) = \frac{\exp\left(\eta \cos(\alpha - \varphi)\right)}{2\pi I_0(\eta)}, \qquad \alpha \in [0, 2\pi], \tag{5}$$

where $\eta \geq 0$ and $\varphi \in \mathbb{R}$ are usually referred to as concentration and centrality parameters respectively. This distribution, also known in the literature as the circular normal distribution or Tikhonov distribution, is widely used in different applications in communications (see [22] and references therein). In particular, it is used to describe the statistics of angles of arrival in wireless systems, or phase error in phased-locked loops (PLLs) just to name a few examples. This model also includes the uniform phase as a particular case when $\eta = 0$.

Since we are interested in modeling hyper-Rayleigh behavior, the centrality parameter is set to $\varphi = \pi$. Thus, for $\eta \neq 0$ the probability of α taking values close to π increases as η increases. The behavior of the GTR fading model with VM distributed phase (GTR-V) is shown in Fig. 2. We observe that as η grows and $\Delta = 1$, the fading falls in the region beyond the Two Ray model; hence, it is also suitable for characterizing very severe propagation conditions.

III. Connection Between Rician and GTR Fading

The approach described in [16] is employed here to establish a connection between the GTR and Rician fading. Conditioning the received signal amplitude on the phase difference between the LOS components we get

$$V_r = \exp(j\phi_1)\left(V_1 + V_2 \exp[j(\alpha)]\right) + V_{\text{diff}}. \tag{6}$$

This problem is equivalent to finding the Rician pdf as there is a single LOS component of phase ϕ_1 and constant amplitude $\bar{V}_1 = \sqrt{V_1^2 + V_2^2 + 2V_1V_2\cos\alpha}$ and $\bar{K} = K\left(1 + \Delta\cos\alpha\right)$. Due to the circular symmetry of V_{diff} the envelope statistics of (6) are independent of the distribution of ϕ_1 and only depend on the phase difference α [23]. Thus, the GTR fading model conditioned on the phase difference α results in the Rician envelope distribution, i.e.

$$f_{\text{GTR}}(r|\alpha) = f_{\text{Rice}}\big(r;\ K[1 + \Delta\cos(\alpha)]\big). \tag{7}$$

The phase difference α could also arise from any arbitrary distribution with pdf $f_\alpha(.)$. The pdf of this GTR fading model with arbitrary phase is given as

$$f_{\text{GTR}}(r) = \int_0^{2\pi} f_{\text{Rice}}\left(r;\, K[1 + \Delta\cos(\alpha)]\right) f_\alpha(\alpha) d\alpha \tag{8}$$

The envelope pdf in GTR-T fading is thus given by

$$f_{\text{GTR-T}}(r) = \frac{1}{2\pi p}\int_{\pi(1-p)}^{\pi(1+p)} f_{\text{Rice}}\left(r;\, K[1 + \Delta\cos(\alpha)]\right) d\alpha. \tag{9}$$

Also, the pdf of the GTR-V fading model is

$$f_{\text{GTR-V}}(r) = \frac{1}{2\pi I_0(\eta)} \times \tag{10}$$
$$\int_0^{2\pi} f_{\text{Rice}}\left(r;\, K[1 + \Delta\cos(\alpha)]\right) \exp\left(-\eta\cos\alpha\right) d\alpha.$$

The following lemma will be of use in employing the connection unveiled above to compute metrics of the GTR fading model:

Lemma 1: Let $H_R(\theta)$ be a general metric of a fading model with parameter θ, expressed as a linear function of its envelope *pdf in the form*

$$H_R(\theta) = \int_a^b f_R(r) g(r) dr, \tag{11}$$

where $0 \leq a \leq\ b \leq \infty$ and $g(\cdot)$ is an arbitrary function defined on $\mathbb{R}$. Then any general metric $H_{GTR}(K, \Delta)$ of the GTR fading model with parameters K, Δ and phase difference α between the LOS components arising from distribution $f_\alpha(.)$ can be expressed in terms of the same metric of the Rician fading model $H_{Rice}(K)$ as

$$H_{\text{GTR}}(K, \Delta) = \frac{1}{2\pi} \int_0^{2\pi} H_{\text{Rice}}\big(K[1 + \Delta \cos(\alpha)]\big) f_\alpha(\alpha) d\alpha. \tag{12}$$

Proof: This is easily verified by changing the order of integration in (11). ∎

This simple approach to derive performance metrics and statistics for the GTR fading model is new in the literature to the best of our knowledge. We note that a similar connection has been recently established between Rayleigh and Hoyt (Nakagami-q) NLOS fading models in [24]; however, in the present work the parameter α has a clear and intuitive interpretation as it is related to the phase difference between the two LOS components.

