The consumption and welfare implications of wage arrears in transition economies [☆]

Dmytro Boyarchuk [a], Lilia Maliar [b,*], Serguei Maliar [b]

[a] EERC at the National University "Kyiv-Mohyla Academy," 04070 Kyiv, Ukraine
[b] Departamento de Fundamentos del Análisis Económico, Universidad de Alicante,
Campus San Vicente del Raspeig, Ap. Correos 99, 03080 Alicante, Spain

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The paper investigates the macroeconomic consequences and the welfare implications of wage arrears in transition economies using a one-sector neoclassical growth model. In our model, a neoclassical firm makes losses during the transition period and uses wage arrears as a survival strategy. At the workers' level, the randomness in the timing and extent of wage payments acts as an idiosyncratic shock to earnings. We calibrate the model to reproduce the characteristics in Ukrainian data and study its quantitative predictions. We find that wage arrears imply substantial social costs, e.g., a consumption loss between 8 and 16%, and a welfare loss from idiosyncratic uncertainty equivalent to an additional consumption loss of 1 to 5%. *Journal of Comparative Economics* **33** (3) (2005) 540–564. EERC at the National University "Kyiv-Mohyla Academy," 04070 Kyiv, Ukraine; Departamento de Fundamentos del Análisis Económico, Universidad de Alicante, Campus San Vicente del Raspeig, Ap. Correos 99, 03080 Alicante, Spain.
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[*] Corresponding author.
E-mail address: maliarl@merlin.fae.ua.es (L. Maliar).

1. Introduction

Wage arrears are wage payments that were not settled at their due date. Typically, wage arrears are not observed in developed market economies because wages are paid to workers punctually. However, the problem of wage arrears is severe in the transition economies of the former Soviet Union.[1] In Ukraine, the average level of wage arrears was around 22% of quarterly output from 1996 to 2001. In 1996, wage arrears constituted around 40% of an average worker's salary, i.e., the average wage debt was equal to five months pay, and they affected more than 60% of the labor force.[2] Only recent economic growth has reduced wage arrears in Ukraine.

Considerable empirical literature investigates the determinants of wage arrears. Alfandari and Schaffer (1996) and Clarke (1998) attribute wage arrears to liquidity problems, e.g., the lack of external finance available to enterprises and non-payment by customers for goods delivered. Earle and Sabirianova (2002) discuss the opportunistic behavior of managers who delay wage payment to pursue their personal interests, e.g., forcing workers to sell their shares in the enterprise. Layard and Richter (1995) cite the willingness of workers to accept wage-cuts to preserve their jobs. Alfandari and Schaffer (1996) focus on managerial strategies to extract tax concessions from the government. Finally, Desai and Idson (2000) discuss survival strategies of loss-making enterprises.

The empirical literature finds that wage arrears play an important role in individual consumption-savings decisions. Desai and Idson (1998, 2000) emphasize that wage arrears not only affect household disposable income in a given period, but also reduce household wealth because they are not indexed to inflation. Lehmann and Wadsworth (2001) find that wage arrears can increase conventional measures of earnings inequality by 20 to 30%. Desai and Idson (2000) provide empirical evidence that Russian families protect themselves from wage delays by borrowing from relatives, selling family assets, reducing savings rates and holding multiple jobs. Skoufias (2004) is an extensive discussion of the literature on the risk management strategies of households in transition economies. Guariglia and Kim (2003) find strong evidence of precautionary savings by households facing wage arrears. However, no theoretical framework for analyzing the impact of wage arrears on consumer behavior has been proposed in the literature.[3]

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[1] Other types of arrears, e.g., arrears between enterprises, arrears of enterprises to banks, tax arrears of enterprises and pension arrears are also prevalent in these transition economies.

[2] Alfandari and Schaffer (1996), Clarke (1998), Desai and Idson (2000) and Earle and Sabirianova (2002) report similar tendencies for wage arrears in Russia.

[3] Earle and Sabirianova (2000) present a formal model in which high and persistent wage arrears arise as an outcome of equilibrium managerial decisions. However, their paper does not consider the consumer side of the economy explicitly.

In this paper, we develop a general equilibrium model that allows us to assess the macroeconomic consequences and the welfare implications of the wage arrears phenomenon, including its consequences for cross-sectional household distributions of income, wealth and consumption. In our model, wage arrears arise because firms that are hit by negative shocks associated with transition cover the losses they incur by the underpayment of wages.[4] We assume that wage arrears depreciate over time because they are not indexed to inflation. At the individual level, randomness in the timing and extent of wage payments act as an idiosyncratic shock to earnings. Markets are incomplete in that agents cannot borrow more than a certain amount and cannot insure themselves against idiosyncratic uncertainty. Thus, the consumer side of our economy is similar to the standard one in a one sector neoclassical growth model, except that fluctuations in individual wages come from wage arrears shocks in our case, while they come from productivity shocks in the standard case, as Aiyagari (1994) and Huggett (1993, 1997) discuss.

The effect of wage arrears on an individual's consumption-savings behavior is characterized by two types of costs. First, delays in wage payments lead to a reduction in expected income from both capital and labor, so that agents lower their consumption. Capital income decreases due to the precautionary savings effect and labor income decreases because of depreciation of wage arrears. Second, the non-regularity of wage payments, together with the borrowing restrictions, leads to variations in the amount of resources available to agents in each period, which induces consumption fluctuations.

We calibrate our model to aggregate and household data from the Ukraine and compute the effects associated with wage arrears. Our model with wage arrears shocks generates approximately the same degrees of wealth, income and consumption inequality as are produced by the standard neoclassical growth model with productivity shocks, e.g., Aiyagari (1994). Regarding the social costs of wage arrears, the reduction in the agent's expected consumption due to wage arrears ranges from 8 to 16% in our experiments. The welfare loss resulting from consumption fluctuations is equivalent to an additional consumption loss, which ranges from 1 to 5% depending on the degree of risk aversion assumed for agents.

The paper is organized as follows. Section 2 presents the empirical evidence on wage arrears in the Ukrainian economy and discusses the relationship between wage arrears and inter-enterprise arrears. Section 3 formulates the model. Section 4 discusses the methodology of the quantitative study and reports the results from simulations. Finally, Section 5 concludes with policy implications and a discussion of possible extensions of the model.

2. Wage arrears in Ukraine

The empirical evidence of arrears in Ukraine is taken from both aggregate and household data. Aggregate time series data come from the Ukrainian European Policy and Legal Advice Center (UEPLAC) and the household data are taken from the Ukraine-96 survey.

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[4] This explanation is consistent with the empirical evidence from Ukraine and with Desai and Idson (2000) who argue that wage arrears would have been eliminated in Russia if nonviable firms were forced into bankruptcy.
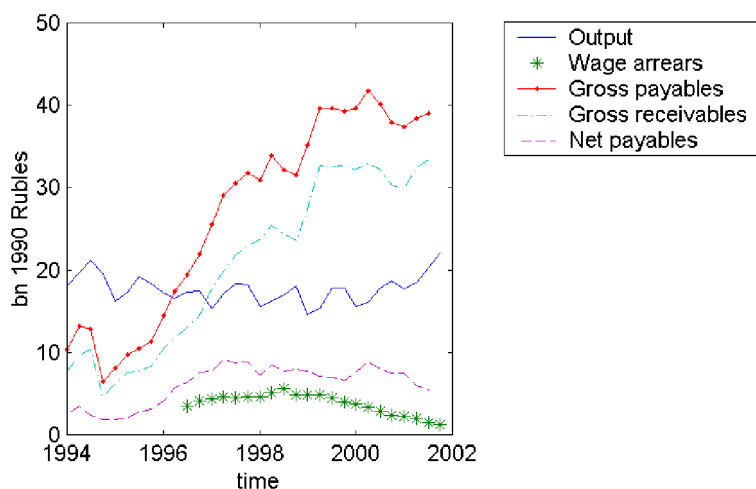
Fig. 1. Aggregate output and arrears.

Appendix A provides a description of the data. Figure 1 depicts the time paths of aggregate arrears in the Ukraine from 1994 to 2001. Gross payables and gross receivables are defined as the sum of the real debt of and to economic agents, respectively. Net payables is the difference between gross payables and gross receivables. Wage arrears is the real wage debt of enterprises to workers.[5] For comparison, we plot the path of real output over this period.

