Information Sale and Competition

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This paper studies the strategic interaction between a monopolistic seller of an information product and a set of potential buyers that compete in a downstream market. Our analysis illustrates that the nature and intensity of competition among the information provider's customers play first-order roles in determining her optimal strategy. We show that when the customers view their actions as strategic complements (such as in Bertrand competition), the provider finds it optimal to offer the most accurate information at her disposal to all potential customers. In contrast, when buyers view their actions as strategic substitutes (for example, when they compete with one another *à la* Cournot), the provider maximizes her profits by either (i) restricting the overall supply of the information product, or (ii) distorting its content by offering a product of inferior quality. We also establish that the provider's incentive to restrict the supply or quality of information provided to the downstream market intensifies in the presence of information leakage.

Key words: information markets, competition, oligopolies.

1. Introduction

Recent advances in information technology have streamlined the process of mining, aggregating, and processing high volume data about economic activity. Arguably, it is widely believed that the availability of more accurate information about the business environment and market conditions can be hugely beneficial to firms across a wide variety of industries. Such a realization has in turn led to a sizable demand for Business-to-Business information services. Several firms ranging from Nielsen to Thomson Reuters and IRI have built their business models around collecting, customizing, and selling information products to other market participants. For example, the market research firm IRI offers its customers a variety of consumer, shopper, and retail market analyses focused on the consumer packaged goods industry, whereas the Economist Intelligence Unit sells industry-wide market analysis reports.

Motivated by the growing interest in the markets for information, this paper studies the problem of the optimal sale of information (such as demand forecasts) to a set of competing firms. We show that the nature and intensity of competition among the information provider's potential customers have a first-order impact on her optimal selling strategy and profits. More specifically, our analysis illustrates that the value the provider can extract from her customers is largely determined by the trade-off between (i) the direct (positive) effect of more precise information on the customers' profits by enabling them to make more informed decisions; and (ii) the strategic effects that arise due to the fact that the provider's customers may interact with one another in other markets.

We present our main findings in the context of an environment that involves a monopolistic information provider who can sell potentially informative signals to a collection of firms that compete with one another in a downstream market. More specifically, we assume that the customer firms face demand uncertainty and that the provider is endowed with a private signal that is (partially) informative about the actual demand realization, thus creating potential gains from trade. Crucially for our argument—and in line with the observation that many real-world information providers offer a variety of information products of varying qualities—we allow for a setting in which the provider can offer information products that are potentially less precise than her private information. In other words, the provider can potentially distort the informativeness of the signal at her disposal by reducing its accuracy.

As our main result, we show that the optimal selling strategy of the provider is largely dependent on the nature and intensity of competition among her potential customers in the downstream market. More specifically, we first show that when firms engage in price competition (Bertrand), the provider finds it optimal to sell her signal with no distortion to the entire set of firms. This is due to the fact that in a Bertrand market, firms' actions are strategic complements and hence, each firm's marginal benefit of procuring a more accurate signal is increasing in the fraction of its competitors that purchase the provider's information product. Therefore, the provider would obtain maximal profits by flooding the market with highly precise signals.

The situation, however, can be dramatically different if the information provider's customers compete with one another in quantities (Cournot). For such a downstream market, we show that the provider may no longer find it optimal to sell an undistorted version of her signal to all firms. Rather, she may find it optimal to either (i) reduce the quality of her information product by selling a signal of a lower precision than the one she possesses; (ii) strategically limit her market share by excluding a subset of her customers from the sale; or (iii) employ both strategies simultaneously by reducing the quality *and* quantity of the products offered. The optimality of these "information-distorting strategies" is due to the fact that in a Cournot market, firms' actions are strategic substitutes, which leads to the emergence of two opposing effects. On the one hand, obtaining additional information about demand directly benefits firms as they can better align their production decisions with underlying market conditions. On the other hand, however, the provider's signal can also serve as a correlating device among her customers' equilibrium actions. In particular, providing the information product to an extra firm can only increase the correlation in the firms' production decisions, an outcome that reduces each firm's profits and hence, can adversely affect the provider's bottom line. Therefore, when downstream competition is intense enough (for example, when firms' products are sufficiently substitutable), this latter, strategic channel would dominate the positive effect of reducing demand uncertainty, implying that the information provider would be better off by restricting the quantity and/or quality of her information products. Interestingly, unlike in Bertrand competition, the provider's profits in a Cournot market are decreasing in the intensity of competition and may end up being significantly lower than in the absence of any competition.

To further clarify the forces that underpin our results, we also discuss a number of extensions to our benchmark setup. First, we let the provider offer a menu of information products with potentially different precisions and at different prices. We provide an explicit characterization of the optimal selling strategy as a function of the nature and intensity of competition and show that when firms compete in quantities and offer substitutable products, there is a continuum of strategies that lead to the same equilibrium profits for the provider. This characterization thus formalizes the trade-off in the provider's incentives for reducing the quantity or quality of her information product. Second, we extend our benchmark framework by allowing for the possibility of information leakage among the provider's customers. In particular, we assume that, by observing the decisions of their competitors, firms can partially infer the information content of the signal purchased by other firms, thus altering their own willingness-to-pay for the information provider's signal. We establish that the provider's incentive for reducing the quality and/or quantity of her information product increases as the extent of information leakage among her customers is intensified. Third, we explore the implications of firm heterogeneity for the provider's selling strategy by considering a setting in which firms differ in their production costs. We show that it is optimal for the provider to sell higher precision information products (at higher prices) to the more efficient firms, i.e., the firms that have lower production costs. Lastly, we establish that our main qualitative insights carry over to a market consisting of finitely many firms and show that our benchmark results can be obtained as the limit of a finite market with the number of firms growing asymptotically large.

Taken together, these findings provide a step towards understanding the intricacies involved in markets for information. Unlike traditional markets for physical goods, it is relatively inexpensive to offer a diverse menu of information products that differ in their precision and pricing. Our results highlight that the value that a given buyer can extract from procuring such products depends not only on the product's characteristics (such as its price and precision) but also on the environment in which the information provider's customers interact with one another.

Our modeling framework provides several qualitative insights on how, in shaping her pricing policy, an information provider may optimally take the strategic interactions among her customers into account. We believe that these insights can be of particular relevance to real-world information markets in which (i) information providers have the ability to sell signals of different precisions (at potentially different prices) to their customers and (ii) information affects the customers' actions and profits through two channels: on the one hand, more precise information enables a firm to take an action that more closely matches the realized uncertainty; on the other hand, the firm also interacts strategically with the rest of the market participants. Potential examples include the multitude of consumer, shopper, and retail market analyses of varying precision offered by firms such as IRI and Nielsen to the consumer packaged goods industry[1] as well as the expansive menus of information products that financial data providers (such as Bloomberg and Thomson Reuters) make available to their customers. Besides the obvious case of differentiating their data based on its granularity (say, its coverage or level of aggregation), financial data providers also use frequency as a dimension to differentiate their information products. For instance, in the context of the U.S. macroeconomic data announcements by various government agencies at prescheduled dates (such as monetary policy announcements by the Federal Reserve or non-farm employment numbers released by the Bureau of Labor Statistics), Kurov et al. (2016) argue that some private data providers release information to exclusive groups of subscribers before making it available to others, with the documented early releases in the range of seconds.[2] Thus, to the extent that slightly out-dated information can be considered as information of lower quality (e.g., due to fast-moving market conditions), such an environment also exhibits the key features of our model.

Related Literature. Our paper is related to the extensive literature that studies firms' strategic considerations in sharing information with one another in oligopolistic markets. For example, Vives (1984), Gal-Or (1985), Li (1985), and Raith (1996) provide conditions under which firms find it optimal to share their private information about market conditions with their competitors. A more recent collection of papers, such as Shin and Tunca (2010), Shamir (2012), Shamir and Shin (2016), Ha and Tong (2008), and Ha, Tong, and Zhang (2011) studies information sharing incentives in vertical supply chains. For instance, Shamir and Shin (2016) determine conditions under which firms can credibly share their demand forecasts with one another, whereas Cui, Allon, Bassamboo, and Van Mieghem (2015) provide a theoretical and empirical assessment of the value

[1] IRI offers an array of information products at different price points. For example, the Basic "Market Advantage Solution" includes a summary of industry sales and a detailed analysis of pricing strategies employed by a firm's competitors. The Premium "Market Advantage Solution", on the other hand, provides a more in-depth analysis of sales and competitors' pricing strategies along with more specialized analytics services. The Basic product is priced around \$10,000 whereas the price for the Premium offering can range between \$100,000 and \$500,000.

[2] Also see The Wall Street Journal (2013) for another example.

of information sharing in two-stage supply chains. In contrast to this literature, which for the most part focuses on firms' incentives to fully share the information at their disposal with one another, we consider a setting in which a *third-party* decides not only the price but also the *accuracy* of the information product(s) she makes available to a set of competing firms. This allows for richer equilibrium outcomes that highlight the interplay between the nature of competition, the optimal selling strategy, and the information provider's profits.

Our paper is also related to the literature, such as Li and Zhang (2008), Anand and Goyal (2009), and Kong, Rajagopalan, and Zhang (2013), that studies the implications of indirect leakage of information in supply chains via firms' actions. Similar considerations have also been studied in the context of financial markets (Admati and Pfleiderer 1990). Building on the framework of Vives (2011), we show how the intensity of information leakage in the market impacts firms' valuation of information and hence alters the provider's incentives in designing her information products.

Our work is also related to the growing theoretical literature on the social and equilibrium value of public information. Morris and Shin (2002) illustrate that public disclosure of information regarding a payoff-relevant parameter may adversely affect social welfare as it may crowd out agents' reliance on their private information. Angeletos and Pavan (2007) extend this framework and provide a complete taxonomy of conditions under which private and public signals are efficiently utilized in equilibrium.[3] Relatedly, Bergemann and Morris (2013) study games of incomplete information with the goal of providing equilibrium predictions that are robust to all possible information structures. Their analysis illustrates that information disclosure policies that involve a partial sharing of a firm's private information may lead to higher equilibrium payoffs.

Also related is the recent work of Myatt and Wallace (2015), who consider a setting in which a set of firms compete in a Cournot market by selling differentiated products to a representative consumer. They characterize the weights firms assign to the private and public signals at their disposal as functions of the signals' precisions, the intensity of the competition, and the extent of product differentiation. They also establish that when signals are costly, firms acquire too much information relative to the socially efficient benchmark. In contrast to their paper, our main focus is on the provider's incentives to reshape the quantity and quality of information sold to the firms.

Finally, our work is related to the more recent work of Bergemann and Bonatti (2015), who explore selling information in the form of cookies in the context of online advertising, as well as Xiang and Sarvary (2013) who consider a market for information with competition on both the demand and supply sides of the market. In a similar context, Babaioff et al. (2012) study the design of optimal mechanisms for a data provider to sell information to a single buyer.

[3] Interestingly, Chen and Tang (2015) study the value of market information for farmers in developing economies.