A. MGF of the GTR Fading Model

The MGF of the SNR of a fading model can be expressed as a linear functional of the envelope pdf [25]. Hence Lemma 1 can be applied to obtain the MGF of the GTR fading models in terms of the MGF of the Rician distribution. The moment generating function (MGF) of the SNR for the Rician fading model is given by

$$\mathcal{M}_{\text{Rice}}(s) = \frac{1 + K}{1 + K - s\bar{\gamma}} \exp\left(\frac{K s\bar{\gamma}}{1 + K - s\bar{\gamma}}\right). \tag{13}$$

The MGF of the GTR-U fading model is thus calculated as

$$\begin{aligned} \mathcal{M}_{\text{GTR-U}}(s) &= \frac{1}{2\pi} \int_0^{2\pi} \frac{1 + K}{1 + K - s\bar{\gamma}} \exp\left(\frac{\bar{K}(\alpha) s\bar{\gamma}}{1 + K - s\bar{\gamma}}\right) d\alpha \\ &= \frac{1 + K}{1 + K - s\bar{\gamma}} \exp\left(\frac{K s\bar{\gamma}}{1 + K - s\bar{\gamma}}\right) I_0\left(\frac{K s\bar{\gamma}\Delta}{1 + K - s\bar{\gamma}}\right). \end{aligned} \tag{14}$$

Interestingly, the MGF of the GTR-V fading model can also be obtained in closed-form. Using Lemma 1 including the pdf of the VM distribution in (5) and (13), we directly obtain:

$$\mathcal{M}_{\text{GTR-V}}(s) = \frac{1 + K}{1 + K - s\bar{\gamma}} \exp\left(\frac{K s\bar{\gamma}}{1 + K - s\bar{\gamma}}\right) \frac{I_0\left(\eta - \frac{K s\bar{\gamma}\Delta}{1+K-s\bar{\gamma}}\right)}{I_0(\eta)}. \tag{15}$$

Hence, we have found a closed-form expression for the MGF of the GTR fading model. Even though the GTR fading pdf cannot be expressed in closed-form, we have shown that the MGF is characterized by a very simple expression. This has two direct implications: first, the moments for the GTR fading model can also be expressed in closed-form, using Leibniz's rule for the derivative of products. Secondly, the MGF is extensively used to characterize performance of digital communication systems [25]. Therefore, expression (14) is useful to analyze some of the scenarios considered in the literature [8–11] without the need for using the approximate pdf in [2].

B. Impact of Phase Distribution

The distributions in (9) and (10) can model the effect of a larger cancellation of the LOS components due to the statistical behavior of the phase difference α. However, we also note that by simply applying a deterministic shift of value π to this phase difference, the resulting distributions would be centered at zero. This implies that the two LOS components would be cancelled with less probability, and hence the fading experienced by the signal would be closer to a Rician behavior than to a hyper-Rayleigh behavior.

This can be seen by deriving the expression for the average SNR of these models using Lemma 1 and using the expression for the first moment of the Rician distribution,

$$\mathbb{E}_{\text{GTR-T}}(\gamma) = \bar{\gamma}\left(1 \mp \Delta \frac{K}{K+1} \text{sinc}(p)\right), \tag{16}$$

$$\mathbb{E}_{\text{GTR-V}}(\gamma) = \bar{\gamma}\left(1 \mp \Delta \frac{K}{K+1} \frac{I_1(\eta)}{I_0(\eta)}\right), \tag{17}$$

where the function $\text{sinc}(p) = \sin(\pi p)/\pi p$ and $\bar{\gamma}$ is the average SNR of the conventional GTR fading model. In these expressions, the negative sign accounts for the cases where the distribution of α is centered at π, whereas the positive sign corresponds to the case where the distributions are centered at zero.