Figure 1 indicates the total amount of debt in the Ukraine increased dramatically over the time period; gross payables rose from one half of quarterly output in 1994 to two quarters of output in 2000. However, the increment in gross payables is not necessarily an indication of poor economic performance. According to estimates by the National Bank of Ukraine, three quarters of gross payables in 1996 were trade credits, i.e., inter-enterprise arrears. Trade credits are cheap and convenient substitutes for bank credits and are common in developed market economies. Trade credits are particularly useful in transition economies because financial institutions are underdeveloped. The current level of trade credits as a percentage of GNP in Ukraine is comparable to that in France or Japan.[6]

The presence of a large net debt is the more relevant problem for the Ukrainian economy. Figure 1 indicates gross payables consistently exceeded gross receivables and average net payables amounted to 35.46% of quarterly output.[7] This indebtedness represents the adverse effects of transition on Ukrainian producers. On the one hand, the typical Ukrainian enterprise faced a reduction in demand for its products because of an increase in foreign

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[5] Except for wage arrears, the net payables include tax arrears, inter-enterprise arrears, arrears of enterprises to banks, and pension arrears.

[6] Alfandari and Schaffer (1996) provide a comparison of the size of trade credits across countries.

[7] Alfandari and Schaffer (1996) report payables and receivables for different countries. For the typical developed market economy, receivables are larger than payables. The largest net receivables are observed in Japan and amount to 15% of annual output.

competition and because of the break-up of the Soviet Union. On the other hand, the firm faced an increase in input prices, especially, energy and primary resources, due to the opening up of the economy. According to the Ukrainian National Bank, 84% of net debt in 1996 was concentrated in manufacturing, agriculture, and the coal sector, whereas 78% of net credits belonged to input suppliers, especially in the energy, oil and gas sectors. Clearly, many enterprises were unable to pay for inputs. UEPLAC (2001) concludes that the shock in relative prices at the onset of transition resulted in much of Ukrainian industry producing negative value added. Hence, the growth of net arrears is attributed to loss-making enterprises that continue in operation by incurring more debt.

Figure 1 indicates wage arrears constituted a large fraction of Ukrainian net arrears during the period 1996 to 2001. On average, wage arrears amounted to 22.44% of quarterly output, which corresponds to two third of the net arrears. Therefore, we conclude that the debt burden of Ukrainian enterprises was laid disproportionately upon the workers through wage arrears.[8] Similarly to the aggregate time series data, household data reveal that Ukrainian wage arrears is both a large-scale and a long-run phenomenon. From the Ukraine-96 household survey, in 1996, wage arrears of the average Ukrainian worker amounted to 39.36% of the annual wage, which implies almost a 5-month wage debt per year and per worker. In addition, wage arrears affected 66.3% of the individuals interviewed and 7.14% of the workers reported wage arrears that exceeded their average annual wage. From the 1999 survey of Ukraine Small and Medium Enterprises, Lukyanenko et al. (2002) report that 66.4% of employees experienced wage arrears. The duration of wage arrears for periods of less than 1 month, 1 to 3 months, 4 to 6 months, 7 to 12 months, and more than 12 months was reported by 4.67, 38.27, 22.36, 14.20, and 18.80% of the employees, respectively.

Lukyanenko et al. (2002) investigate the determinants of wage arrears using a probit model and report the following regularities. The individual characteristics of workers, e.g., age and gender, do not have a statistically significant impact on the incidence of wage arrears. Alternatively, the firms' characteristics, e.g., sectorial affiliation and ownership, are statistically significant. Specifically, finance, trade, catering, transport, and health and education have incidences of wage arrears that are lower by 46.5, 34.5, 26.3, 17.3, and 15.1%, respectively, than in agriculture. Privately-owned firms have an incidence of wage arrears that is 30% lower than that of state-owned firms. Furthermore, the incidence of wage arrears varies across regions; the difference between the Kiev region having the lowest incidence of wage arrears in Ukraine and the Rivne region with the highest incidence is around 45%. The authors also consider other variables, e.g., the size of the firm, the number of hours worked and an urban-rural dummy, but these are not statistically significant.[9]

To summarize, we find the following key regularities in the Ukrainian data. During the transition, many Ukrainian enterprises incurred losses and their debt burden was shifted

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[8] UNECE (2004) also reaches the same conclusion arguing that labor was the least-protected factor of production during the transition in Ukraine because the drop in real wages attributable to wage arrears exceeded the reduction in output.

[9] The characteristics of the incidence of wage arrears in the Russian Federation are similar, as Lehmann et al. (1999), Earle and Sabirianova (2000, 2002), and Lehmann and Wadsworth (2001) report.

mainly to workers.[10] For a considerable period of time, the Ukrainian economy has had a large and roughly stationary stock of wage arrears.[11] Wage arrears affect all workers, independently of age and gender; however, the incidence of wage arrears across firms varies by region, sector and ownership type. From a worker's perspective, wage arrears have essentially the same effect as idiosyncratic shocks to wages because workers do not know when and how much they will be paid.[12] In the next section, we present a one-sector neoclassical growth model consistent with this evidence.[13]

3. The model

Our model follows Bewley (1986) by considering ex ante identical agents who become heterogeneous due to uninsurable idiosyncratic uncertainty. Our economy consists of a continuum of consumers and a continuum of firms. Both consumers and firms are uniformly distributed on a closed interval [0, 1]. Time is discrete and the horizon is infinite, so that $t \in T$ is a time index with $T = \{0, 1, 2, \ldots\}$.[14]

Each agent supplies one unit of non-valued time to production inelastically. The agent's contractual wage is denoted W.[15] However, due to the presence of wage arrears, the effective wage, denoted ω_t, does not usually coincide with the contractual one. The effective wage is equal to a fraction of the wage debt carried over from the previous period and a fraction of the contractual wage. For the sake of simplicity, we assume both fractions are equal, so that we have:

$$\omega_t = \big[q_{t-1}(1 - d_q) + W\big](1 - v_t), \tag{1}$$

where q_{t-1} denotes the stock of wage arrears at the end of period $t - 1$, $d_q \in (0, 1]$ is the depreciation rate of wage arrears, and $(1 - v_t)$ is the fraction of the total wage bill paid to the agent in period t. We specify v_t to be an agent-specific random variable, which is independently and identically distributed across agents on the interval [0, 1], and refer to this variable as an idiosyncratic shock to wage arrears.

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[10] Delays in wage payments also help enterprises to reduce effective wages because wage arrears are not fully indexed by inflation. Layard and Richter (1995) argue that workers were willing to accept wage cuts to preserve jobs.

[11] Lehmann and Wadsworth (2001) also find stationary wage arrears in Russia. They report that the stock of wage arrears has been approximately at the steady state equivalent to two month wage bill since 1996. Hence, during this time, contractual wages not paid to some workers in any given month are approximately equal to the amount of wage debts paid back to some other workers in that month.

[12] The random nature of wage arrears is emphasized by Earle and Sabirianova (2002) who argue that arrears generate uncertainty about the timing and extent of the eventual payment. According to Lehmann and Wadsworth (2001), the wage distribution that would have been observed if all workers had been paid in full is very similar to the one that is actually observed for workers who are paid in full. Hence, these authors argue that wage arrears are drawn reasonably uniformly from the entire wage distribution.

[13] More precisely, we can account for all of the above evidence, except the systematic variation of wage arrears across firms having certain specific characteristics.

[14] This assumption implies that average and aggregate quantities coincide.

[15] Variables having common equilibrium values for all agents are denoted by capital letters and those having agent-specific values are denoted by small letters. The same connection is applied to the firms.

The assumption that the stock of wage arrears depreciates allows us to account for the fact that wage arrears are not indexed to inflation in transition economies. Hence, a delay in wage payment results in a reduction of the real value of wage debt. Furthermore, some wage arrears are not paid at all because firms go bankrupt or because workers quit their jobs and loose the right to claim the debt. The underpaid (overpaid) wages are added to (subtracted from) the stock of wage arrears so that the evolution of the debt of the firm to the worker is described by:

$$q_t = \big[q_{t-1}(1 - d_q) + W\big]v_t. \tag{2}$$

We assume that q_t is bounded, i.e., $q_t \in \mathbf{Q} \equiv [0, \bar{q}] \subset \mathbf{R}$ for all t.