2. Model

Firms: Consider an economy consisting of a unit mass of firms indexed by $i \in [0,1]$ that compete with one another in a downstream market. Each firm i takes an action $a_i \in \mathbb{R}$ in order to maximize its profit which is given by the following expression

$$\pi(a_i, A, \theta) = \gamma_0 a_i \theta + \gamma_1 a_i A - \frac{\gamma_2}{2} a_i^2, \tag{1}$$

where $A = \int_0^1 a_i di$ denotes the aggregate action taken by the firms, $\theta \in \mathbb{R}$ is an unknown payoff-relevant parameter, and $\{\gamma_0, \gamma_1, \gamma_2\}$ are some exogenously given constants. Depending on the context, action a_i may represent the quantity sold or the price set by firm i. As we will show in Subsection 2.2, the above framework nests Cournot and Bertrand competition as special cases. For the time being, however, we find it more convenient to work with the general setup above without taking a specific position on the mode of competition.

The unknown parameter θ is randomly drawn by nature before firms choose their actions. As we will discuss in the following subsections, this parameter can represent the intercept of the (inverse) demand curve in the downstream market. All firms hold a common prior belief on θ, which for simplicity we assume to be the (improper) uniform distribution over the real line.[4] Even though firms do not know the realization of θ, each firm i observes a noisy private signal

$$x_i = \theta + \epsilon_i \quad , \quad \epsilon_i \sim N(0, 1/\kappa_x),$$

with κ_x capturing the precision of the private signal observed by each firm. The noise terms ϵ_i are independently distributed across firms. Given firm i's profit function in (1), we let

$$\beta = -\frac{\partial^2 \pi}{\partial a \partial A} \Big/ \frac{\partial^2 \pi}{\partial a^2} = \frac{\gamma_1}{\gamma_2}, \tag{2}$$

denote the degree of strategic complementarity in firms' actions. Note that $\beta > 0$ corresponds to an economy in which firms' actions are strategic complements: the benefit of taking a higher action to firm i increases the higher the actions of other firms are. In contrast, when $\beta < 0$, firms face a game of strategic substitutes, where i's incentives for taking a higher action decrease with the aggregate action A. Finally, $\beta = 0$ corresponds to a market in which firms face no strategic interactions.

Throughout the paper, we assume that $\gamma_2 > \max\{2\gamma_1, 0\}$. This assumption, which implies that $\beta \in (-\infty, 1/2)$, is made to guarantee that firm i's profits are strictly concave in a_i and that i's marginal profit is more sensitive to its own action a_i than to the aggregate action A.

[4] More formally, suppose that θ is distributed according to a Gaussian distribution with mean 0 and variance σ_θ^2. By letting $\sigma_\theta \to \infty$, we obtain a distribution with full support over $(-\infty, \infty)$ that, in the limit, assigns the same probability to all intervals that have the same Lebesgue measure.

Information Provider: In addition to the competing firms, the economy contains a monopolist who possesses some private information about the realization of the unknown parameter θ that it can potentially sell to the firms before they take their actions. The provider has access to a private signal z with precision κ_z given by

$$z = \theta + \zeta \quad , \quad \zeta \sim N(0, 1/\kappa_z),$$

where the noise term ζ is independent of ϵ_i's. Given that our main focus is on the market for information, we assume that this signal has no intrinsic value to the provider and that she can only benefit from the signal by selling it to the firms.

The key feature of our model is that the provider has control over both the "quantity" and "quality" of information sold to the firms: the information provider not only chooses the set of firms $I \subseteq [0,1]$ that she decides to trade with, but can also choose the precision of the signal offered to the firms. More specifically, she offers a signal

$$s_i = z + \xi_i \quad , \quad \xi_i \sim N(0, 1/\kappa_\xi),$$

to firm $i \in I$ at price p_i, where ξ_i is independent from z and $1/\kappa_\xi$ captures the variance of the noise introduced by the provider into s_i. This specification thus captures the idea that the provider can control the quality of the information sold to the firms: by choosing a smaller κ_ξ, the provider can "damage" the signals offered to the firms.[5] Throughout the paper, we refer to s_i as the *market signal* sold to firm i.

In general, the noise added to different firms' signals by the provider may be correlated with one another. To capture this idea formally, we assume that in addition to their precision κ_ξ, the provider can also determine the correlation between different firms' market signals by setting $\rho_\xi = \text{corr}(\xi_i, \xi_j) \in [0,1]$. Our specification thus accommodates situations in which the provider offers identical or conditionally independent signals to any subset of the firms as special cases.

Putting the above together, the market signal s_i offered to firm $i \in I$ can be rewritten as

$$s_i = \theta + \eta_i \quad , \quad \eta_i \sim N(0, 1/\kappa_s) \quad \text{and} \quad \text{corr}(\eta_i, \eta_j) = \rho,$$

where $\kappa_s = (1/\kappa_z + 1/\kappa_\xi)^{-1}$ is the signal's precision and $\rho = (\kappa_\xi + \rho_\xi \kappa_z)/(\kappa_\xi + \kappa_z)$. By construction, signals sold by the provider cannot be more precise than the information she possesses, i.e., $\kappa_s \leq \kappa_z$.

We remark that given firms' *ex ante* symmetry, we can assume, without loss of generality, that $I = [0, \lambda]$, where $\lambda \in [0,1]$ captures the fraction of firms that the information provider decides to

[5] Note that in our baseline setting, the provider offers a signal of the same precision to all firms $i \in I$; that is, κ_ξ is independent of i. We relax this assumption in Section 4 and show that all our insights are robust to this assumption.

trade with. Also note that even though we assume that the seller chooses the fraction of firms she wants to trade with before offering them her information products, as we show in Section 4, our setting is isomorphic to an environment in which the provider announces the features of her product(s)—i.e., price and precision—and firms subsequently decide whether to purchase them.

Finally, with some abuse of terminology, we refer to the firms who purchase the market signal s_i as *informed firms*, whereas firms that were denied the signal or decided not to purchase it from the information provider are simply referred to as being *uninformed*.

2.1. Contracts and Equilibrium

Once the seller's and the firms' private signals are realized, the former has the option to sell potentially informative signals about θ to the latter. To capture this idea formally, we assume that the information provider makes a take-it-or-leave it offer $(\kappa_\xi, \rho_\xi, p_i)$ to a fraction λ of the firms, where κ_ξ captures the quality of the market signal offered to firm i and p_i is the corresponding firm-specific price.

Following the seller's offer, each firm $i \in [0, \lambda]$ then decides whether to accept $(b_i = 1)$ or reject $(b_i = 0)$ its corresponding offer. This stage is then followed by the *competition subgame* between the firms in which they choose their actions a_i. Note that whereas the strategy of an uninformed firm i is a mapping from its private signal x_i to an action, the strategy of an informed firm maps the pair (x_i, s_i) to an action. We have the following standard solution concept:

Definition 1. A perfect Bayesian equilibrium consists of a strategy $(\lambda, \kappa_\xi, \rho_\xi, \{p_i\}_{i\in[0,\lambda]})$ for the information provider, acceptance/rejection decisions $b_i \in \{0, 1\}$ for each firm i, a posterior belief μ_i for each firm i, firm-specific strategies a_i, and an aggregate action A such that

(i) the information provider chooses $(\lambda, \kappa_\xi, \rho_\xi, \{p_i\}_{i\in[0,\lambda]})$ to maximize her expected profit;
(ii) firm $i \in [0, \lambda]$ accepts the information provider's offer only if doing so maximizes its profit;
(iii) each firm's posterior belief on θ is obtained via Bayes rule, conditional on its information set;
(iv) given its posterior belief, each firm i maximizes its expected payoffs in the competition subgame, taking the strategies of all other firms as given;
(v) the aggregate action A is consistent with individual firm-level actions.

2.2. Examples

As already mentioned, Cournot and Bertrand competition can be derived as special cases of our general framework above. This feature of the model enables us to provide a comparison of the optimal information selling strategies in markets with different modes and intensities of competition. The following simple examples illustrate how in the presence of linear demand functions, various forms of competition can induce quadratic profit functions in the form of Equation (1). We will use these examples in the subsequent sections to discuss the implications of our results for the optimal trading strategies of the information provider.

Example 1 (Cournot competition). Consider a market in which firms sell a possibly differentiated product to a downstream market and compete by setting quantities. Firm i faces an inverse demand function given by

$$r_i = \gamma_0 \theta - (1-\delta)Q - \delta q_i, \tag{3}$$

where q_i is the quantity sold by firm i, $Q = \int_0^1 q_i di$ is the aggregate quantity sold to the downstream market, and θ is a "demand shifter" that captures the intercept of the (inverse) demand curve. In this setting, $\delta \in [0,1]$ represents the degree of product differentiation among firms, as a smaller δ corresponds to a more homogenous set of products.[6] Assuming that firms' marginal cost of production is zero, it is then immediate that their profit function $\pi_i = r_i q_i$ is simply a special case of our framework in (1), with action a_i representing the quantity sold by firm i.

Note that in this environment, the degree of strategic complementarity defined in (2) is equal to $\beta = (\delta-1)/2\delta < 0$, thus implying that firms face a game of strategic substitutes. Parameter β also captures the intensity of competition between the firms. In particular, given that β is increasing in δ, a larger β corresponds to a market in which products are more differentiated. In the extreme case that $\beta \to 0$, the products are no longer substitutes and each firm essentially becomes a monopolist in its own market. At the other extreme, as $\beta \to -\infty$, the products become perfect substitutes and the oligopoly converges to a perfectly competitive market.

Example 2 (Bertrand competition). Next, consider a market in which firms compete in prices and face a linear demand function given by $q_i = \gamma_0\theta + (\phi-1)R - \phi r_i$, where r_i is the price set by firm i and $R = \int_0^1 r_i di$ is the average price in the market. Note that this demand system can be obtained by inverting (3) and setting $\phi = 1/\delta > 1$. Once again, it is immediate that firm i's profit function $\pi_i = r_i q_i$ would coincide with (1), where action a_i now represents the price set by firm i. Furthermore, it is straightforward to verify that, in this environment, $\beta = (\phi-1)/2\phi > 0$, thus implying that the competition game between the firms exhibits strategic complementarities, the degree of which is increasing in ϕ.

Example 3. Once again consider the Cournot competition setting described in Example 1, but instead suppose that firms produce homogeneous products, i.e., $\delta = 0$, and have quadratic production costs given by $c(q_i) = q_i^2/2$. The profit of firm i is then given by $\pi(q_i, Q, \theta) = \gamma_0 q_i \theta - q_i Q - q_i^2/2$, which again fits within our general framework.

We conclude this section by remarking that even though, for the sake of tractability and expositional simplicity, we focus on an environment consisting of a continuum of firms, as we show in Section 7, all our results and insights carry over to a setting consisting of finitely many firms. We also

[6] See Myatt and Wallace (2015) for micro-foundations for this demand system.

note that, when dealing with a continuum of firms, we assume that a variant of the "exact law of large numbers" guarantees that the cross-sectional average of firm-level variables (such as firms' quantity or price decisions) coincide with the corresponding variables' expectations almost surely.[7]

3. Optimal Sale of Information

In this section, we present our main results and characterize the information provider's optimal information selling strategy. Our results show that the seller's strategy is highly sensitive to the mode and intensity of competition in the downstream market as expressed by β.

3.1. Competition Subgame

We start our analysis by studying the equilibrium in the competition subgame between the firms once the contracts offered by the information provider are accepted or rejected. Without loss of generality, let $[0,\ell]$ denote the set of firms who accept the seller's offer, where, clearly, $\ell \leq \lambda$.