It is interesting to observe how the average received SNR is in three circumstances: (1) when the two LOS components tend to have similar magnitudes (i.e. increasing Δ), (2) when the LOS power is larger (i.e. increasing K) and (3) when the phase α is more concentrated towards π (i.e. reducing p or increasing η). It is easy to see how in the limiting cases of the three parameters (i.e. $\Delta \to 1$, $K \to \infty$ and $p \to 0$ or $\eta \to \infty$), the average SNR tends to zero. However, by simply concentrating the phase α towards zero, we cause the average SNR to be increased by the same magnitude. In the limiting case previously discussed, we would be increasing the average SNR by a factor of 2.

IV. Ergodic Capacity

The effect of fading on the maximum rate of data transmission over a wireless link has been a matter of interest in communication and information theory for many years, considering different adaptation policies at the transmitter and receiver, as well as for different configurations in terms of the number of antennas. Specifically, the work by Alouini and Goldsmith [26] provided the first analytical results for the capacity of adaptive transmission with diversity-combining techniques in Rayleigh fading. However, extensions of these results to other types of fading are often more challenging and do not lend themselves to analytically tractable solutions.

TABLE I

ASYMPTOTIC RESULTS FOR THE ERGODIC CAPACITY IN GTR-U FADING (PERFECT CSI AT THE RECEIVER) IN THE HIGH-SNR REGIME.

Rice	$C_{\text{ora}}\|_{\bar{\gamma}\Uparrow} \approx \nu \cdot \bar{\gamma}(dB) + \log_2 e \left\{ \log\left(\frac{K}{K+1}\right) + \Gamma(0,K) \right\}$
GTR-U	$C_{\text{ora}}\|_{\bar{\gamma}\Uparrow} \approx \nu \cdot \bar{\gamma}(dB) + \log_2 e \left\{ \log\left(\frac{K}{K+1}\right) + \log\left(\frac{1+\sqrt{1-\Delta^2}}{2}\right) + \mathcal{J}(K,\Delta) \right\}, \quad \mathcal{J}(K,\Delta) = \int_1^\infty \frac{e^{-tK}}{t} I_0(tK\Delta)dt.$
GTR-U$_{(K\cdot\Delta>>1)}$	$C_{\text{ora}}\|_{\bar{\gamma}\Uparrow} \approx \nu \cdot \bar{\gamma}(dB) + \log_2 e \left\{ \log\left(\frac{K}{K+1}\right) + \log\left(\frac{1+\sqrt{1-\Delta^2}}{2}\right) + \sqrt{\frac{2}{\pi}} \left[\frac{e^{-K(1-\Delta)}}{\sqrt{K\Delta}} - \sqrt{\left(\frac{1}{\Delta}-1\right)} \text{erfc}\left(K(1-\Delta)\right) \right] \right\}$
GTR-U$_{(K>>1,\Delta=1)}$	$C_{\text{ora}}\|_{\bar{\gamma}\Uparrow} \approx \nu \cdot \bar{\gamma}(dB) + \log_2 e \left\{ \log\left(\frac{K}{K+1}\right) - \log 2 + \sqrt{\frac{2}{\pi K}} \right\}$
C loss GTR-U	$\delta_C(K,\Delta) = \log_2 e \left\{ \Gamma(0,K) - \log\left(\frac{1+\sqrt{1-\Delta^2}}{2}\right) - \mathcal{J}(K,\Delta) \right\}$
C loss Two-Ray	$\delta_C(K \to \infty, 1) = 1$

Inspired by the general framework for the average error probability analysis based on the MGF [25], an alternative formulation for the analysis of the ergodic capacity in fading channels in terms of the MGF of the received SNR was recently proposed in [19], and was then further complemented in [20]. If the MGF of interest has an analytical closed-form solution, the capacity can be evaluated using a single integral over the MGF.