The agent is assumed to save in the form of real assets. Income from assets is equal to Ra_t, where R is the interest rate and a_t is the individual's current asset holdings. Assets are restricted to the set $\mathbf{A} = [\underline{a}, \bar{a}] \subset \mathbf{R}$, where $\underline{a} \leqslant 0$ and $\bar{a} > 0$, so that the agent is allowed to borrow only up to a certain amount $\underline{a}$. The agent seeks to maximize the expected discounted sum of one-period utilities by choosing an optimal consumption path denoted $\{c_t\}_{t=0}^{\infty}$. The utility function $u(c)$ is continuously differentiable, strictly increasing, strictly concave and satisfies the Inada condition $\lim_{c \to 0} u'(c) = \infty$. Consumption is restricted to being non-negative. Consequently, the consumer's problem is given by:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t), \tag{3}$$

subject to

$$c_t + a_{t+1} = \omega_t + (1 + R)a_t, \tag{4}$$

$$a_{t+1} \geqslant \underline{a}, \tag{5}$$

where (a_0, q_{-1}, v_0) represents the initial condition and is given, and ω_t is defined by (1).[16] In (3), E_t denotes the expectation, conditional on all information about the agent's wage payments available at t, and $\delta \in (0, 1)$ is the discount factor. To ensure the existence of a solution to the consumer's problem, we shall require $\delta(1 + R) < 1$.

The firms are assumed to have identical production technologies that convert capital and labor into output. Each firm chooses its capital input, K_t, and its labor input, N_t, to maximize expected period-by-period profit, taking the interest rate, R, and wage, W, as given. The level of output is assumed to depend on an additive idiosyncratic shock, denoted θ_t. Thus, the firm solves the following problem:

$$\max_{K_t, N_t} E\big[F(K_t, N_t) - dK_t - RK_t - WN_t - \theta_t\big], \tag{6}$$

where E is an unconditional expectation, F is the production function and $d \in (0, 1]$ is the depreciation rate of capital. The production function exhibits constant returns to scale; it is strictly increasing, strictly concave, continuously differentiable and satisfies the appropriate Inada conditions.

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[16] Non-negativity of consumption is guaranteed by the Inada condition.

We assume that the firm cannot exit the market even if it incurs losses.[17] The first-order conditions from (6) imply that factor prices are equal to their corresponding marginal products. Hence, we have:

$$R = F_1(K_t, N_t) - d, \qquad W = F_2(K_t, N_t), \tag{7}$$

where F_1 and F_2 are the first-order partial derivatives of the production function F with respect to capital and labor inputs, respectively. We assume that the firm first pays for capital and only afterwards for labor. In addition, we assume the revenue from output is always sufficient to cover depreciation of capital and its rental price, i.e., $F(K_t, N_t) - \theta_t \geqslant (d + R)K_t$.[18] Thus, the effective wage per unit of labor is:

$$\omega_t = \big[F(K_t, N_t) - \theta_t - (d + R)K_t\big]/N_t = W - \theta_t/N_t. \tag{8}$$

We assume that each firm hires one worker so that $N_t = 1$. Wages that are underpaid (overpaid) to a worker are added to (subtracted from) the stock of wage arrears yielding:

$$q_t = q_{t-1}(1 - d_q) + \theta_t. \tag{9}$$

Conditions (8) and (9) describing the evolution of the stock of the firm's wage arrears are equivalent to conditions (1) and (2) describing the evolution of the stock of its worker's wage arrears, if the shocks θ_t and v_t are related by:

$$\theta_t \equiv W v_t - q_{t-1}(1 - d_q)(1 - v_t). \tag{10}$$

To capture the fact that the typical firm in a transition economy is a loss-maker, we assume that the idiosyncratic shock, θ_t, is, on average, positive, i.e., $E[\theta_t] > 0$. By assumption, the process for θ_t is exogenous and cannot be affected by the firm's actions. However, even if we allowed the firm to reduce θ_t at zero cost, it would have no incentive to do so because it shifts all losses to the workers and makes the same effective zero profit, independently of the amount of wage arrears. Consequently, the rationale for the firm to pay any wages must be established. Hence, we assume that shocks to production can be perfectly and costlessly monitored by workers, trade unions and the government. Workers will tolerate wage arrears only if they arise as a consequence of a negative shock. If wage arrears are created artificially, workers will report this abuse to the government and the managers of the firm will be penalized. The empirical literature describes numerous examples in which managers delayed paying wages intentionally.[19] However, the Russian and Ukrainian legislations explicitly prohibit this practice making it illegal.[20]

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[17] This assumption allows us to account for the fact that many loss-making enterprises were not closed down and continued operating with the government support in the transition economies. Indeed, Russia and Ukraine still maintain almost full employment in spite of the existence of unprofitable enterprises.

[18] This assumption guarantees that capital arrears do not arise in our model. In the actual transition economies, capital arrears (e.g., arrears of enterprises to banks) were present although they were of a much smaller order of magnitude than wage arrears, see UEPLAC (2001). We abstract from the capital arrears phenomenon therefore.

[19] Alfandari and Schaffer (1996) discuss managers using wage arrears for extracting tax concessions from the government. Earle and Sabirianova (2002) explain that Russian managers threaten wage arrears to make the workers sell their shares in the firm.

[20] Lukyanenko et al. (2002) discuss the Ukrainian legislation concerning wage arrears.

In actual transition economies, wage arrears is a transitory phenomenon. Arrears accumulated at the beginning of the transition, have remained roughly constant during the transition period and, presumably, will disappear when the transition is over. However, modeling such a non-stationary pattern of wage arrears in a framework of heterogeneous agents is difficult. Hence, we restrict our attention to a more simple, stationary model in which the aggregate economy is always in a steady state with a constant amount of wage arrears. Under this assumption, we do not model the transition between states with and without wage arrears. However, this simplification does not distort our results because, under standard parameterization of preferences and production, the convergence from one steady state to another in a neoclassical growth model is typically fast.[21] In other words, if agents experience wage arrears of approximately the same size for several years in our model, they behave as if this situation is stationary.

We focus on a recursive solution to the agent's problem given by (1) through (5), so that the agent makes consumption-savings decisions according to the same decision rule in all periods. Although the consumer's problem has three state variables, i.e., a_t, q_{t-1} and v_t, we can reduce their number to two with the transformations:

$$\hat{a}_t = a_t - \underline{a}, \quad \text{and} \tag{11}$$

$$z_t = \omega_t + (1+R)\hat{a}_t + R\underline{a}. \tag{12}$$

The variable z_t can be interpreted as the total amount of resources available to the agent in period t. Using (11) and (12), we can rewrite constraints (4) and (5) as follows:

$$c_t + \hat{a}_{t+1} = z_t, \quad \text{and} \tag{13}$$

$$\hat{a}_t \geqslant 0. \tag{14}$$

Denote the optimal value function for the agent with the total resources z_t and wage arrears q_t by $V(z_t, q_t)$. Without time subscripts, the recursive formulation of the problem specified by (1) through (5) is given by:

$$V(z, q) = \max_{\hat{a}' \in [0, \bar{a} - \underline{a}]} \left\{ u(z - \hat{a}') + \delta E\big[V(z', q') \mid q\big] \right\} \tag{15}$$

subject to

$$z' = \omega' + (1+R)\hat{a}' + R\underline{a}, \tag{16}$$

where $\omega' = q(1 - d_q) + W - q'$, with q' being the exogenous driving process for wage arrears given by (9) and (10). Note that q' together with the agent's saving decision, $\hat{a}'$, determines the amount of future resources, i.e., z'. The problem given by (15)–(16) defines the optimal asset demand function, $\hat{a}' \equiv A(z, q)$.

By calculating the Kuhn–Tucker conditions of this problem, we obtain the following Euler equation:

$$u'(c) \geqslant \delta E\big\{ u'(c')(1+R) \big\}, \tag{17}$$

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[21] Maliar and Maliar (2004a) plot impulse-response functions for a neoclassical growth model with permanent productivity shocks and show that, after a shock, the transition to a new steady state occurs essentially in one period.

where $c = z - \hat{a}'$ and u' denotes the marginal utility of consumption. The Euler equation holds with equality if the borrowing restriction is non-binding, $a' > \underline{a}$, and it holds with inequality if the limit on borrowing is reached, i.e., $a' = \underline{a}$.