PROPOSITION 1. *The competition subgame between the firms has a unique Bayes-Nash equilibrium in linear strategies. Furthermore, the equilibrium strategies of the firms are given by*

$$a_i = \begin{cases} \alpha[(1-\omega)x_i + \omega s_i] & \text{if } i \in [0,\ell] \\ \alpha x_i & \text{if } i \in [\ell,1] \end{cases},$$

where

$$\omega = \frac{\kappa_s}{(1-\beta\ell\rho)\kappa_x + \kappa_s} \qquad \text{and} \qquad \alpha = \gamma_0/(\gamma_2 - \gamma_1).$$

Proposition 1, which is in line with Angeletos and Pavan (2007) and Myatt and Wallace (2015), provides a characterization of the firms' equilibrium strategies in the competition subgame and serves as a preliminary result for the rest of the results in the paper. It states that the equilibrium action of an informed firm is a weighted sum of its original private signal and the signal it obtains from the information provider. More importantly, however, it shows that the weights firm i assigns to its two signals not only depend on their relative precisions, but also on the fraction of informed firms, ℓ, as well as correlation ρ in the market signals. Furthermore, the equilibrium weight that each informed firm assigns to the market signal s_i is increasing in the degree of strategic complementarities β, regardless of the values of ρ and ℓ. This is due to the fact that in the presence of stronger strategic complementarities, firms have stronger incentives to coordinate with one another, and as a result, rely more heavily on their market signals, which can function as (imperfect) coordination devices. On the other hand, in the absence of strategic considerations

[7] We provide the formalism and the required conditions for such a variant of the law of large numbers in the electronic companion of the paper. For a thorough treatment of the subject, see Sun (2006) and Sun and Zhang (2009).

(i.e., when $\beta = 0$), the optimal strategy of all firms would be independent of ℓ and ρ, making the weight assigned to each signal proportional to its relative precision.

Relatedly, Proposition 1 also establishes that for a given positive (negative) β, the equilibrium weight that informed firms assign to their market signals is increasing (decreasing) in ℓ and ρ. To see the intuition underlying this, suppose that $\beta > 0$ (the argument for $\beta < 0$ is identical). In such an environment, firms face a game of strategic complements, as for example would be the case if they compete *à la* Bertrand. Given that firms value coordinating their actions, an informed firm i assigns a higher weight to its market signal—above and beyond what its relative precision would justify—the more other firms base their own decisions on the signal sold by the provider (i.e., higher ℓ) and the more informative s_i is about the signals of other firms (i.e., higher ρ).

With Proposition 1 in hand, in the remainder of this section, we turn to the the seller's problem and characterize her optimal information selling strategy as a function of the mode and intensity of competition in the downstream market. In order to present our results in the most transparent manner, we study Bertrand and Cournot competition separately.

3.2. Bertrand Competition

First, consider the case in which firms compete with one another *à la* Bertrand. As already mentioned in Example 2, such a market corresponds to a special case of our general framework with $\beta > 0$. Also, recall that the information provider needs to choose the fraction of firms with whom she trades (λ), the precision of the signal offered to the firms (κ_s), and the correlation induced in the noise terms (ρ_ξ). We have the following result:

Proposition 2. *If $\beta > 0$, the information provider sells her signal without any distortions to all firms; that is, $\kappa_s^* = \kappa_z$ and $\lambda^* = 1$. Furthermore, the provider's expected profit is given by*

$$\Pi^* = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z + \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2}. \tag{4}$$

The above result thus establishes that under Bertrand competition, it is always optimal for the provider to sell her signal z to the entire set of firms without any additional noise. To understand the intuition underlying this result, recall that in a Bertrand market, the firms' actions are strategic complements: setting a lower price becomes more attractive the lower the prices of other competing firms are. Such strategic complementarities induce a strong coordination motive among the firms. Therefore, providing the market signal to an additional marginal firm, not only increases the profits of the seller directly (via sales to that new marginal firm), but also increases the surplus of all other firms who have already acquired the signal. This extra surplus can thus be appropriated by the seller via higher prices, leading to even higher profits. Consequently, the information provider always finds it optimal to sell to the entire market of firms. An identical argument then shows that the provider would not distort the signal either: sharing a more precise signal with a new firm increases the value of the market signal to the rest of the informed firms.

Proposition 2 also characterizes the expected profit of the seller. From (4), it is easy to verify that Π^* is increasing in the quality of information available to the monopolist (κ_z), but is decreasing in the precision of the firms' private signals (κ_x). The intuition underlying these observations is simple. Given that the information provider always has the option to reduce the precision of the signals it offers to the firms, her profits can never decrease by having access to a more precise signal. On the other hand, however, the extra benefit of the market signal to the firms is lower the more informed they are to begin with, thus reducing the provider's expected profits.

More importantly, however, (4) also shows that the monopolist's expected profit increases in the degree of strategic complementarities β. Recall from Example 2 that $\beta=(\phi-1)/2\phi$, where $1/\phi=\delta$ is the degree of product differentiation among the firms. Therefore, increasing β is essentially equivalent to a lower degree of product differentiation, and hence, more intense competition. Thus, as β increases, coordination becomes more important to the firms, increasing the value of the seller's signal which in turn leads to higher expected profits.

As a final remark, note that since it is never optimal for the information provider to add noise to the signals, the correlation $\rho_\xi=\text{corr}(\xi_i,\xi_j)$ is immaterial for her profits.

3.3. Cournot Competition

We next focus on the case in which firms compete with one another *à la* Cournot. Recall from Example 1 that such a market is a special case of our general setup with $\beta<0$. In this case, firms choose quantities and their actions are strategic substitutes. Note that, unlike the case of Bertrand competition, firms no longer value coordination *per se*. The following two propositions provide a characterization of the optimal information selling strategy of the monopolist as a function of the degree of strategic substitutability among the actions of downstream firms.

PROPOSITION 3. *If $-(1+\kappa_z/\kappa_x)\leq\beta<0$, the information provider sells her signal without any distortions to all firms; that is, $\kappa_s^*=\kappa_z$ and $\lambda^*=1$. Furthermore, the provider's expected profit is*

$$\Pi^*=\alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z+\kappa_x}{[(1-\beta)\kappa_x+\kappa_z]^2}. \tag{5}$$

Thus, in a Cournot market with a weak enough intensity of competition, the seller finds it optimal to follow the same strategy as in a Bertrand market: sell an undistorted version of her signal to the entire set of firms. The intuition underlying this result is straightforward: acquiring information about the demand intercept (θ) allows each firm i to better match its supply decision to the underlying demand and as a consequence, to increase its profit. The monopolist can then appropriate the increase in i's sales by demanding a higher price for her signal. Therefore, the provider is always better off by making the most precise version of her signal available to all firms.

Even though the seller follows the same strategy as in the Bertrand market, comparing expressions (4) and (5) implies that her expected profit is lower under Cournot competition ($\beta < 0$). This is due to the fact that unlike Bertrand competition, firms do not have an incentive to coordinate their actions, undermining the role of the market signal as a coordination device.

Interestingly, the predictions of Propositions 2 and 3 no longer hold if the intensity at which downstream firms compete in a Cournot market is high. We have the following result:

PROPOSITION 4. *If $\beta < -(1+\kappa_z/\kappa_x)$, the information provider maximizes her expected profit by following any information selling strategy that is a solution to the following equation:*

$$(\kappa_z + \beta\lambda^* \kappa_s^*)\kappa_x + \kappa_z \kappa_s^* = 0. \tag{6}$$

Furthermore, her expected profit is given by

$$\Pi^* = -\alpha^2 \left(\frac{\gamma_2}{2}\right) \frac{\kappa_z}{4\beta\kappa_x^2}. \tag{7}$$

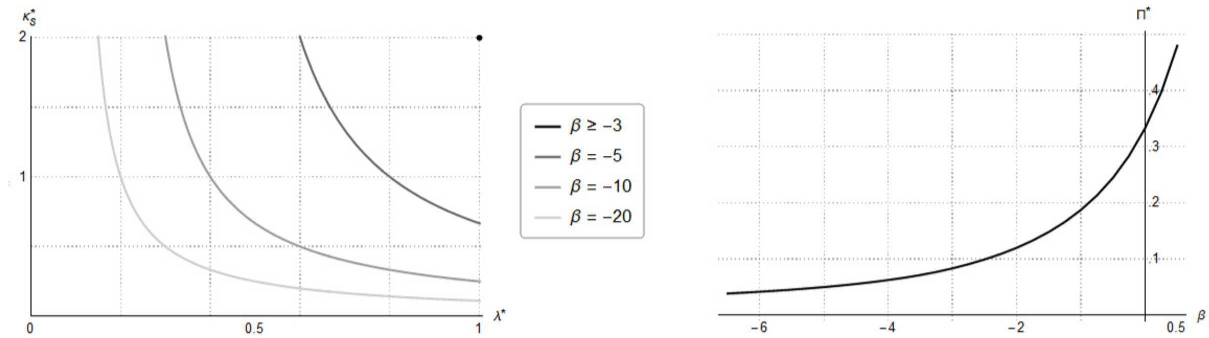
The key observation here is that the pair $\kappa_s^* = \kappa_z$ and $\lambda^* = 1$ does not satisfy (6), leading to the following corollary:

COROLLARY 1. *Suppose that $\beta < -(1+\kappa_z/\kappa_x)$. Then, either $\kappa_s^* < \kappa_z$ or $\lambda^* < 1$.*

Therefore, when firms compete with one another *à la* Cournot and offer goods that are strong substitutes—corresponding to a large enough negative β—it is optimal for the seller to distort the information ($\kappa_s^* < \kappa_z$) and/or exclude a fraction of the firms from the sale ($\lambda^* < 1$).

To see the intuition underlying the above result, recall that in a Cournot market, firms' actions are strategic substitutes, i.e., increasing a firm's supply leads to higher marginal profit the lower the supply decisions of its competitors are. Therefore, providing the market signal to an additional firm i affects its profit through two distinct channels. On the one hand, a more precise market signal enables i to better match its supply to the realized demand. On the other hand, however, making such a signal available to i increases the correlation in the firms' actions, as now i's action would be more correlated with the market parameter θ. The presence of this second effect implies that the strategic value of the seller's signal to firm i, and consequently, i's willingness-to-pay for it are decreasing in the fraction of firms that accept the provider's offer. In the presence of sufficiently intense competition (i.e., when the firms offer sufficiently substitutable products), this strategic effect dominates the first effect, thus making it profitable for the information provider to restrict her offer to a strict subset of the firms ($\lambda^* < 1$).

Figure 1 Optimal selling strategy for different levels of β (left); Equilibrium profit as a function of β (right). We use the following set of parameters for this example: $\alpha = \gamma_2 = 1$ and $\kappa_x = 1, \kappa_z = 2$.



By Proposition 4, an alternative optimal strategy for the monopolist would be to distort the information she sells to the market. In fact, as equation (6) suggests, the fraction λ of the firms that the monopolist trades with and the precision κ_s of the signal offered to the firms are substitutes: as the monopolist increases her market share, she finds it optimal to increasingly distort the signals.

Note that equation (7) in Proposition 4 indicates that the information provider's expected profit decreases in the degree of strategic substitutability ($|\beta|$) of the firms' actions. This is a consequence of the fact that the strategic value of the seller's signal, and hence, a firm's willingness-to-pay decrease as the market becomes more competitive. This is in contrast with the case of Bertrand competition where the seller's expected profit increases with the intensity of competition, as her customers have a stronger incentive to purchase the market signal and coordinate their actions.