As an application of this method for evaluating the Shannon capacity in fading channels, we will consider an optimal rate adaptation (ORA) policy with constant transmit power. This is the capacity of the fading channel when the channel state information is only available at the receiver side. According to [19, eq. 7], the capacity per unit bandwidth is given in terms of the MGF of the SNR at the receiver side as

$$C_{\text{ora}} = \log_2 e \int_0^\infty E_i(-s)\mathcal{M}_\gamma^{(1)}(-s)ds, \tag{18}$$

where $E_i(\cdot)$ denotes the Exponential integral function [27, eq. 2.325.1] and $\mathcal{M}_\gamma^{(1)}(s)$ indicates the first derivative of the MGF with respect to s. Assuming a multi-antenna receiver with L independent branches using MRC detection, we have

$$\mathcal{M}_\gamma^{(1)}(s) = \sum_{l=1}^{L} \mathcal{M}_{\gamma_l}^{(1)}(s) \times \prod_{\substack{k=1\\k\neq l}}^{L} \mathcal{M}_{\gamma_k}(s). \tag{19}$$

We will, for the remainder of this section focus on the GTR-U fading model. Since we have a closed-form expression for the MGF of the received SNR per branch for this model, we can also compute its first derivative in closed-form as

$$\mathcal{M}_{\gamma_l}^{(1)}(s) = \frac{(1+K_l)\bar{\gamma}_l}{(1+K_l-s\bar{\gamma}_l)^2} \exp\left(\frac{K_l s\bar{\gamma}_l}{1+K_l-s\bar{\gamma}_l}\right) \times \tag{20}$$
$$\left[I_0\left(\tfrac{K_l s\bar{\gamma}_l\Delta_l}{1+K_l-s\bar{\gamma}_l}\right)\left(1+\tfrac{K_l(1+K_l)}{1+K_l-s\bar{\gamma}_l}\right) + \tfrac{K_l\Delta(1+K_l)}{1+K_l-s\bar{\gamma}_l} I_1\left(\tfrac{K_l s\bar{\gamma}_l\Delta_l}{1+K_l-s\bar{\gamma}_l}\right)\right]$$

where $I_1(\cdot)$ is the modified Bessel function of the first kind and order one. Hence, the expression for the ergodic capacity in GTR fading channels using ORA policy and MRC detection can be computed by plugging (20) and (19) into (18).

Using [19, eq. 12], we find a simple asymptotic approximation for the capacity in the low-SNR regime as

$$C_{\text{ora}}|_{\bar{\gamma}\Downarrow} \approx \log_2 e\mathcal{M}_\gamma^{(1)}(s)|_{s=0} = \bar{\gamma}\log_2 e, \tag{21}$$

where we assumed that the received SNRs per branch are i.i.d. and $\bar{\gamma} = L\bar{\gamma}_l$. Interestingly, we observe that (21) is independent of Δ for GTR fading.

An asymptotic expression for capacity in the high-SNR[1] can also be obtained from the first derivative of the n^{th} moment [28, eq. 8] or [29, eq. 22] as

$$C_{\text{ora}}|_{\bar{\gamma}\Uparrow} \approx \log_2 e \cdot \frac{\partial}{\partial n}\mathbb{E}\left[\gamma^n\right]|_{n=0}. \tag{22}$$

In Table I, we summarize the asymptotic results (high-SNR) for the capacity in GTR fading and a single-branch receiver. The derivations of these results are omitted due to space constraints. The asymptotic capacity is given in the form $C_{\text{ora}}|_{\bar{\gamma}\Uparrow} = \nu \cdot \bar{\gamma}(dB) + \mu$, where $\nu = 0.1\log(10)\log_2(e)$, μ is a constant value independent of the average SNR, and the average SNR $\bar{\gamma}(dB) = 10\log_{10}\bar{\gamma}$ is given in dB.

The capacity loss or the difference between the asymptotic capacity of Rice and GTR, given by $\delta_C = C_{\text{ora}}^{\text{Rice}} - C_{\text{ora}}^{\text{GTR-U}}$, is

$$\delta_C(K,\Delta) = \log_2 e \left\{ \Gamma(0,K) - \log\left(\tfrac{1+\sqrt{1-\Delta^2}}{2}\right) - \mathcal{J}(K,\Delta) \right\}. \tag{23}$$

It is easy to verify that $\delta_C > 0$. In the hyper-Rayleigh zone of the GTR fading model, we have that the capacity loss is

$$\delta_C(K \to \infty, \Delta = 1) = 1 \tag{24}$$

with respect to the AWGN case (i.e. Rician with $K \to \infty$). This implies that the capacity loss in the most severe fading condition modeled by GTR fading is only 1 bps/Hz with respect to the AWGN case (i.e. no fading).