To describe equilibrium, we take the individual state space in our economy to be $\mathbf{Z} \times \mathbf{Q} = [\underline{z}, \overline{z}] \times [0, \overline{q}]$, where $\underline{z}$ and $\overline{z}$ are the lower and the upper bounds on the total amount of resources implied by (16), i.e., $\underline{z} = W - \overline{q} + R\underline{a}$ and $\overline{z} = \overline{q}(1 - d_q) + W + (1 + R)\overline{a} + R\underline{a}$. Let $\mathbf{B}$ be the Borel σ-algebra of the set $\mathbf{Z} \times \mathbf{Q}$ and let $P : \mathbf{Z} \times \mathbf{Q} \times \mathbf{B} \to [0, 1]$ be a transition function on the measurable space $(\mathbf{Z} \times \mathbf{Q}, \mathbf{B})$ so that:

$$P(z, q, B) = \text{Prob}\big(\{q' \in \mathbf{Q} : [z', q'] \in B\} \mid q\big) \quad \text{for } B \in \mathbf{B},$$

in which, for given values of q' and $A(z, q)$, the value of z' is uniquely determined by $q(1 - d_q) + W - q' + (1 + R)A(z, q) + R\underline{a}$. For each $(z, q) \in \mathbf{Z} \times \mathbf{Q}$, $P(z, q, \cdot)$ is a probability measure on $(\mathbf{Z} \times \mathbf{Q}, \mathbf{B})$ and, for each $B \in \mathbf{B}$, $P(\cdot, B)$ is a $\mathbf{B}$-measurable function. In addition, let $\lambda_t(B)$ be a probability measure, defined on $\mathbf{B}$, that represents the mass of agents whose individual states lie in $B \in \mathbf{B}$ at time t. Since $\lambda_t(B)$ is a probability measure, the total mass of the agents equals one. Under the transition function $P(z, q, B)$, the law of motion of λ_t can be written as: $\lambda_{t+1}(B) = \int_{\mathbf{Z} \times \mathbf{Q}} P(z, q, B) \, d\lambda_t$ for all $t \in T$ and all $B \in \mathbf{B}$.

The continuum of agents guarantees that the mass of agents with the shock q' at $t + 1$ and the shock q at t is equal to the conditional probability given by $\text{Prob}(q' \mid q)$. Since the stock of wage arrears follows a first-order Markov process, this probability depends only on the recent past and is the same in all periods. Hence, the aggregate amount of wage arrears is constant. We study only the equilibria in which the period-$(t + 1)$ probability measure λ_{t+1} is the same as the period-t probability measure λ_t, for all $t \in T$. In this case, the probability measure, denoted λ^* is stationary. Stationarity implies that the aggregate capital stock is constant, $K = \int_{\mathbf{Z} \times \mathbf{Q}} a_t \, d\lambda^*$ for all t, even though the asset holdings of each agent vary stochastically over time.

Given this structure, we have the following definition of equilibrium. A stationary equilibrium is defined as a stationary probability measure λ^*, an optimal asset function $A(z, q)$, and positive real numbers (K, Q, R, W) such that:

(1) λ^* satisfies $\lambda^* = \int_{\mathbf{Z} \times \mathbf{Q}} P(z, q, B) \, d\lambda^*$ for all $B \in \mathbf{B}$;
(2) $A(z, q)$ solves (16)–(17) for a given pair of prices (R, W);
(3) (R, W) satisfy the profit maximization conditions in (7) with $N_t = 1$;
(4) Q is the average of the agents' wage arrears: $Q = \int_{\mathbf{Q}} q_t \, d\lambda^*$;
(5) K is the average of the agents' asset holdings: $K = \int_{\mathbf{Z} \times \mathbf{Q}} a_t \, d\lambda^*$.

The existence and uniqueness of such an equilibrium follow from Theorems 1 and 2 in Huggett (1993). Specifically, Theorem 1 establishes the existence and uniqueness of the optimal policy function $A(z, q)$ for the consumer's problem given by (15)–(16), and Theorem 2 shows the existence and uniqueness of the equilibrium probability measure λ^*.

After the appropriate transformation of variables, we have shown that the individual state in our economy can be characterized in terms of only two state variables, (z_t, q_t), rather than three state variables, (a_t, q_{t-1}, v_t). Given this result, the consumer's problem with wage arrears becomes identical to the one in the standard one-sector neoclassical growth model analyzed by Aiyagari (1994), with the exception that fluctuations in the

individual wage are a consequence of delays in wage payments and not of idiosyncratic shocks to productivity.[22] The similarity between the two models arises because the effect of wage arrears on the individual consumption-savings behavior is similar to the impact of idiosyncratic productivity shocks in that both lead to randomness in the amount of wage payments. Therefore, the difference between the two models lies therefore in the stochastic properties of wages.

In our model, the idiosyncratic delays in wage payments lead to the heterogeneity in income, assets, and consumption among agents. To smooth consumption fluctuations, risk-averse agents hold precautionary savings. There are two types of social costs associated with wage arrears. First, as a result of the depreciation of wage arrears, wage debts are never fully repaid so that average income and average consumption are reduced. Second, due to idiosyncratic uncertainty about wage payments, individual consumption is volatile and the utility of risk averse agents is reduced. In the next section, we evaluate the macroeconomic consequences and the welfare implications of wage arrears in a calibrated version of our model.

4. Quantitative analysis

The model's period is one year. We calibrate the model to reproduce several basic characteristics of the Ukrainian economy. Regarding the process for wage arrears given by (2), we assume that the idiosyncratic shock is drawn from a normal distribution, i.e., $v_t \sim N(V, \sigma_v^2)$. With the assumption of stationary distributions of wages and wage arrears, Eqs. (1) and (2) yield:

$$V = \frac{\Omega}{\Omega + Q}, \tag{18}$$

$$d_q = 1 - \sqrt{1 + 2\,\text{corr}(q, w)\frac{\sigma_\omega}{\sigma_q} + \frac{\sigma_\omega^2}{\sigma_q^2}}, \tag{19}$$

where Ω and σ_ω (Q and σ_q) are the mean and the standard deviation of the wage distribution (the distribution of wage arrears), respectively, and $\text{corr}(q, w)$ is the correlation coefficient between variables q_t and w_t. We compute the above statistics by using the data from the Ukraine-96 household survey and find the corresponding values of the parameters V, σ_v and d_q by simulation, as described in Appendix A. The results are reported in panel A of Table 1.

We assume a Cobb–Douglas production function, $F(K, N) = K^\alpha N^{1-\alpha} = F(K, 1) = K^\alpha$. We calibrate the remaining parameters $\{\alpha, d, \delta\}$ so that, in the non-stochastic steady state, the model reproduces the following four statistics of the Ukrainian economy: the share of labor income in production, denoted Y^L/Y, the consumption to output ratio, denoted C/Y, the capital to output ratio, denoted K/Y, and the wage arrears to output ratio, denoted Q/Y. In the absence of wage arrears, $\alpha = 1 - (Y^L/Y)$. With wage

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[22] The individual state in the model in Aiyagari (1994) is characterized by two variables (z_t, s_t), where z_t is the total amount of resources and s_t is an idiosyncratic productivity shock.

Table 1
Selected statistics for Ukrainian economy and the implied parameters in the model

A. Ukrainian household data for 1996 and the implied model's parameters

Q, krb	σ_q, krb	Prob($q_t > 0$)	Ω, krb	σ_ω, krb	corr(q, w)	d_q	V	σ_v
18674.6	31574.5	0.6630	46430.5	24432.2	−0.8870	0.5245	0.2868	1.1129

B. Ukrainian aggregate time series data from 1994 to 2001

	1994	1995	1996	1997	1998	1999	2000	2001
Y^L/Y	0.3967	0.4789	0.4793	0.4616	0.4604	0.4392	0.4253	0.4404
C/Y	0.8024	0.7810	0.6882	0.7268	0.6769	0.6660	0.6973	0.7702
K/Y	–	–	4.2546	–	3.4871	2.7519	2.1718	–
Q/Y	–	–	0.2190	0.2612	0.3042	0.2822	0.1850	0.0920

C. Averages of the Ukrainian aggregate time series and the implied parameters in the model

C/Y	K/Y	Y^L/Y	Q/Y	d	α	δ
0.7261	3.1663	0.4477	0.2244	0.0865	0.4578	0.9451

Sources: UEPLAC (2001), Derzhcomstat (2000), and Ukraine-96 Project, Kiev International Institute of Sociology.

arrears, agents do not receive all income earned but are paid only a fraction, denoted $\Omega/W = \Omega/(\Omega + d_q Q)$. Hence, we have:

$$\alpha = 1 - \left(1 + \frac{d_q Q}{\Omega}\right)(Y^L/Y). \tag{20}$$

Taken together Eqs. (4), (7), (8) and (9) yield:

$$d = \frac{1 - (C/Y) - d_q(Q/Y)}{(K/Y)}. \tag{21}$$

Finally, the Euler equation (17) implies that:

$$\delta = \frac{1}{1 - d + \alpha(Y/K)}. \tag{22}$$

In panel B of Table 1, we report the ratios for Y^L/Y, C/Y, K/Y and Q/Y computed from Ukrainian time-series data from 1994 to 2001. The first two ratios, i.e., Y^L/Y and C/Y, appear to be roughly stationary over the period. However, the third ratio, i.e., K/Y, declines considerably over time because capital was used to reduce the impact of demand shocks on wages and because owners were allowed to extract rent and export capital. The last ratio, i.e., Q/Y, appears to be stationary except for the last period in which it decreases sharply due to the overall improvement in the Ukrainian economic situation. We compute the averages of these four ratios over the sample period and find the corresponding values of α, d and δ from Eqs. (20), (21) and (22), respectively. The results are found in panel C of Table 1.