We also remark that regardless of the value of β and the strategy adopted by the information provider, she never has an incentive to introduce correlation into market signals, i.e., it is always optimal to set $\rho_\xi^* = 0$. Increasing the correlation in the signals provided to downstream firms would invariably increase the correlation among their actions and lead to lower profits for the seller.

Finally, note that the threshold $-(1+\kappa_z/\kappa_x)$ at which the seller finds it optimal to limit her market share and/or strategically distort the market signal is decreasing in the ratio κ_z/κ_x, implying that the more informed the information provider is relative to her customers, the more likely it is that she will be able to fully exploit her informational advantage by selling it to the entire market of firms without distortion.

Figure 1 illustrates the optimal selling strategy and the equilibrium profit of the information provider for the following set of parameters: $\alpha = \gamma_2 = 1$, $\kappa_x = 1$, and $\kappa_z = 2$. For these parameters, it is immediate to verify that the threshold at which the seller finds it optimal to strategically distort the market signal is equal to $-(1+\kappa_z/\kappa_x) = -3$. Indeed, as the left panel of Figure 1 illustrates, for values of β greater than this threshold, the provider sets the precision of the market signal to

	$\beta = 0$	$\beta = -3$	$\beta = -5$	$\beta = -10$	$\beta = -20$
Π^*_β/Π^*_0	1	.250	.150	.075	.038
$\Pi^{\textbf{no-dist}}_\beta/\Pi^*_0$	1	.250	.141	.053	.017
Increase in Profits (%)	0%	0%	6.67%	40.83%	120.42%

Table 1 Profits under the optimal information selling strategy over selling the signal undistorted to the market.

$\kappa^*_s = \kappa_z = 2$, i.e., she does not distort the information she has at her disposal, and does not exclude any firms from the sale ($\lambda^* = 1$). On the other hand, for $\beta < -3$, the seller finds it optimal to distort the information she sells and limit her market share. The right panel of Figure 1 illustrates how the provider's profit varies with the intensity of competition. Note that the seller is better off when firms view their actions as strategic complements ($\beta > 0$) as opposed to strategic substitutes.

We conclude this section by exploring the extent to which an information provider can increase her profits by strategically distorting the information she provides to her downstream customers and/or limiting her market share. Table 1 provides a comparison of the provider's profit under the optimal selling strategy (Π^*_β) to the profits of a provider who sells her signal to the entire market with no distortion ($\Pi^{\text{no-dist}}_\beta$). We benchmark Π^*_β and $\Pi^{\text{no-dist}}_\beta$ against the profits for a provider that follows her optimal strategy in the absence of competition, i.e., when $\beta = 0$. The first two rows of the table highlight the effect of competition intensity on the providers's profits. More importantly, however, as the bottom row of the table indicates, the provider earns significantly higher profits under competition when she distorts her market signal and/or limits her market share: the increase in her profits by following the strategy characterized in Proposition 4 ranges from 6.67% to 120.42% as the extent to which firms view their actions as strategic substitutes increases.

4. Information Quality Discrimination

In our baseline model presented in Section 2 and analyzed in Section 3, we assumed that the information provider can only offer a single product to the entire market, in the sense that she offers a market signal of the same precision to all firms. In this section, we relax this assumption by allowing the seller to offer signals that potentially differ in both price and precision.

Formally, we assume that the information provider offers (κ_{si}, p_i) to each firm $i \in [0,1]$, specifying the signal precision κ_{si} and price p_i. The seller cannot offer a signal of a higher precision than her own private signal, that is, $\kappa_{si} \leq \kappa_z$ for all i. The following result, which generalizes Propositions 2–4, shows that all our earlier insights remain valid under this more general specification.

Proposition 5. *The information provider's optimal strategy is as follows:*

(a) If $\beta \geq -(1 + \kappa_z/\kappa_x)$, the provider offers her signal undistorted to all firms at price

$$p^* = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z + \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2}.$$

(b) If $\beta < -(1+\kappa_z/\kappa_x)$, she offers a signal of precision κ^*_{si} to firm i, where $\{\kappa^*_{si}\}_{i\in[0,1]}$ solve

$$\int_0^1 \frac{\kappa^*_{si}}{\kappa_x+\kappa^*_{si}} di = -\frac{\kappa_z}{\beta\kappa_x}, \tag{8}$$

at price $p^*_i = \alpha^2\left(\frac{\gamma_2}{2}\right)\frac{\kappa^*_{si}}{4\kappa_x(\kappa_x+\kappa^*_{si})}$.

Statement (a) of the above result shows that the information provider offers an undistorted version of her signal to all firms in the downstream market if either they compete *à la* Bertrand, or alternatively, if the intensity of the Cournot competition is not strong enough. In this sense, this result generalizes Propositions 2 and 3, establishing that the seller has no incentive to discriminate among the firms in either price or information quality.

Statement (b) of Proposition 5 considers the setting in which firms' actions are strong strategic substitutes, for example, when they compete *à la* Cournot and produce goods that are highly substitutable. Consistent with the discussion in Subsection 3.3, this result shows that the information provider finds it optimal to either distort the signals sold to the downstream firms or strategically restrict her market share. In particular, it is easy to verify that $\kappa^*_{si} = \kappa_z$ for all i does not satisfy the optimality condition (8). The intuition underlying this result parallels those behind Proposition 4 and Corollary 1: providing high quality signals to all firms increases the induced correlation in their actions, which in turn reduces their profit when their actions are strong strategic substitutes. Thus, the monopolist would be better off by limiting her market share or reducing the quality of the signals sold to the firms. Note, however, that the optimal strategy of the information provider is not unique. Rather, any signal precision profile $\{\kappa^*_{si}\}$ that satisfies (8) would lead to the same expected profit. Nevertheless, irrespective of the strategy chosen by the monopolist, her incentive to lower the precision of the market signals increases as firms' actions become stronger strategic substitutes. In particular, as $\beta \to -\infty$, the downside of coordination among firms that trade with the monopolist is so strong that essentially no trade takes place in equilibrium: the information provider offers a completely uninformative signal $\kappa^*_{si} \to 0$ to all firms at price $p^*_i \to 0$.

Example 4 (Selling two products). Consider a Cournot market in which $\beta < -(1+\kappa_z/\kappa_x)$ and suppose that the information provider can offer two information products: a premium product of precision $\bar{\kappa}_s$ at price $\bar{p}$ and an inferior one of precision $\underline{\kappa}_s < \bar{\kappa}_s$ at price $\underline{p}$. Let $\bar{\lambda}$ and $\underline{\lambda}$ denote the fraction of firms offered the premium and inferior products, respectively, where by construction $\bar{\lambda} + \underline{\lambda} \leq 1$. Condition (8) implies that it is optimal for the seller to design her information products such that $\bar{\lambda}\left(\frac{\bar{\kappa}_s}{\kappa_x+\bar{\kappa}_s}\right) + \underline{\lambda}\left(\frac{\underline{\kappa}_s}{\kappa_x+\underline{\kappa}_s}\right) = -\frac{\kappa_z}{\beta\kappa_x}$. This equation highlights the trade-off between information quality and quantity faced by the information provider in designing her menu of products. In particular, increasing the precision $\bar{\kappa}_s$ of the premium product requires either a reduction in its supply $\bar{\lambda}$, or alternatively, a reduction in the precision or the supply of the inferior product.

We end by remarking that the ability to discriminate on quality does not offer the seller any advantage compared to our benchmark model of Sections 2 and 3. In particular, equation (8) always has a solution such that $\kappa_{si} = \kappa_s$ for a fraction λ of the firms and $\kappa_{si} = 0$ for the rest. In other words, offering two products, one with non-zero precision at a strictly positive price and another with zero precision at zero price, is sufficient for the seller to maximize her expected profit.

5. Information Leakage

Thus far, we assumed that purchasing a signal from the information provider is the only channel available to the firms for acquiring information about the unknown parameter θ. Firms, however, can also infer potentially valuable information by observing their competitors' actions. For instance, a firm's price or quantity decisions can (partially or fully) reveal the information it has at its disposal to other firms. In this section, we extend our baseline model to allow for the possibility of such indirect "information leakage" and study the information provider's optimal selling strategy when her customers can potentially free-ride on the information purchased by other firms.

We capture the possibility of information leakage by allowing firms to condition their actions on an extra piece of information that is informative about their competitors' actions. More specifically, we assume that, in addition to its signal x_i and the market signal s_i (if purchased from the information provider), firm i can also condition its action on a *leakage signal*,

$$S_i = A + \nu_i \qquad , \qquad \nu_i \sim N(0, 1/\kappa_\nu), \tag{9}$$

where $A = \int_0^1 a_i di$ denotes the aggregate action and the noise terms ν_i are independently distributed across the firms. The key observation is that as long as firms' actions are based (even in part) on the information at their disposal, signal S_i would be informative about such information. As such, the precision κ_ν can serve as a proxy for the extent of information leakage in the market: S_i is perfectly informative about the aggregate action A when $\kappa_\nu = \infty$, whereas as κ_ν decreases, the information content of the leakage signal is reduced. In the extreme case that $\kappa_\nu = 0$, signal S_i does not convey any payoff-relevant information. It is immediate to see that this latter case reduces to the no-leakage setting in our benchmark model.[8]

To formally model firms' ability to incorporate any information leaked through the market into their decisions, we follow Vives (2011) and extend the firms' strategy space by assuming that firm i's strategy is a contingent schedule $a_i(\cdot, S_i)$ that maps its private and market signals, (x_i, s_i), to an

[8] Recall from the payoff function (1) that each firm i cares about the actions of other firms only insofar as these actions impact the aggregate action A. This observation thus implies that any (symmetric) setting in which firm i observes noisy signals about other firms' individual actions can be mapped into an isomorphic setting in which firm i only observes a signal about the aggregate action, as in (9).

action depending on the realization of the leakage signal S_i.[9] Thus, the equilibrium of the subgame between firms requires (i) each firm i to choose $a_i(x_i, s_i, S_i)$ in order to maximize its expected profit conditional on its information set (that is, $\mathbb{E}[\pi_i|x_i, s_i, S_i]$), taking the strategies of all other firms as given; and (ii) the aggregate action to be consistent with the realization of the firms' individual actions, that is, $A = \int_0^1 a_i(x_i, s_i, S_i)\, di$.

We remark that despite the slightly more complex nature of the firms' strategies, this modeling approach enables us to directly incorporate information leakage into our benchmark model without resorting to a multi-period, dynamic model of interaction between firms. Crucially, it also enables us to study how the provider's optimal strategy and profits vary as a function of the intensity of information leakage in the market. We have the following result:

Proposition 6. *For sufficiently small $\kappa_\nu > 0$,*

(a) The provider's profit decreases in the extent of information leakage; that is, $\partial \Pi^* / \partial \kappa_\nu < 0$;

(b) There exists $-(1 + \kappa_z/\kappa_x) < \bar{\beta} < 0$ such that $\kappa_s^* < \kappa_z$ for all $\beta \in (-(1 + \kappa_z/\kappa_x), \bar{\beta})$.