V. Implications for System Design

The preceding analysis allows us to gain new insights on the behavior of the GTR fading model. In this section, we evaluate the derived expressions for the Shannon capacity in GTR fading channels with perfect CSI at the receiver in some scenarios of interest using (18) and (19). We will consider that α is uniformly distributed in $[0, 2\pi]$.

First, we consider an L-branch receiver with MRC reception, and we assume a LOS power ratio $K = 10$. In Fig. 3,

[1]Note that at high-SNR, the capacity with ORA policy is the same as the capacity with optimal power and rate allocation (OPRA) policy, which considers that CSI is available at both the transmitter and receiver sides [26].

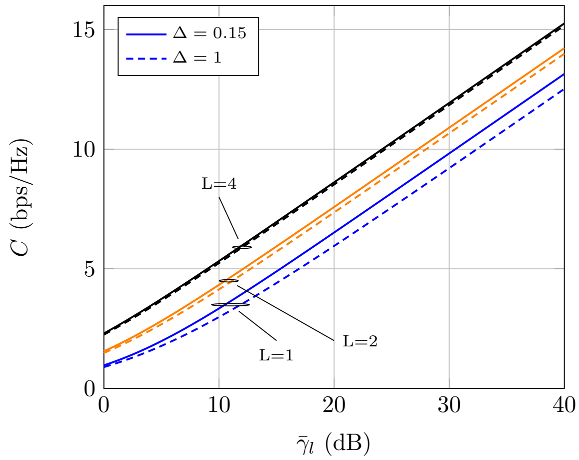

Fig. 3. Capacity vs average SNR per branch $\bar{\gamma}_l$, for different fading conditions and numbers of receive antennas L. Parameter value $K = 10$. Solid lines correspond to $\Delta = 0.15$, dashed lines correspond to $\Delta = 1$.

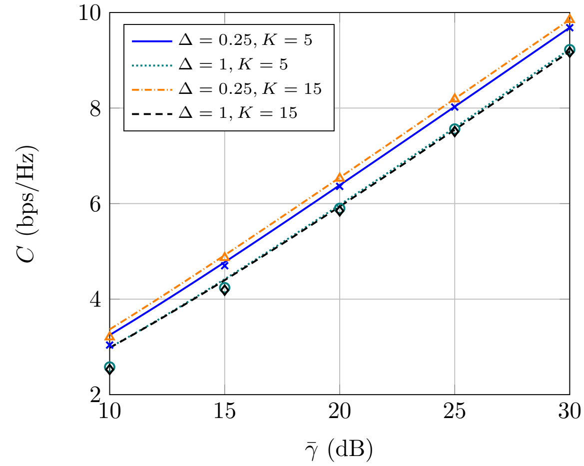

Fig. 5. Capacity vs average SNR $\bar{\gamma}$ in the high-SNR regime, for different fading conditions. Markers indicate the asymptotic result given by (22).

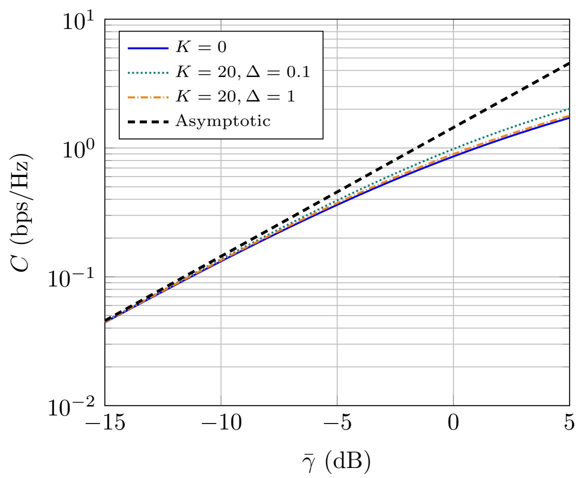

Fig. 4. Capacity vs average SNR $\bar{\gamma}$ in the low-SNR regime, for different fading conditions.