The debt limit is set at zero, i.e., $\underline{a} = 0$. We assume that the agent's momentary utility function exhibits a constant relative risk aversion (CRRA) so that $u(c) = (c^{1-\gamma} - 1)/(1 - \gamma)$ with $\gamma > 0$. Since the coefficient of relative risk-aversion, γ, is not

identified by our calibration procedure, we consider four alternative values for this parameter, namely, $\gamma \in \{0.5, 1, 3, 10\}$. The benchmark value for γ is 1, which corresponds to the limiting logarithmic case, yielding $u(c) = \ln(c)$. To compute numerical solutions, we approximate the process for wage arrears (9) by a seven-state Markov chain, for which the distribution of v_t is truncated such that $v_t \in [0, 1]$. The corresponding unconditional probabilities of states are reported in Table 2. A description of the solution procedure is elaborated in Appendix B. We report the statistics generated by the model in Table 3.

Beginning with our benchmark model, denoted BM, in the first column of Table 3, we provide several inequality measures of the distributions of wages, assets, and consumption to illustrate its distributional predictions. The table indicates, idiosyncratic uncertainty associated with wage arrears leads to a significant dispersion in effective wages across workers. In particular, the bottom 40% wage earners receives only 16.8% of total wages, while the top 20% earns 33.3% of total wages. The Gini coefficient of the wage distribution is 0.31. In our model, the wage distribution coincides with the earnings distribution given that each agent supplies all endowed labor to production. The computed Gini coefficient is somewhat lower than the ones reported by UNECE (2004); in 1993, the Gini coefficient of the earnings distribution in Ukraine was 0.36 and by 2001, it had risen to 0.45.

Inequality in assets and consumption occurs as a result of wage inequality but the dispersion in assets is much higher than in consumption. Specifically, the bottom 40% of asset owners holds 22.3% of total assets, while the top 20% holds 34.7%. In comparison, the respective consumption groups consume 33.6 and 25.0% of total consumption. Moreover, the normalized standard deviations and the Gini coefficients are 0.481 and 0.266, respectively, for the asset distribution, and 0.152 and 0.096, respectively, for the consumption distribution.[23] Hence, our model predicts that risk-averse agents smooth consumption fluctuations by accumulating assets in high-wage states and dissaving in low-wage states. Although the household survey does not contain information about individual consumption and asset holdings, it does provide information about the respondents' own evaluations of their material status relative to that of other people. In Fig. 2, we plot the joint distribution of this reported variable and wage arrears. Figure 3 depicts the simulated distribution of assets and wage arrears based on our benchmark model. A comparison of the two figures demonstrates their similarity.

Our finding that agents use assets to smooth consumption is consistent with the empirical literature on transition economies. Desai and Idson (2000) find that in Russia, agents who were owed wages were more likely to engage in dissaving activities including borrowing, selling family consumer durable items and other assets, and using accumulated savings. Investigating a decrease in cash consumption in Russia, Klugman and Kolev (2001) report that capital income, i.e., income from rents, property and capital investments, was 4.5 and 8.3% of cash income for the bottom and the top consumption–expenditure quintiles, respectively, in 1994 but increased to 8.6 and 9.3%, respectively, in 1995. Their regression analysis shows that capital income has a positive and significant impact on cash

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[23] The degrees of wealth, income, and consumption inequality in our model with wage arrears are very similar to those generated by a standard neoclassical growth model with productivity shocks in Aiyagari (1994).

Table 2
Markov chain and unconditional actual and fitted distributions of arrears

$q'\vert q$	$q_1 = 0$	$q_2 = 0.2801$	$q_3 = 0.5602$	$q_4 = 0.8403$	$q_5 = 1.1205$	$q_6 = 1.4006$	$q_7 = 1.6807$	Prob(q_i) fitted	Prob(q_i) actual
$q_1 = 0$	0.4447	0.4385	0.4337	0.4297	0.4265	0.4238	0.4215	0.4374	0.4476
$q_2 = 0.2801$	0.1554	0.1519	0.1251	0.1227	0.1210	0.1028	0.1015	0.1404	0.2785
$q_3 = 05602$	0.1503	0.1491	0.1240	0.1224	0.1213	0.1033	0.1023	0.1376	0.1312
$q_4 = 0.8403$	0.1356	0.1386	0.1176	0.1178	0.1179	0.1013	0.1008	0.1277	0.0746
$q_5 = 1.120$	0.1140	0.1219	0.1068	0.1094	0.1113	0.0968	0.0973	0.1125	0.0251
$q_6 = 1.4006$	0	0	0.0928	0.0980	0.1020	0.0902	0.0918	0.0408	0.0129
$q_7 = 1.6807$	0	0	0	0	0	0.0819	0.0848	0.0036	0.0302

Notes. (i) The grid points for the stock of wage arrears are expressed in units of the contractual wage, W. (ii) The column designated Prob(q_i) fitted is an unconditional probability distribution of wage arrears generated by the constructed Markov chain. (iii) The column designated Prob(q_i) actual is an unconditional probability distribution of wage arrears generated by Ukrainian household data.
Source: Ukraine-96 Project, Kiev International Institute of Sociology.

Table 3
Selected statistics

Statistic	BM	$\gamma = 0.5$	$\gamma = 3$	$\gamma = 10$	$\alpha = 0.36$	$\delta = 0.96$	$d = 0.1$	$d_q = 0.477$	$V = 0.316$	$\sigma_v = 1.22$
Aggregate statistics										
R, %	5.6155	5.7146	5.1876	3.6001	5.5192	4.0383	5.6081	5.6203	5.6179	5.6183
ΔK, %	2.5157	1.2148	8.4386	35.7675	3.2143	1.8734	2.3869	2.4521	2.4838	2.4785
$\Delta(RK)$, %	−0.8986	−0.4291	−3.1604	−15.8594	−1.9353	−1.2642	−1.1539	−0.8754	−0.8870	−0.8851
ΔW, %	1.1440	0.5543	3.7787	15.0267	1.1454	0.8534	1.0858	1.1153	1.1296	1.1272
$\Delta\Omega$, %	−20.7653	−21.2272	−18.7013	−9.8898	−20.7642	−20.9929	−20.8109	−18.0488	−20.9642	−20.4773
ΔI, %	−15.7327	−15.9587	−14.7645	−11.4020	−17.2935	−16.7435	−16.1559	−13.6985	−15.8783	−15.5142
ΔC, %	−13.6338	−13.8611	−12.6704	−9.4579	−14.7878	−14.8278	−14.0855	−11.5864	−13.8134	−13.4434
$\Delta^u C$, %	−0.9202	−0.4801	−2.3205	−5.1610	−0.5201	−0.6801	−0.7602	−0.8402	−0.9202	−0.9202
The distribution of wages, ω										
std$(\omega)/\omega$	0.5244	0.5241	0.5266	0.5415	0.5717	0.5363	0.5229	0.5012	0.5216	0.5201
corr(ω, q)	0.2788	0.2806	0.2706	0.2374	0.3432	0.2469	0.3016	0.2516	0.2823	0.2841
0–40%	16.8493	16.8493	16.8493	16.8493	16.8493	16.8493	16.8493	19.0644	17.1068	17.2261
80–100%	33.2770	33.2770	33.2770	33.2770	33.2770	33.2770	33.2770	31.5543	33.4944	33.3497
90–95%	8.5280	8.5280	8.5280	8.5280	8.5280	8.5280	8.5280	8.0342	8.5855	8.5500
95–99%	7.2468	7.2468	7.2468	7.2468	7.2468	7.2468	7.2468	6.7604	7.3026	7.2665
99–100%	1.9026	1.9026	1.9026	1.9026	1.9026	1.9026	1.9026	1.7710	1.9185	1.9079
Gini(ω)	0.3124	0.3124	0.3124	0.3124	0.3124	0.3124	0.3124	0.2828	0.3116	0.3094
The distribution of assets, a										
std$(a)/a$	0.4816	0.4941	0.4395	0.3231	0.4402	0.4808	0.4662	0.4695	0.4807	0.4799
corr(a, q)	−0.1620	−0.1588	−0.1727	−0.2094	−0.2629	−0.1461	−0.1830	−0.1699	−0.1598	−0.1597
0–40%	22.3413	21.9209	23.8187	27.9793	23.8495	22.4206	22.9304	22.7780	22.3844	22.4101
80–100%	34.7273	35.0959	33.3908	29.7084	33.2699	34.7482	34.1950	34.3454	34.7003	34.6574
90–95%	8.7852	8.8853	8.4409	7.4954	8.4076	8.7905	8.6537	8.6882	8.7802	8.7701
95–99%	8.2745	8.3884	7.8842	6.8011	7.8547	8.2966	8.1304	8.1719	8.2711	8.2630
99–100%	2.6079	2.6531	2.4607	2.0522	2.4603	2.6254	2.5553	2.5728	2.6074	2.6051
Gini(a)	0.2663	0.2730	0.2435	0.1803	0.2429	0.2656	0.2577	0.2596	0.2658	0.2654