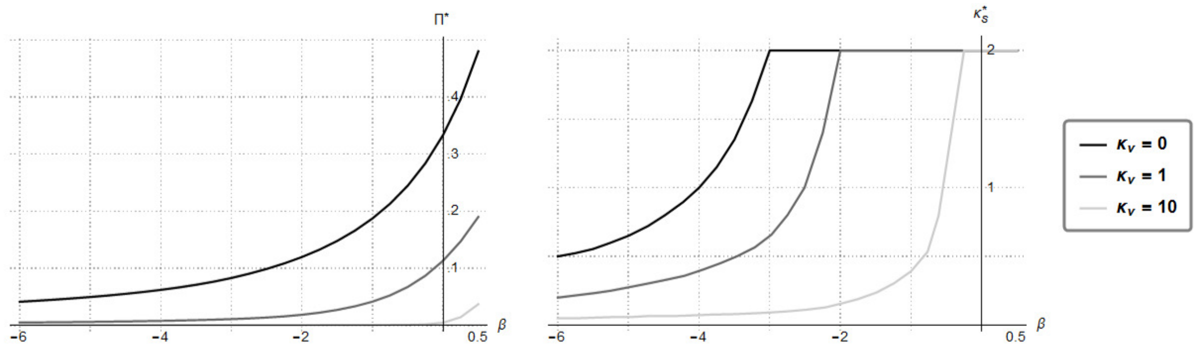
Therefore, Proposition 6 establishes that regardless of whether actions are strategic substitutes or complements (and hence, regardless of the mode of competition), the information provider's profits decrease as the extent of information leakage is intensified. This is due to the fact that firms' willingness-to-pay for an extra piece of information reduces whenever they can free-ride on the information purchased by their competitors. Given that more information leakage would only intensify this free-riding incentive, the information provider is forced to charge lower prices for her signal, thus making less profits.

More importantly, however, the above result establishes that the range of β's for which the information provider finds it optimal to distort the market signal offered to her customers widens in the presence of information leakage. Recall from Corollary 1 and Proposition 5 that, with no information leakage, the information provider would reduce the quality of the market signal if and only if $\beta < -(1 + \kappa_z/\kappa_x)$. In contrast, part (b) of Proposition 6 shows that, no matter how small the extent of leakage, the provider would offer distorted signals for some $\beta > -(1 + \kappa_z/\kappa_x)$. This is due to the fact that the provider's ability to extract surplus from the firms by increasing the precision of s_i is hindered in the presence of leakage. That said, the fact that $\bar{\beta} < 0$ means that, regardless of the presence or absence of information leakage, distorting the signal sold to the firms is never optimal when firms' actions are strategic complements (for example, as in Bertrand competition).

Figure 2 illustrates the provider's equilibrium profits (left panel) and her optimal distortion strategy (right panel) for different levels of information leakage. As the left panel indicates, the

[9] It is immediate to see that the setting in which firms' actions cannot be contingent on the realization of S_i reduces to our benchmark model.

Figure 2 The provider's equilibrium profits (left) and her optimal selling strategy (right) as functions of β for different levels of information leakage. We use the following set of parameters for this example: $\alpha = \gamma_2 = 1$ for the firms' payoff functions and $\kappa_x = 1, \kappa_z = 2$ for the signal precisions of the firms' private signals and the provider's information respectively. We plot the provider's profits and the precision of the signal she sells to the downstream market (κ_s^*) as a function of β for three levels of information leakage $\kappa_\nu = 0$ (no leakage), $\kappa_\nu = 1$, and $\kappa_\nu = 10$.



provider's profits are decreasing in the leakage intensity irrespective of the value of β. As for the precision of the signal offered to the firms, the right panel clearly illustrates the two key observations mentioned above: (i) as leakage is intensified (higher κ_ν), the provider finds it optimal to sell signals of lower quality for a wider range of β's; and (ii) distortion is never optimal in the presence of strategic complementarities ($\beta > 0$) irrespective of the value of κ_ν.

Table 2 presents the results of a numerical simulation for the effect of information leakage on the provider's optimal strategy and equilibrium profits for different values of β, with $\kappa_\nu = 0$, $\kappa_\nu = 1$, and $\kappa_\nu = 10$ corresponding, respectively, to a scenario with no, low, and high levels of leakage intensity. The last column of the table indicates that, at $\beta = 1/3$, the equilibrium profit in the high leakage regime is only 5% of the corresponding profit in the benchmark case with no information leakage. Finally, the lower panel of Table 2 indicates that for certain values of β (say, $\beta = -2$), the optimal strategy may entail selling a signal with maximal precision when leakage is absent or low, whereas the seller finds it optimal to dramatically decreases the signal precision to only 8% of her best signal precision in the high leakage regime.

6. Heterogeneous Firms

In this section, we discuss how our results are affected by introducing heterogeneity among the firms (in terms of their production costs).

We generalize the setting described in Section 2 along two dimensions. First, we allow for heterogeneity in firms' production costs and, second, we introduce a transaction cost borne by the information provider whenever she trades with a downstream firm. We mostly focus on the case that firms view their actions as strategic substitutes and, thus, interpret our results in the

	Leakage Level	$\beta=-4$	$\beta=-3$	$\beta=-2$	$\beta=-1$	$\beta=0$	$\beta=1/3$
Equilibrium Profits	$\kappa_\nu=0$	.0625	.0833	.1200	.1875	.3333	.4218
	$\kappa_\nu=1$	.0082	.0014	.0189	.0420	.1134	.1606
	$\kappa_\nu=10$	.0002	.0003	.0005	.0008	.0050	.0209
Optimal Precision	$\kappa_\nu=0$	1	2	2	2	2	2
	$\kappa_\nu=1$	.40	.65	2	2	2	2
	$\kappa_\nu=10$	.08	.12	.16	.36	2	2

Table 2 The provider's equilibrium profits and optimal precision for different levels of leakage at different levels of β. The other parameter values are the same as in the example of Figure 2.

context of Cournot competition with quadratic production costs (Example 3 from Subsection 2.2). Specifically, we assume that downstream firms are heterogeneous with respect to their costs of production: firm i faces a quadratic production cost of $C_i(q_i)=c_iq_i^2/2$, where q_i is the quantity produced by i and $c_i>0$. The firm's profit is thus given by

$$\pi_i(q_i,Q,\theta)=\gamma_0q_i\theta+\gamma_1q_iQ-\frac{1}{2}c_iq_i^2, \tag{10}$$

where Q denotes the aggregate quantity in the market and $\gamma_1<0$ is some constant. Note that even though the above expression is similar to (1), the extent of strategic complementarities can no longer be captured by a single parameter β, as now firms face different production costs.

As for transaction costs, we assume that the seller incurs a cost equal to $v\kappa_{si}$ whenever she sells a signal of precision κ_{si} to firm i, where $v>0$. This cost can, for example, capture the idea that the firm cannot provide verifiable and/or credible information to her customers at no cost. Rather, it needs to spend resources to ensure her customer that the market signal is indeed as informative as claimed. Alternatively, it can be thought of as the cost associated with customizing the provider's information to meet the customer's informational needs. As in Section 4, we allow the seller to discriminate along both signal precision and price. We have the following result:

PROPOSITION 7. *There exist $\overline{v}>\underline{v}$ such that*

(a) if $v>\overline{v}$, the information provider does not transact with any of the firms; i.e., $\kappa^*_{si}=0$ for all i.

(b) if $v<\underline{v}$, the information provider sells her signal with no distortion to all firms.

(c) for any $v\in(\underline{v},\overline{v})$, then there exist c^* such that

$$\kappa^*_{si}=\begin{cases}0 & \textit{if } c_i>c^* \\ \kappa_z & \textit{if } c_i<\dfrac{\kappa_x^2}{(\kappa_x+\kappa_z)^2}c^* \\ \kappa_x\left(\sqrt{c^*/c_i}-1\right) & \textit{otherwise.}\end{cases}.$$

The above result thus establishes that the information provider finds it optimal to follow an information selling strategy that involves offering a signal to firm i with a precision that is decreasing in the firm's cost c_i, i.e., the provider sells higher quality signals to more efficient firms. Formally, κ_{si}^* is always non-increasing in c_i. However, note that this does not mean that the monopolist sells her best available information to all firms, even when transactions are costless. Rather, due to the presence of strategic interactions between downstream firms (and in line with our earlier results), the provider may either sell distorted signals to some firms or simply even exclude them by offering non-informative signals $\kappa_{si}^* = 0$ altogether. Thus, Proposition 7 generalizes Propositions 4 and 5 to the case in which firms face heterogeneous production costs.

Finally, note that depending on the parameter values, the threshold $\underline{v}$ in the above result may be negative, thus ruling out the case in which the information provider sells an undistorted signal to all firms. In fact, as the proof of the proposition highlights, $\underline{v} < 0$ whenever:

$$\int_0^1 \frac{1}{c_i} di < -\frac{1}{\gamma_1}(1 + \kappa_z/\kappa_x),$$

which reduces to the condition of Proposition 4 when firms face identical production costs.

Cost dispersion and optimal information selling strategy: To further clarify the impact of firm heterogeneity on the provider's equilibrium strategy and profits, consider a special case consisting of two types of firms $i \in \{1, 2\}$, with production costs $C_i(q_i) = c_i q_i^2/2$, where

$$\frac{1}{c_1} = \frac{1}{c} + \delta, \qquad \text{and} \qquad \frac{1}{c_2} = \frac{1}{c} - \delta,$$

for some $\delta > 0$. Also, assume that both types have mass equal to $1/2$. It is immediate to see that in such a setting δ measures the cost dispersion in the market. We have the following corollary:

Corollary 2. *Let κ_{s1}^* and κ_{s2}^* be the optimal signal precisions offered to firms of type 1 and type 2 respectively. Then, for any $\delta < 1/\left(c\sqrt{2}\right)$,*

$$\frac{\partial \kappa_{s1}^*}{\partial \delta} \geq 0, \qquad \textit{and} \qquad \frac{\partial \kappa_{s2}^*}{\partial \delta} \leq 0.$$

Corollary 2 thus establishes that as the cost dispersion among the downstream firms increases, the provider finds it optimal to sell increasingly more accurate signals to the efficient type while she decreases the accuracy of the signals she sells to the type that has high production costs. This change in selling strategy occurs despite the fact that the average cost in the downstream market remains constant. Table 3 reports the results of a numerical simulation that quantifies the effect of cost dispersion on the provider's optimal information selling strategy and equilibrium profits. As the table indicates, when the dispersion between firms' production costs is sufficiently high ($\delta = 4$ in this case), the optimal strategy of the information provider requires excluding the less efficient firms from trade altogether. It is also immediate to see that the provider's profits are increasing in the cost dispersion parameter δ, with her profits roughly 20% higher when $\delta = 4$ compared to the benchmark case with no dispersion.