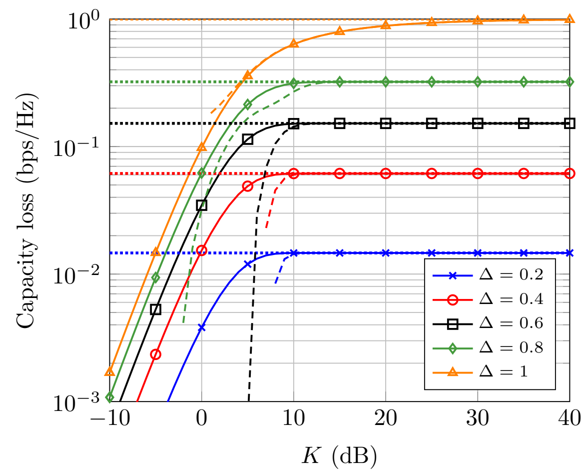

Fig. 6. Capacity loss in GTR-U fading with respect to Rician fading, as a function of the LOS power factor K (in dB). Solid lines correspond to the exact evaluation of $\mathcal{J}(K, \Delta)$, dashed lines correspond to the approximate evaluation of $\mathcal{J}(K, \Delta)$ for large $K \cdot \Delta$, and dotted lines correspond to the asymptotic case of $K \to \infty$.

we represent the ergodic capacity as a function of the average SNR per branch $\bar{\gamma}_l$ for different values of the parameter Δ. For the sake of simplicity, we assume i.i.d. fading on the receive branches.

We notice that the capacity is reduced as Δ grows, leading to a gap for high SNR of around 2 dB when single antenna reception is used. However, as the number of receive antennas is increased, we see that the capacity is barely affected by the value of Δ. Hence, in very severe fading conditions the use of diversity reception techniques allows for an increase in the capacity.

We now study the behavior of capacity in the low-SNR and high-SNR regimes. First, in Fig. 4 we investigate the capacity in the low-SNR regime using the asymptotic approximation given in (21), as a function of the average SNR $\bar{\gamma}$ with $L = 1$. In the low-SNR regime, we observe that the capacity is asymptotically independent of K and Δ, as suggested by equation (21).

In Fig. 5, the high-SNR regime is considered. The asymptotic capacity results are given by (22) and the expressions are summarized in Table I. We see that the asymptotic capacity (represented with markers) is very tight for values of $\bar{\gamma} > 15$ dB and is even more accurate for low values of Δ.

Fig. 6 represents the asymptotic capacity loss of GTR fading channels with respect to the case of Rician fading (i.e., $\Delta = 0$). This metric $\delta_C(K\Delta)$ is independent of $\bar{\gamma}$, and indicates how the capacity is reduced due to the non-zero probability of the two LOS components partially cancelling, dependent on the parameter Δ. We represent this capacity loss as a function of the LOS power ratio parameter K, for different values of Δ.

As K is increased, the capacity loss grows to a maximum degradation value given by

$$\delta_C(K \to \infty, \Delta) = 1 - \log_2\left(1 + \sqrt{1 - \Delta^2}\right) \quad (25)$$

that corresponds to the capacity reduction with respect to the AWGN case. We see how the approximate expression for $\mathcal{J}(K,\Delta)$ is very accurate for reasonably large values of $K\cdot\Delta$. In the limiting case of the hyper-Rayleigh fading condition (i.e. $K\rightarrow\infty$ and $\Delta=1$), we see that the capacity loss is only 1 bps/Hz.

VI. Conclusion

We have provided an analytical approach to the characterization of Generalized Two Ray fading channels, and systems operating over them. The class of GTR fading models was proposed as a natural generalization of TWDP fading. This model considers an arbitrary phase difference between the LOS components, can characterize a wider set of propagation conditions than previous models. By observing that the GTR fading conditioned on the difference in phase between the two LOS components results in Rician fading, any linear metric of the GTR fading can be expressed in terms of a simple finite integral of the corresponding metric of the Rice fading model. This simple yet powerful approach has allowed us to derive a closed-form expression for the MGF of the GTR fading model.

We then used this general MGF-based technique to investigate the capacity limits of communication systems affected by GTR-U fading and observed that the asymptotic capacity penalty per unit bandwidth in the extreme case of hyper-Rayleigh fading with respect to the AWGN case is only 1 bps/Hz in the high-SNR regime, when perfect CSI is available at the receiver. The empirical validation of this new model with field measurements, as well as the performance limits of communication systems in the new extremely-severe fading condition denoted as hyper-Two Ray fading, will be a matter of future work.

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