Table 3 (*continued*)

Statistic	BM	$\gamma = 0.5$	$\gamma = 3$	$\gamma = 10$	$\alpha = 0.36$	$\delta = 0.96$	$d = 0.1$	$d_q = 0.477$	$V = 0.316$	$\sigma_v = 1.22$
The distribution of consumption, c										
std$(c)/c$	0.1517	0.1556	0.1377	0.0987	0.1137	0.1307	0.1395	0.1444	0.1517	0.1509
corr(c, q)	−0.1107	−0.1083	−0.1189	−0.1460	−0.1901	−0.0999	−0.1262	−0.1193	−0.1088	−0.1083
0–40%	33.5688	33.3775	34.1982	35.8524	35.0084	34.4698	34.1131	33.9209	33.6068	33.6402
80–100%	24.9627	25.1188	24.4512	23.0279	23.6455	24.2747	24.5180	24.7157	24.9537	24.9372
90–95%	6.2774	6.3170	6.1423	5.7769	5.9350	6.0982	6.1638	6.2118	6.2738	6.2657
95–99%	5.4147	5.4578	5.2615	4.8403	5.0186	5.2189	5.2865	5.3426	5.4131	5.4036
99–100%	1.5229	1.5391	1.4647	1.3036	1.3714	1.4525	1.4753	1.4973	1.5226	1.5197
Gini(c)	0.0962	0.0990	0.0865	0.0610	0.0739	0.0827	0.0878	0.0911	0.0958	0.0952

Notes. (i) The statistics in the column labeled BM are those generated by the model under the benchmark parameterization: $\gamma = 1$, $\alpha = 0.4578$, $\delta = 0.9451$, $d = 0.0865$, $d_q = 0.5245$, $z_q = 0.2868$, $\sigma_q = 1.1129$. (ii) The statistics in each subsequent column are obtained when the parameter identified at the top of the column is set to the given value, while the other parameters are set to their benchmark values. (iii) The statistic ΔX is the percentage difference between the mean of variable X_t and its non-stochastic steady state value, X^{ss}, i.e., $\Delta X \equiv (X - X^{\text{ss}})/X^{\text{ss}} \times 100\%$. (iv) The statistic $\Delta^u C$ is the cost of the computed consumption fluctuations. (v) The percentages refer to shares of aggregate quantities owned by ranked population subgroups. (vi) For the last three columns, we recompute the corresponding Markov chain.
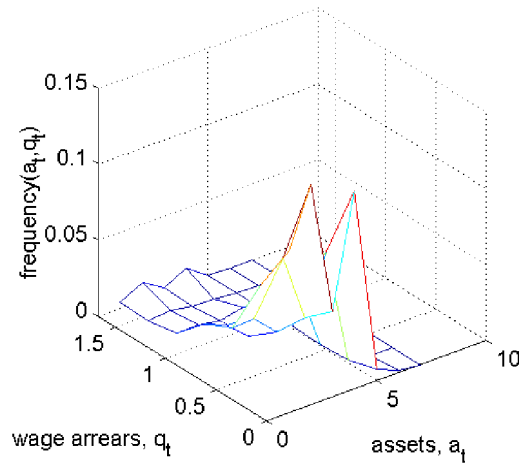
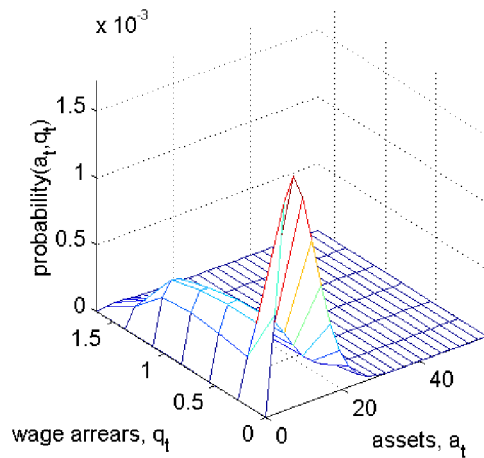Fig. 2. Empirical distribution of assets and wage arrears.

Fig. 3. Simulated distribution of assets and wage arrears.

consumption for all consumption–expenditure quintiles. These findings indicate that capital income played an important role in consumption smoothing not only for rich but also for poor households in Russia. Finally, Skoufias (2004) provides conclusive evidence about the importance of borrowing from formal and informal credit markets for Bulgarian households facing idiosyncratic fluctuations in income.

   In our model, agents hold precautionary savings to self-insure against shocks in wage arrears even if they do not currently suffer from wage arrears. This result is consistent with Guariglia and Kim (2003) who find compelling evidence of precautionary savings by Russian households the heads of which have a high probability of suffering from wage arrears one year ahead. In other words, real-world agents foresee the possibility of suffering from wage arrears and react by increasing their precautionary savings.

To investigate the effects of wage arrears on the aggregate variables in the model, we consider average income, which is the sum of capital and labor income, given by:

$$I = \int_{\mathbf{Z} \times \mathbf{Q}} (Ra_t + \omega_t)\, d\lambda^* = RK + \Omega. \tag{23}$$

Regarding capital income, i.e., RK, uncertainty about the time and amount of wage payments, together with the restriction on borrowing, induces the agents to increase their savings. As Table 3 indicates, the resulting increase in the aggregate capital stock due to precautionary savings, denoted ΔK, is 2.5157%.[24] Since the interest rate is inversely related to the aggregate capital stock, i.e., $R = \alpha K^{\alpha-1} - d$, the interest rate is lower in our economy with wage arrears than in one without wage arrears.[25] The total effect of wage arrears on capital income is negative as $\Delta(RK) = -0.8986\%$ from Table 3.

According to Eqs. (8) and (9), average labor income is $\Omega = W - d_q Q$, where $W = (1-\alpha)K^{1-\alpha}$. Two effects arise. First, the marginal product of labor goes up because the capital stock rises due to precautionary savings, i.e., $\Delta W = 1.1440\%$. Second, the wage decreases by the amount of $d_q Q$ because of the depreciation of wage arrears. Table 3 indicates that the second effect dominates because $\Delta \Omega = -20.7653\%$.