	$\delta = .5$	$\delta = 1$	$\delta = 2$	$\delta = 4$
κ^*_{s1}	1.334	1.505	1.819	2
κ^*_{s2}	.976	.789	.409	0
Profits	1.067	1.076	1.114	1.251

Table 3 Optimal information selling strategy and equilibrium profits as a function of the cost dispersion between the two types of firms. For this example, we use $\kappa_x = 1,\ \kappa_z = 2,\ c = 1/6,\ \gamma_1 = 3/5,$ **and** $\gamma_0 = 10.$

7. Finite Markets

To simplify the exposition and allow for a tractable analysis, most of the paper focused on an environment with a continuum of firms. In this section, we show that our qualitative insights regarding the monopolist's optimal information selling strategy carry over to a market consisting of finitely many firms. In particular, we focus on a Cournot market, in which n firms compete with one another in quantities, with the inverse demand function given by $r = \gamma_0\theta + \gamma_1 Q$, where $Q = \frac{1}{n}\sum_{i=1}^n q_i$ is the average quantity in the market, q_i is the quantity produced by firm i, r denotes the market price, and $\gamma_1 < 0$ is some constant. Assuming that firm i faces quadratic production costs $c(q_i) = \gamma_2 q_i^2/2$, its profits can be expressed as

$$\pi_i = \gamma_0 q_i \theta + \frac{n-1}{n}\gamma_1 q_i Q_{-i} - \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) q_i^2,$$

where $Q_{-i} = \frac{1}{n-1}\sum_{j\neq i} q_j$ is the average quantity of i's competitors. It is immediate to see that the above expression is similar to the firms' profit function (1) for a market consisting of a continuum of firms. The degree of strategic complementarity among firms' actions can also be defined as

$$\beta_n = -\frac{\partial^2 \pi_i}{\partial q_i \partial Q_{-i}} \Big/ \frac{\partial^2 \pi_i}{\partial q_i^2} = \left(\frac{n-1}{n}\right)\frac{\gamma_1}{\gamma_2 - 2\gamma_1/n}.$$

As in the environment with a continuum of firms, we assume that each firm i observes a noisy private signal x_i about the realization of θ and that the information provider can offer a market signal s_i to firm i. Let K denote the set of firms that the information provider trades with, where $|K| = k \le n$. Lemma EC.1 in the Appendix provides a complete characterization of the equilibrium of the competition subgame for any k, which can be viewed as the discrete analog of Proposition 1 in Section 3. However, as we argued in Section 3, there always exist an equilibrium in which the provider offers the market signal to all firms. Thus, without loss of generality we can restrict our attention to the case of $k = n$. We have the following result:

Proposition 8. *The optimal information selling strategy is given as follows:*

(a) If $\beta_n \ge -(1 + \kappa_z/\kappa_x)$*, the provider offers an undistorted version of her signal to all firms.*

(b) If $\beta_n < -(1 + \kappa_z/\kappa_x)$*, she offers a signal of precision* $\kappa_s^* = -\kappa_z/(\beta_n + \kappa_z/\kappa_x) < \kappa_z$*.*

Furthermore, the seller's expected profit is given by

$$\Pi^* = \begin{cases} n\ \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right)\left(\frac{\kappa_z}{\kappa_x}\right) \frac{\kappa_z + \kappa_x}{[(1-\beta_n)\kappa_x + \kappa_z]^2} & \text{if } \beta_n \geq -(1+\kappa_z/\kappa_x) \\ n\ \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) \frac{\kappa_z}{-4\beta_n \kappa_x^2} & \text{otherwise} \end{cases},$$

where $\alpha_n = \gamma_0 / \left(\gamma_2 - \frac{n+1}{n}\gamma_1\right)$.

Proposition 8 thus illustrates that the insights underlying our main results remain unchanged when the downstream market is composed of a finite number of firms. Additionally, it is straightforward to verify that as n grows to infinity, the expressions characterizing the provider's optimal strategy and her expected profits (normalized by the total number of firms n) reduce to those we obtained in Section 3.3 for a market consisting of a continuum of firms. Finally, the fact that β_n is decreasing in n implies that the range of parameters over which the monopolist finds it optimal to distort the information she sells to the market grows with the number of firms n.

Proposition 8 also illustrates that, when $\beta_n < -(1+\kappa_z/\kappa_x)$, the gain to the provider from optimally distorting the information can be obtained by comparing

$$\Pi_n^{no-dist} = \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) \frac{\kappa_z}{\kappa_x} \frac{\kappa_z + \kappa_x}{[(1-\beta_n)\kappa_x + \kappa_z]^2}$$
$$\Pi_n^* = \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) \frac{\kappa_z}{-4\beta_n \kappa_x^2},$$

where $\Pi_n^{no-dist}$ is the provider's expected profit per customer under no information distortion and Π_n^* is the normalized expected profit under optimal distortion. Thus, the gain from distorting the information, normalized by the number of firms n, is given by

$$\Delta_n = \frac{(n-1)|\gamma_1|\alpha_n^2}{8n\beta_n^2} \frac{\kappa_z[(1+\beta_n)\kappa_x + \kappa_z]^2}{\kappa_x^2[(1-\beta_n)\kappa_x + \kappa_z]^2}, \tag{11}$$

where $\alpha_n = \gamma_0 / \left(\gamma_2 - \frac{n+1}{n}\gamma_1\right)$ and $\beta_n = \left(\frac{n-1}{n}\right) \frac{\gamma_1}{\gamma_2 - 2\gamma_1/n}$ and recall that $\gamma_1, \gamma_2, \kappa_x$, and κ_z are model primitives that do not depend on n. Expression (11) thus leads to two key observations. First, it illustrates that, consistent with the findings of Proposition 8, the gain from distorting the information is positive for all values of $\beta_n < -(1+\kappa_z/\kappa_x)$. Second, taking the market size n and intensity of competition β_n (which only depends on parameters γ_1 and γ_2) as given, the gain Δ_n can be made arbitrarily large by increasing the value of γ_0 (and hence α_n). This observation thus illustrates that, even though the *percentage change* in profit gain $(\Pi_n^* - \Pi_n^{no-dist})/\Pi_n^* = [(1+\beta_n)\kappa_x + \kappa_z]^2/[(1-\beta_n)\kappa_x + \kappa_z]^2$ is maximized when the intensity of competition is maximized ($\beta_n \to -\infty$), the *level* of gain Δ_n can be large even away from the competitive limit.

8. Conclusions

This paper considers the problem of selling information to a set of firms that compete in a downstream market. We establish that both the information provider's optimal selling strategy as well as her profits depend critically on the environment in which her customers operate. In particular, our results highlight that the extent of strategic substitutability and complementarity in the latter's actions has a first-order impact on the former's optimal strategy: when the firms' actions are strategic complements, the provider finds it optimal to sell an undistorted version of her information to the entire market, whereas if their actions are strategic substitutes, the optimal strategy involves offering an inferior information product and/or limiting the supply of information.

Our results are largely driven by the following trade-off: on the one hand, information about market conditions, e.g., demand realization, always has a direct positive effect on firms' profits as they can better align their actions with the underlying environment. On the other hand, however, in the presence of strategic substitutability among the firms, the provider's signal may have an additional (adverse) effect by increasing the correlation between the firms' actions. It turns out that this latter effect may dominate the former when firms' view their actions as strong strategic substitutes, in which case the provider finds it optimal to degrade the quality of her information products and/or exclude a subset of the firms from the sale.

We showcase the implications of our results in the context of Bertrand and Cournot competition thus complementing the extensive prior literature in operations management that explores vertical and horizontal information sharing in a supply chain. In addition, we discuss how the extent of information leakage in the market can affect the provider's selling strategy and profits. Finally, we extend our findings to the case when firms differ in their production costs and establish that the optimal selling strategy involves offering several information products with varying precisions and at different prices. We also show that in equilibrium, the information provider offers more precise signals to the more efficient firms at higher prices in order to maximize her profit.

Taken together, our findings illustrate that the optimal provision and pricing of information products cannot be decoupled from the market structure in which the firm's potential customers operate. They also uncover a potential rationale for why information markets typically feature several versions of essentially the same information product, but of varying qualities and price tags. Identifying the prevalence of the said mechanism and its relative importance compared to other potential explanations (such as price discrimination driven, for example, by the heterogeneity in the willingness-to-pay among potential buyers) in various contexts is an important question, with both positive and normative insights for the pricing of information products.

To facilitate our analysis, we focused on an environment with a monopolistic provider of information interacting with a market of competing firms. Extending our framework to incorporate competition among information providers is an interesting direction for future research. Even though the basic mechanism we identify—i.e., that the extent of strategic complementarities/substitutabilities in the downstream market matters for products firms should offer—will be present regardless of market conditions, departures from our baseline framework can potentially impact the optimal degree of distortion in the quality or quantity of information.

Appendix : Proofs

With the exception of our results in Section 6, firms in our model are assumed to be *ex ante* symmetric. Therefore, unless otherwise noted, we assume without loss of generality that the price offered by the provider to the firms is non-decreasing in the firms' index; that is, $p_i \geq p_j$ for $i > j$. Given that excluding a firm i from trade is equivalent to offering a price $p_i = \infty$, the above assumption also implies that the set of firms that are offered a contract by the provider is of the form $[0, \lambda]$ for some $\lambda \in [0,1]$. Let ℓ denote the fraction of firms who accept the provider's offer. In view of the above assumption, it is immediate that $\ell = \sup\{i \in [0,\lambda] : b_i = 1\}$, and that $b_i = 1$ for all $i \leq \ell$.

Proof of Proposition 1

The first-order optimality condition for firm i's problem with respect to action a_i is given by

$$\mathbb{E}\left[\frac{\partial}{\partial a_i}\pi(a_i, A, \theta)\middle|\mathcal{I}_i\right] = 0,$$

where $\mathcal{I}_i = \{x_i\}$ if $i \in [\ell, 1]$, i.e., the firm is uninformed, and $\mathcal{I}_i = \{x_i, s_i\}$ if $i \in [0, \ell]$, i.e., the firm is informed. Consequently, $a_i = \mathbb{E}\left[\beta A + (1-\beta)\alpha\theta|\mathcal{I}_i\right]$, where $\beta = \gamma_1/\gamma_2$ is the degree of strategic complementarity in the downstream market as defined in (2) and $\alpha = \gamma_0/(\gamma_2 - \gamma_1)$. Thus, the firms' equilibrium actions are given by

$$a_i = \begin{cases} \mathbb{E}\left[\beta A + (1-\beta)\alpha\theta|x_i\right] & \forall i \in [\ell, 1], \\ \mathbb{E}\left[\beta A + (1-\beta)\alpha\theta|x_i, s_i\right] & \forall i \in [0, \ell]. \end{cases}$$

Noticing that $\mathbb{E}\left[\theta|x_i\right]$ is linear in x_i and $\mathbb{E}\left[\theta|x_i, s_i\right]$ is linear in x_i and s_i, we conjecture that equilibrium strategies are linear functions of x_i and s_i and then verify our hypothesis. In particular, we conjecture that

$$a_i = \begin{cases} c_0 x_i & \forall i \in [\ell, 1] \\ c_1 x_i + c_2 s_i & \forall i \in [0, \ell] \end{cases},$$

for some constants $c_0, c_1, c_2 \in \mathbb{R}$. Replacing the candidate equilibrium strategy of an uninformed firm $i \in (\ell, 1]$ in its first-order optimality condition yields

$$\begin{aligned} c_0 x_i &= \mathbb{E}\left[\beta\left(\int_0^\ell c_1 x_j + c_2 s_j dj + \int_\ell^1 c_0 x_j dj\right) + (1-\beta)\alpha\theta\middle|x_i\right] \\ &= [\beta\ell(c_1 + c_2) + \beta(1-\ell)c_0 + (1-\beta)\alpha]x_i, \end{aligned} \tag{12}$$

where we are using the fact that $\mathbb{E}[\theta|x_i] = \mathbb{E}[x_j|x_i] = \mathbb{E}[s_j|x_i] = x_i$. Similarly, the first-order optimality condition for the optimization problem of an informed firm $i \in [0, \ell]$ yields

$$c_1 x_i + c_2 s_i = \mathbb{E}\left[\beta\left(\int_0^\ell (c_1 x_j + c_2 s_j)dj + \int_\ell^1 c_0 x_j dj\right) + (1-\beta)\alpha\theta\middle|x_i, s_i\right]. \tag{13}$$