These decreases in both capital and labor income lead to a significant reduction in total income given by $\Delta I = -15.7327\%$. The main determinant of income loss is the large depreciation rate applied to wage arrears, i.e., $d_q = 0.5245$. Our calculations imply that about half of the wage bill, which is not paid to the agent in time, is lost after a one-year delay. This high depreciation rate is explained by the high rate of inflation in Ukraine, which averages 63.4% annually from 1994 to 2001.[26]

Wage arrears cause social welfare loss for two reasons: First, aggregate consumption declines due to the depreciation of wage arrears. Second, randomness in the amount of wage payments induces consumption fluctuations, which reduces the welfare of risk-averse agents. Table 3 indicates, aggregate consumption loss is $\Delta C = -13.6338\%$ in the benchmark case. To measure the cost of consumption fluctuations, we use the approach proposed by Lucas (1987) for measuring the cost of business cycle fluctuations. Specifically, we compute a consumption premium, denoted $\Delta^u C$, which must be given to the agent to maintain a utility level equal to the one that would be derived from expected consumption so that:

$$E\big[u\big(c_t + \Delta^u C\big)\big] = u\big(E[c_t]\big). \tag{24}$$

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[24] We denote by ΔX the percentage difference between the mean of variable X in the model with wage arrears and its non-stochastic steady state value, X^{ss}, i.e., $\Delta X \equiv \frac{X - X^{ss}}{X^{ss}} \times 100$. The non-stochastic steady state can be computed analytically. For example, the steady state capital stock, K^{ss}, is computed from the steady state expression of the Euler equation as $1 = \delta(1 - d + \alpha(K^{ss})^{\alpha-1})$. The other aggregate variables can be computed in a similar manner.

[25] This implication accords with the empirical evidence. During the transition period, the Ukrainian capital stock decreased. For example, from 1996 to 2000 period, the capital stock declined from 291 to 148 in billions of 1990 rubles. Over a similar period, the real interest rate increased as it was negative until 1996, except for 1994, but remained positive from 1997 to 2001, except for 2000.

[26] We compute the average inflation rate, denoted π, using $CPI_{1994}(1+\pi)^7 = CPI_{2001}$, where CPI_{1994} and CPI_{2001} are the consumer price indices in the years 1994 and 2001, respectively, as reported by UEPLAC (2001).

The table indicates, the cost of consumption fluctuations equals 0.9202% in the benchmark case.

To investigate the sensitivity of our results to the value of the risk aversion coefficient, we examine the results in Table 3 for various specifications of γ. As agents become more risk averse, i.e., γ goes up, precautionary savings increase, which decreases aggregate capital income and increases aggregate labor income. The total effect of this change on aggregate income and aggregate consumption is positive in that the income and consumption losses are reduced. In contrast, the cost of consumption fluctuations goes up. The magnitude of these effects can be considerable. For example, as γ increases from 1 to 10, precautionary savings increase from 2.5157 to 35.7675%, aggregate income and consumption losses are reduced from 15.7327 and 13.6338% to 11.4020 and 9.4579%, respectively, and the welfare loss from consumption fluctuations rises from 0.9202 to 5.1610%.[27]

To study the robustness of the model's predictions, we set the parameters equal to values that are standard in macroeconomic literature, namely, $\alpha = 0.36$, $\delta = 0.96$ and $d = 0.1$. We also consider variations in the other parameters relative to their benchmark values, specifically, a 10% decrease in d_q, a 10% increase in V, and a 10% increase in σ_v. As the final six columns of Table 3 indicate, the quantitative implications of our model are relatively robust.

However, our calibration procedure neglects the effects of permanent heterogeneity in skills, i.e., workers' productivities, by assuming that, in the absence of wage arrears, all agents earn the same wage. Since we have data on effective but not on contractual wages, this assumption is necessary. To examine the impact of neglecting skill heterogeneity on the model's predictions, we use two alternative calibration procedures. First, we split the sample into groups by education and weight the wages and wage arrears of agents by coefficients reflecting wage differentials across these educational groups. Second, we adjust the wages and wage arrears of the agents according to wage differentials across qualification-profession groups. Our main findings are robust to these modifications and the magnitude of the effects associated with wage arrears is comparable to that in the benchmark calibration.

Our welfare estimates depend on the fundamental stationarity assumption. In our model, aggregate wage arrears are constant; however, in transition economies, wage arrears are hump-shaped in that they increase sharply at first, remain constant for some time, and gradually disappear. If we extend the model to include an unexpected appearance of wage arrears, we would compute higher welfare losses in the beginning of transition than those predicted by our stationary model because precautionary savings are not present initially. In contrast, including the possibility of an eventual reduction and disappearance of wage arrears at the end of transition would imply lower welfare losses than those produced by our stationary model. Hence, our model tends to understate short-run and to overstate long-run welfare losses due to the stationarity assumption.

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[27] If we allow for different types of agents to reflect systematic variations of wage arrears across sectors, regions and firm ownership, the social welfare loss from wage arrears can be much larger than that calculated in the present paper. Since the utility function is concave, a reduction in either the mean or the variance of wage arrears of some agents at the expense of other agents increases the utility levels of the former agents less than it decreases the utility levels of the latter agents.

Finally, welfare losses from wage arrears are affected by other market imperfections in transition economies that we do not model. First, estimated welfare losses would be reduced if workers suffering from wage arrears are more protected from a drop in their personal income than are unemployed agents. Unlike unemployed agents, workers with wage arrears continue to receive non-wage benefits associated with their job, e.g., housing, child care, and medical treatment. Such job-specific insurance is particularly important in transition economies because outside insurance possibilities are limited. Bank borrowing is not available and unemployment benefits are meager. Second, our welfare-loss estimates would be reduced if enterprises distribute wage arrears among workers to reduce wage discrimination, as Gerry et al. (2004) discuss for gender differences.

5. Conclusion

Wage arrears is a characteristic particular to many transition economies. However, this phenomenon can be investigated in a standard neoclassical growth model, traditionally used to study economic issues relevant to developed market economies. We show that the effect of wage arrears on individual consumption-savings behavior is similar to that of idiosyncratic shocks to productivity. We distinguish two types of costs associated with wage arrears. First, effective wages are reduced because of depreciation of wage arrears; second, a welfare loss arises due to consumption fluctuations. In a version of the model calibrated to Ukrainian data, we find substantial costs in that consumption falls by 8 to 16%, and the welfare loss resulting from idiosyncratic uncertainty is equivalent to an additional consumption loss of 1 to 5%.

Wage arrears is often a survival strategy of loss-making firms during the transition process. In our framework, firms cannot influence the amount of the losses so that wage arrears continue forever. However, even if firms were able to eliminate the loss-making activities at no cost and, hence, resolve the problem of wage arrears, this would not necessarily be the chosen strategy because firms have no incentive to reduce losses as long as they can shift them to their workers. This insight may explain why wage arrears have existed in Ukraine and other former Soviet Union countries for such a long period of time. Following the neoclassical growth model with uninsurable idiosyncratic shocks to earnings, we abstract from the effect of aggregate dynamics on consumption-savings decisions of heterogeneous agents by assuming that the aggregate economy is always in a stochastic steady state. Our model explains how wage arrears are created on impact of negative productivity shocks, why wage arrears perpetuate, and what effects they have on the individual and aggregate allocations. However, we draw no implications concerning the origins of negative productivity shocks in transition economies or the evolution of these shocks over time. Our model generates no transitional dynamics between steady states with and without wage arrears; rather it predicts that the economy jumps instantaneously from the former to the latter as soon as the negative productivity shock disappears.

These limitations result from oversimplified way of modeling the institutional environment and the production side of transition economies. We assume the existence of well-functioning markets and standard profit-maximizing neoclassical firms. Therefore, the only reason for poor economic performance is the presence of exogenous negative

shocks. Hence, these shocks represent all of the institutional shortcomings and production difficulties of transition economies. To gain intuition into the origins of transitory phenomena, we should model explicitly those factors that are responsible for poor economic performance in transition countries. Such factors include underdeveloped financial institutions, opportunistic behavior of managers, and labor-market imperfections. A richer model of transition in which firms can reduce their losses by restructuring or modernizing might have different implications concerning wage arrears. In particular, wage arrears may be a socially desirable phenomenon in that they are essentially a zero-interest loan by workers to firms and, therefore, may stimulate higher levels of investment. Furthermore, making profits and having an outstanding wage debt at the same time is problematic so that profits may be a sufficient incentive for firms to eliminate loss-making activities.