Note that

$$\mathbb{E}[\theta|x_i, s_i] = \mathbb{E}[x_j|x_i, s_i] = \delta_1 x_i + (1-\delta_1)s_i \qquad \text{and} \qquad \mathbb{E}[s_j|x_i, s_i] = \delta_1(1-\rho)x_i + [1-\delta_1(1-\rho)]s_i,$$

where $\delta_1 = \kappa_x/(\kappa_x + \kappa_s)$. Consequently, we can rewrite (13) as

$$\begin{aligned} c_1 x_i + c_2 s_i =& [\beta\ell c_1\delta_1 + \beta\ell c_2\delta_1(1-\rho) + \beta(1-\ell)c_0\delta_1 + (1-\beta)\alpha\delta_1]x_i \\ &+ [\beta\ell c_1(1-\delta_1) + \beta\ell c_2(1-\delta_1(1-\rho)) + \beta(1-\ell)c_0(1-\delta_1) + (1-\beta)\alpha(1-\delta_1)]s_i. \end{aligned} \tag{14}$$

From Equation (12) we have for the equilibrium strategy coefficients: $[c_0 = \beta\ell(c_1+c_2)+\beta(1-\ell)c_0+(1-\beta)\alpha$, for any admissible $\ell \in [0,1]$. In turn, this implies that $c_0 = \alpha$ and $c_1 + c_2 = \alpha$. Replacing $c_0 = \alpha$ and $c_2 = \alpha - c_1$ in Equation (14) implies that equilibrium coefficient c_1 must satisfy $c_1 = \beta\ell c_1\delta_1\rho + \alpha\delta_1(1-\beta\ell\rho)$. Solving for c_1 yields $c_1 = \alpha\frac{(1-\beta\ell\rho)\kappa_x}{(1-\beta\ell\rho)\kappa_x+\kappa_s}$, and hence, $c_2 = \alpha - c_1 = \alpha\frac{\kappa_s}{(1-\beta\ell\rho)\kappa_x+\kappa_s}$. Combining the above, we conclude that firms' actions at equilibrium are given by

$$a_i = \begin{cases} \alpha x_i & \forall i \in [\ell, 1] \\ \alpha\dfrac{(1-\beta\ell\rho)\kappa_x}{(1-\beta\ell\rho)\kappa_x+\kappa_s}x_i + \alpha\dfrac{\kappa_s}{(1-\beta\ell\rho)\kappa_x+\kappa_s}s_i & \forall i \in [0, \ell] \end{cases},$$

completing the proof. Q.E.D.

Two Auxiliary Lemmas

We state and prove two lemmas that we use in the remainder of the appendix. The first lemma characterizes the expected surplus of an informed firm, whereas the second lemma shows that, for any given λ, the provider always finds it optimal to charge a constant price to all firms $i \in [0, \lambda]$.

Lemma 1. *The expected surplus of each firm from buying the market signal is given by*

$$\Delta(\ell, \kappa_s, \rho, \kappa_x) = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{\kappa_s+\kappa_x}{[(1-\beta\ell\rho)\kappa_x+\kappa_s]^2}, \tag{15}$$

where ℓ denotes the fraction of informed firms.

Proof: Let $a_i^1 := \alpha\dfrac{\kappa_s s_i + (1-\beta\ell\rho)\kappa_x x_i}{\kappa_s + (1-\beta\ell\rho)\kappa_x}$ denote the equilibrium action of an informed firm and let $a_i^0 := \alpha x_i$ denote the equilibrium action of an uninformed firm. Recall that ℓ denotes the fraction of informed firms, and thus the aggregate equilibrium action is $A = \int_0^\ell a_i^1 di + \int_\ell^1 a_i^0 di$. By replacing the equilibrium actions in the expressions for the firms' payoffs and then taking the expectations conditional on θ, we get

$$\mathbb{E}\left[\pi(a_i^1, A, \theta)|\theta\right] = \alpha^2\left(\frac{\gamma_2}{2}\right)\left[\theta^2 + \frac{2\beta\ell\rho\kappa_s}{[(1-\beta\ell\rho)\kappa_x+\kappa_s]^2} - \frac{(1-\beta\ell\rho)^2\kappa_x+\kappa_s}{[(1-\beta\ell\rho)\kappa_x+\kappa_s]^2}\right], \tag{16}$$

and

$$\mathbb{E}\left[\pi(a_i^0, A, \theta)|\theta\right] = \alpha^2\left(\frac{\gamma_2}{2}\right)\left[\theta^2 - \frac{1}{\kappa_x}\right]. \tag{17}$$

Next note that we can use the two conditional expectations (16) and (17) to compute the (unconditional) expectation for a firm's surplus given by

$$\Delta := \mathbb{E}\left[\pi(a_i^1, A, \theta)\right] - \mathbb{E}\left[\pi(a_i^0, A, \theta)\right].$$

Applying the law of total expectation yields

$$\Delta = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{\kappa_s+\kappa_x}{[(1-\beta\ell\rho)\kappa_x+\kappa_s]^2},$$

which completes the proof of the lemma. Q.E.D.

Lemma 2. *The provider sets $p_i = p^*(\lambda)$ for all $i \in [0, \lambda]$, where $p^*(\lambda)$ is equal to the expected equilibrium surplus of an informed firm when the fraction of informed firms is λ. Furthermore, $p^*(\lambda)$ is such that all firms that receive the provider's offer accept in equilibrium, thus $\ell = \lambda$.*

Proof: Consider the simultaneous game of accepting/rejecting the provider's offer. Recall that in such game each firm $i \in [0, \lambda]$ accepts the offer if her expected surplus is bigger than her individual price p_i while taking the decisions of the rest of the firms as given.

We suppose that a fraction $\ell \in [0, \lambda]$ of firms has accepted the provider's offer, and we write the optimal decision of each firm $i \in [0, \lambda]$ as a function of firm's i individual price. We have

$$b_i(p_i) = \begin{cases} 1 & \text{if} \quad \Delta(\ell) > p_i \\ 0 & \text{if} \quad \Delta(\ell) < p_i \\ \in \{0,1\} & \text{if} \quad \Delta(\ell) = p_i \end{cases},$$

where $\Delta(\ell)$ is given by equation (15) and denotes the expected surplus of an informed firm when a fraction ℓ is informed. We can write the provider's optimization problem as follows

$$\begin{aligned} \max_{\{p_i\}_{i\in[0,\lambda]}} \quad & \int_0^\lambda p_i b_i(p_i) di \\ s.t. \quad & b_i(p_i) = \begin{cases} 1 & \text{if} \quad \Delta(\ell) > p_i \\ 0 & \text{if} \quad \Delta(\ell) < p_i \\ \in \{0,1\} & \text{if} \quad \Delta(\ell) = p_i \end{cases}, \quad \forall i \in [0,\lambda]. \end{aligned} \tag{18}$$

Before solving for the provider's optimal selling strategy, we rewrite the set of constraints (18) as

$$\begin{cases} \sup_{i\in[0,\lambda]} p_i \leq \Delta(\lambda) & \text{if} \quad \ell = \lambda \\ \inf_{i\in[0,\lambda]} p_i \geq \Delta(0) & \text{if} \quad \ell = 0 \\ \int_0^\lambda \mathbb{I}_{\{p_i < \Delta(\ell)\}} di \leq \ell \leq \int_0^\lambda \mathbb{I}_{\{p_i \leq \Delta(\ell)\}} di & \text{if} \quad \ell \in (0,\lambda) \end{cases},$$

Recall that without loss of generality the pricing schedule $p : [0, \lambda] \to \mathbb{R}_+$ is non-decreasing, thus we can further simplify the set of constraints as

$$\begin{cases} p_\lambda \leq \Delta(\lambda) & \text{if} \quad \ell = \lambda & (19a) \\ p_0 \geq \Delta(0) & \text{if} \quad \ell = 0 & (19b) \\ p_\ell \leq \Delta(\ell) \quad \text{and} \quad p_{\ell+} \geq \Delta(\ell) & \text{if} \quad \ell \in (0,\lambda). & (19c) \end{cases}$$

The proof proceeds by showing that for any equilibrium of the subgame that results from a fraction ℓ of the firms accepting the provider's offer, there exists an optimal pricing schedule such that $p_i = \Delta(\ell)$ for all $i \leq \ell$ and $p_i = \infty$ for all $i > \ell$. There are the following three cases to consider:

(i) For case (19a), the problem simplifies to

$$\begin{aligned} \max_{\{p_i\}_{i\in[0,\lambda]}} \quad & \int_0^\lambda p_i di \\ s.t. \quad & p_\lambda \leq \Delta(\lambda). \end{aligned}$$

In this case a fraction $\ell = \lambda$ of firms accepts and as we show below it is optimal for the provider to set $p_i = \Delta(\lambda)$ for all $i \in [0, \lambda]$. Suppose, for the sake of contradiction, that p is optimal but $u := \sup\{i \in [0, \lambda] : p_i < \Delta(\lambda)\} \geq 0$. If $u = 0$, then we have $p_i = \Delta(\lambda)$ except for a set of measure 0, so this case is immaterial. If $u > 0$, the maintained assumption that p is non-decreasing implies that

$$p_i < \Delta(\lambda), \ \forall i < u \quad \text{and} \quad p_i = \Delta(\lambda), \ \forall i \geq u.$$

This implies that we can construct pricing schedule p' such that

$$p_i < p'_i \leq \Delta(\lambda),\ \forall i < u \quad \text{and} \quad p'_i = p_i,\ \forall i \geq u,$$

that is feasible and achieves a higher objective value. Thus, it must be that $p_i = \Delta(\lambda)$ for all $i \leq \lambda$.

(ii) For case (19b), $\ell = 0$ and the objective function is always equal to 0. Thus, p can be chosen such that $p_i = \infty$ for all $i \in [0, \lambda]$.

(iii) Finally, for case (19c), the problem simplifies to

$$\begin{aligned} \max_{\{p_i\}_{i\in[0,\lambda]}} \quad & \int_0^\ell p_i di \\ s.t. \quad & p_\ell \leq \Delta(\ell) \text{ and } p_{\ell^+} \geq \Delta(\ell). \end{aligned}$$

First, we show that the provider can always set $p_i = \infty, \forall i > \ell$. Note that the individual price of each firm $i > \ell$ does not affect the objective function of the provider. This implies that all feasible solutions p that differ only on $(\ell, \lambda]$ attain the same objective value, so it is without loss of generality to focus on solutions that are such that $p_i = \infty$ for all $i > \ell$. Next, we show that $p_i = \Delta(\ell), \forall i \leq \ell$. Suppose, for the sake of contradiction, that p is optimal but $u := \sup\{i \in [0, \ell] : p_i < \Delta(\ell)\} \geq 0$. If $u = 0$ we have $p_i = \Delta(\ell)$, except for a set of measure 0. If $u > 0$, the assumption that p is non-decreasing implies that

$$p_i < \Delta(\ell),\ \forall i < u \quad \text{and} \quad p_i = \Delta(\ell),\ \forall i \geq u,$$

which in turn implies that we can construct a pricing schedule p'' such that

$$p_i < p''_i \leq \Delta(\ell),\ \forall i < u \quad \text{and} \quad p''_i = p_i,\ \forall i \geq u,$$

that is feasible and achieves a higher objective value. Thus, it must be that $p_i = \Delta(\ell)$ for all $i \leq \ell$.
Thus, there exists an optimal pricing schedule such that $p_i = \Delta(\ell)$ for all $i \leq \ell$ and $p_i = \infty$ for all $i > \ell$, which implies that only a fraction ℓ of firms accepts the provider's offer and the latter's optimal profit is $\ell \cdot \Delta(\ell)$. Without loss of generality the provider sets $\lambda = \ell$ and $p_i = \Delta(\lambda)$ for all $i \in [0, \lambda]$. Thus, all firms accept her offer and the provider's profit is given by $\lambda \cdot \Delta(\lambda)$. Setting $p^*(\lambda) = \Delta(\lambda)$ completes the proof. Q.E.D.