To extend our model to include aggregate transitory effects, we should introduce an explicit relationship between individual and aggregate dynamics. Computing the equilibrium of such a model is complex. To reduce computational expense, Krusell and Smith (1998) employ the numerical aggregation technique, which models the effect of aggregate dynamics on individual allocations by taking a few moments of the wealth distribution. As an alternative method, Maliar and Maliar (2003) combine the analytic aggregation technique with numerical methods. Both approaches rely on the assumption of stationarity, which is not likely to prove adequate for modeling transitory phenomena. Methods for solving non-stationary models are not yet developed, so that these extensions must be left for future research.

Appendix A. The data

The household survey referred to as Ukraine 96 is carried out by Kiev International Institute of Sociology and contains information on 5403 Ukrainian households. From the whole sample, we select a subsample in which household heads are employed. We restrict our attention to wages and wage arrears for the household head from the main job, i.e., the one that yields the largest income. We define the agent's monthly wage as the sum of both monetary and non-monetary compensation, as estimated by the respondents. Specifically, we have:

$$\omega^{\text{month}} = ZH28 + ZH31 + ZH34,$$

where $ZH28$ is net working compensation, $ZH31$ is the estimated cost of goods received, and $ZH34$ is the estimated cost of privileges.

We construct the distribution of yearly wages by bootstrapping. Specifically, we compute the yearly wage as a sum of 12 random draws from the constructed distribution of

monthly wages, i.e., $\omega^{\text{year}} \equiv \sum_{i=1}^{12} \omega_i$, where $\omega_i \sim \{\omega^{\text{month}}\}$. The mean and standard deviation of wages, which we report in Table 1, are computed over the sample with 1,000,000 observations. The mean and standard deviation of wage arrears in Table 1 are for the variable $Z25$, which is the amount of money that the owner or the administration owes to the household head for the main job.

To compute σ_v and corr(q, w), we apply the following iterative procedure. Fix some values of these parameters, $\hat{\sigma}_v$ and $\widehat{\text{corr}}(q, w)$, compute the corresponding value of d_q from (19) and simulate the process $\{q_t, w_t\}_{t=1}^{1,000,000}$ according to (1) and (2). Use the obtained series to compute the standard deviation of q_t and the correlation coefficient between q_t and w_t and denote the results as $\tilde{\sigma}_q$ and $\widetilde{\text{corr}}(q, w)$, respectively. Iterate on the parameters σ_v and corr(q, w) until two criteria are satisfied. First, $\widehat{\text{corr}}(q, w) = \widetilde{\text{corr}}(q, w)$ and, second, $\tilde{\sigma}_q$ coincides with the standard deviation of wage arrears in the Ukrainian household data from Table 1.

To draw the empirical distribution of wealth and wage arrears in Fig. 2, we use the variable $I20$, which is the subjective evaluation of an individual's material status relative to someone else in the city, village, or town. This variable is coded between 1 and 7, to correspond to the following evaluations: "much lower than average," "lower than average," "a bit lower than average," "average," "a bit higher than average," "higher lower than average" and "much higher than average." To construct the educational and profession-qualification groups used for the sensitivity experiments, we chose the variables $E3$ and $ZH18$, which are the education and profession or qualification of the household head, respectively.

Ukrainian aggregate data, e.g., GNP, personal consumption, wages, government expenditures, wage arrears, gross payables, gross receivables, population, CPI, and PPI are quarterly time series from UEPLAC (2001). All the series, except for the one for wage arrears, range from the first quarter of 1994 to the fourth quarter of 2001; the series for wage arrears ranges from the first quarter of 1996 to the fourth quarter of 2001. To convert nominal series into real ones, we use the CPI. We use GNP to measure output in the model, Y.

We define aggregate consumption C as the sum of personal and government consumption. Although time series data on government consumption are not available, UEPLAC (2001) provides a detailed budget of the Ukrainian government for the year 1998, which allows us to estimate the division of total government expenditures between consumption and investment. We define government investment as the sum of government expenditures on R&D, education, construction, health care, telecommunication, transportation and reserve funds. Government consumption is defined as the residual. Using this approach, 48% of government expenditures is investment and 52% is consumption. Therefore, we construct the series for consumption by summing up personal consumption and 52% of total government expenditures. Aggregate labor income is defined as the aggregate wage bill excluding wages paid by collective agricultural enterprises. Estimates of the Ukrainian capital stock are available for the years 1996, 1998, 1999 and 2000 from Derzhcomstat (2000). We convert this variable into real terms by using the PPI.

Appendix B. The numerical algorithm

To solve for the individual asset demand function, we use an algorithm that computes a solution to the Euler equation given by (17) on a grid of pre-specified points. We restrict wage arrears to belong to the interval $q \in [\underline{q}, \bar{q}]$ and split this interval into H equally-spaced points $\{q_1, \ldots, q_H\}$. Then we compute the transitional probabilities associated with the autoregressive process for wage arrears in (9), i.e., the Markov chain. For each state $q \in \{q_1, \ldots, q_H\}$, we parameterize the asset demand by a function of the currently available resources z. The grid for the resources consists of M equally spaced points in the range $[\underline{z}, \bar{z}]$. The minimum value for resources, $\underline{z}$, is obtained when the agent has no asset holdings and receives the minimal possible labor income, $\underline{\omega}$. Hence, the agent faces the maximum possible increase in wage arrears from q_1 to q_H. The maximum value for resources is $\bar{z} = \bar{\omega} + (1 + R)\bar{a}$, where $\bar{a}$ is the maximum sustainable capital stock, i.e., the solution to $F(\bar{a}) = d\bar{a}$, and $\bar{\omega}$ is the maximal possible labor income, i.e., the maximum possible decrease in wage arrears from q_H to q_1. Our construction implies that individual asset holdings are restricted to the range $[0, \bar{a}]$. Our baseline parameterization is $H = 7$ and $M = 100$. To evaluate the asset function outside the grid, we use a cubic polynomial interpolation.

We use an algorithm iterating on the Euler equation. By substituting consumption from (17) to the budget constraint (13), we obtain the following Euler equation:

$$\hat{a}' \leqslant z - \left[\delta \sum_{q \in \{q_1, \ldots, q_H\}} \frac{(1 + R)\,\mathrm{Prob}(q' \mid q)}{(A(z, q)(1 + R) + \omega' - A(A(z, q), q'))^\gamma} \right]^{-1/\gamma}, \qquad \text{(B.1)}$$

where $\omega' = q(1 - d_q) + W - q'$.

Then, we implement the following iterative procedure. At the first step, we fix some asset function on the grid, $A(z, q)$. In the second step, we use the assumed decision rule for assets to calculate the right side of the Euler equation given by (B.1) in each point on the grid. The left-hand side of the Euler equation is the new asset function, $\widetilde{A}(z, q)$. In the third step, we compute the asset function for the next iteration $\widetilde{\widetilde{A}}(z, q)$ by using updating in the following manner:

$$\widetilde{\widetilde{A}}(z, q) = \eta \widetilde{A}(z, q) + (1 - \eta) A(z, q), \quad \eta \in (0, 1].$$

For each point, such that $\widetilde{\widetilde{A}}(z, q)$ does not belong to $[0, \bar{z}]$, we set $\widetilde{\widetilde{A}}(z, q)$ equal to the corresponding boundary value.[28] The procedure continues by iteration of these three steps until a fixed point is achieved with a given degree of precision, $\|\widetilde{\widetilde{A}}(z, q) - A(z, q)\| < 10^{-10}$, where $\|\cdot\|$ is the L^2 distance.

Following Ríos-Rull (1999), we solve for the interest rate and the wage corresponding to a given asset function $A(z, q)$ by computing an invariant probability distribution of the

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[28] Maliar and Maliar (2004b) show that by imposing a lower bound on the asset demand function, we guarantee non-negativity of the Lagrange multiplier associated with the borrowing constraint. An upper bound should be sufficiently high not to affect the properties of the solution.

total resources and wage arrears, $\text{Prob}(z, q)$, given by:

$$\text{Prob}(z', q') = \sum_{q \in \{q_1, \ldots, q_H\}} \text{Prob}\big(A^{-1}(\hat{a}', q), q\big) \cdot \text{Prob}(q' \mid q),$$

$$\hat{a}' = \frac{z' - (q(1 - d_q) + W - q')}{1 + R},$$

where $A^{-1}(\hat{a}', q) = \{z, \hat{a}' = A(z, q)\}$ is the inverse of the asset demand function given by $A(z, q)$. Finally, to solve for the equilibrium fixed-point interest rate, i.e., the stochastic steady state, we use a bisection method proposed in Aiyagari (1994).

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