Proof of Proposition 2

By Lemma 2, the provider's problem simplifies to choosing λ, κ_y and ρ in order to maximize the expected profit $\Pi := \lambda \cdot p^*(\lambda, \kappa_s, \rho, \kappa_x) = \lambda \cdot \Delta(\lambda, \kappa_s, \rho, \kappa_x)$, subject to the constraints imposed by the information structure. Replacing the expected surplus (15) into the objective function yields

$$\Pi(\lambda, \kappa_s, \rho, \kappa_x) = \lambda\ \alpha^2 \left(\frac{\gamma_2}{2}\right) \left(\frac{\kappa_s}{\kappa_x}\right) \frac{\kappa_s + \kappa_x}{[(1 - \beta\lambda\rho)\kappa_x + \kappa_s]^2}, \tag{20}$$

and thus the provider's problem can be rewritten as

$$\begin{aligned} \max_{\rho, \kappa_s, \lambda} \quad & \Pi(\lambda, \kappa_s, \rho, \kappa_x) \\ s.t. \quad & \frac{\kappa_s}{\kappa_z} \leq \rho \leq 1, \text{ and } 0 \leq \lambda \leq 1. \end{aligned} \tag{21}$$

Note that the partial derivative of Π with respect to ρ, i.e.,

$$\frac{\partial \Pi}{\partial \rho} = \lambda\ \alpha^2 \gamma_2 \frac{\beta\lambda\kappa_s(\kappa_x + \kappa_s)}{[(1 - \beta\lambda\rho)\kappa_x + \kappa_s]^3}, \tag{22}$$

is positive for $\beta \in (0, 1/2)$; thus, $\rho^* = 1$. Replacing this into (20) and differentiating with respect to λ yields

$$\frac{\partial \Pi}{\partial \lambda} = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{(\kappa_x + \kappa_x)[(1+\beta\lambda)\kappa_x + \kappa_s]}{[(1-\beta\lambda)\kappa_x + \kappa_s]^3}. \tag{23}$$

Similarly, the partial derivative with respect to κ_s is given by

$$\frac{\partial \Pi}{\partial \kappa_s} = \lambda\ \alpha^2 \left(\frac{\gamma_2}{2}\right)\frac{(1-\beta\lambda)\kappa_x + (1-2\beta\lambda)\kappa_s}{[(1-\beta\lambda)\kappa_x + \kappa_s]^3}. \tag{24}$$

In addition, note that (23) and (24) are positive for $\beta \in (0, 1/2)$, so the provider finds it optimal to set $\lambda^* = 1$ and $\kappa_s^* = \kappa_z$. Replacing ρ^*, λ^* and κ_s^* into (20) yields $\Pi^* = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z + \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2}$. Q.E.D.

Proof of Proposition 3

Consider the provider's expected profit (20) and her profit-maximization problem (21), and let $-(1+\kappa_z/\kappa_x) \leq \beta < 0$. In this case, the partial derivative of Π with respect to ρ given in (22) is negative, which implies that the provider finds it optimal to set the level of correlation to its minimum, i.e., $\rho_\xi^* = 0$ or $\rho^* = \kappa_s/\kappa_z$. Replacing this into (20) and differentiating with respect to κ_s yields

$$\frac{\partial \Pi}{\partial \kappa_s} = \lambda\ \alpha^2 \left(\frac{\gamma_2}{2}\right)\frac{(1+\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s}{[(1-\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s]^3}, \tag{25}$$

while differentiating with respect to λ yields

$$\frac{\partial \Pi}{\partial \lambda} = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{(\kappa_x + \kappa_s)[(1+\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s]}{[(1-\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s]^3}. \tag{26}$$

The assumption on β implies that (25) and (26) are positive, which results in $\lambda^* = 1$ and $\kappa_s^* = \kappa_z$. The proof follows by replacing the optimal values for ρ^*, λ^*, and κ_s^* into expression (20). Q.E.D.

Proof of Proposition 4

Consider the provider's expected profit (20) and her profit-maximization problem (21), and let $\beta < -(1 + \kappa_z/\kappa_x)$. First, note that in this case (22) is negative, so the provider finds it optimal to set $\rho^* = \kappa_s/\kappa_z$. Replacing this into (20) and differentiating with respect to κ_s and λ we again obtain (25) and (26) respectively. Both (25) and (26) are equal to 0 if and only if $(\kappa_z + \beta\lambda\kappa_s)\kappa_x + \kappa_z\kappa_s = 0$. Moreover, Π is unimodal in both κ_s and λ, which implies that the set of optimal allocations (κ_s^*, λ^*) is given by the solutions to above equation. Finally, replacing $\kappa_s^*, \rho^* = \kappa_s^*/\kappa_z$ and $\lambda^* = \frac{(\kappa_x + \kappa_s^*)\kappa_z}{-\beta\kappa_x\kappa_s^*}$ into (20) yields $\Pi^* = -\alpha^2\left(\frac{\gamma_2}{2}\right)\frac{\kappa_z}{4\beta\kappa_x^2}$. Q.E.D.

Proof of Proposition 5

We solve the game by backward induction, i.e., first, we characterize the firms' equilibrium actions in the competition subgame that results from a (subset) of them obtaining the provider's information signal; then, we solve for their acceptance/rejection decisions; and, finally, we turn to the provider's problem and complete the proofs of parts (a) and (b) of the proposition. Recall that the provider possesses a signal $z = \theta + \zeta$, with $\zeta \sim N(0, 1/\kappa_z)$, and offers to firm $i \in [0, 1]$ a signal $s_i = z + \xi_i$ with $\xi_i \sim N(0, \kappa_{\xi i})$. Without loss of generality we assume that the provider does not add any correlation to the signal she sells, i.e., $\mathrm{corr}(\xi_i, \xi_j) = 0$. The market signal s_i offered to firm $i \in [0, 1]$ can be rewritten as $s_i = \theta + \eta_i$, with $\eta_i \sim N(0, 1/\kappa_{si})$, where $\kappa_s = (1/\kappa_z + 1/\kappa_{\xi i})^{-1}$ and $\mathrm{Cov}(s_i, s_j) = 1/\kappa_z$. We have the following auxiliary lemma.

LEMMA 3. *The competition subgame has a unique Bayes-Nash equilibrium in linear strategies, given by* $a(\kappa_{si}, \kappa_{\mathbf{s-i}}) = \alpha[(1-\omega_i)x_i + \omega_i s_i]$ *for all* $i \in [0,1]$, *where*

$$\omega_i = \left(\frac{\kappa_{si}}{\kappa_x + \kappa_{si}}\right) \Big/ \left(1 - \beta \frac{\kappa_x}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right) \qquad \textit{and} \qquad \alpha = \gamma_0/(\gamma_2 - \gamma_1).$$

Proof: The first-order optimality condition of firm i with respect to action a_i implies that in equilibrium

$$a_i = \mathbb{E}[\beta A + (1-\beta)\alpha\theta | x_i, s_i]. \tag{27}$$

Assume that each firm $i \in [0,1]$ uses a linear strategy $c_i x_i + h_i s_i$, for constants $c_i, h_i \in \mathbb{R}$. Then, we can rewrite the equilibrium condition (27) as $c_i x_i + h_i s_i = \mathbb{E}\left[\beta \int_0^1 (c_j x_j + h_j s_j) dj + (1-\beta)\alpha\theta \Big| x_i, s_i\right]$. Using equations

$$\mathbb{E}[s_j|x_i, s_i] = \frac{\kappa_x(1-\kappa_{si}/\kappa_z)}{\kappa_x + \kappa_{si}} x_i + \frac{\kappa_{si}(1+\kappa_x/\kappa_z)}{\kappa_x + \kappa_{si}} s_i, \text{ and } \mathbb{E}[\theta|x_i, s_i] = \mathbb{E}[x_j|x_i, s_i] = \frac{\kappa_x}{\kappa_x + \kappa_{si}} x_i + \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} s_i,$$

which are obtained by the conditional expectation of Gaussian random vectors, we have

$$\begin{aligned} c_i x_i + h_i s_i =& \beta \left[\left(\frac{\kappa_x}{\kappa_x + \kappa_{si}} x_i + \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} s_i\right) \int_0^1 c_j dj + \left(\frac{\kappa_x(1-\kappa_{si}/\kappa_z)}{\kappa_x + \kappa_{si}} x_i + \frac{\kappa_{si}(1+\kappa_x/\kappa_z)}{\kappa_x + \kappa_{si}} s_i\right) \int_0^1 h_j dj\right] \\ &+ (1-\beta)\alpha \left(\frac{\kappa_x}{\kappa_x + \kappa_{si}} x_i + \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} s_i\right). \end{aligned}$$

Note that the equilibrium coefficients $(c_i, , h_i)$ for $i \in [0,1]$, must solve the following sets of equations

$$c_i = \beta \frac{\kappa_x}{\kappa_x + \kappa_{si}} \int_0^1 c_j dj + \beta \frac{\kappa_x(1-\kappa_{si}/\kappa_z)}{\kappa_x + \kappa_{si}} \int_0^1 h_j dj + (1-\beta)\alpha \frac{\kappa_x}{\kappa_x + \kappa_{si}} \quad \forall i \in [0,1], \tag{28}$$

and

$$h_i = \beta \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} \int_0^1 c_j dj + \beta \frac{\kappa_{si}(1+\kappa_x/\kappa_z)}{\kappa_x + \kappa_{si}} \int_0^1 h_j dj + (1-\beta)\alpha \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} \quad \forall i \in [0,1]. \tag{29}$$

Integrating over $[0,1]$ in (28) and (29) yields a linear-system of two equations, which implies that

$$\int_0^1 c_i di = \alpha \left(1 - \left(1 + \beta \frac{\kappa_x}{\kappa_z}\right) \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right) \Big/ \left(1 - \beta \frac{\kappa_x}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right)$$

and

$$\int_0^1 h_i di = \alpha \left(\int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right) \Big/ \left(1 - \beta \frac{\kappa_x}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right).$$

Thus, we can rewrite (28) and (29) as $c_i = \alpha \frac{\kappa_x}{\kappa_x + \kappa_{si}} \left(1 - \beta \frac{\kappa_x + \kappa_{si}}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right) \Big/ \left(1 - \beta \frac{\kappa_x}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right)$ and $h_i = \alpha \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} \Big/ \left(1 - \beta \frac{\kappa_x}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right)$. Finally, noting that $c_i + h_i = \alpha$ and setting $h_i = \alpha\omega_i$ completes the proof. Q.E.D.

The next step in our analysis involves studying the firms' acceptance/rejection decisions that precede the competition subgame. We restrict attention to subgame perfect equilibria in which all firms accept the provider's offers. This is without loss of generality, since the case in which there is a firm i that rejects the provider's offer is surplus-equivalent to the case in which the provider offers a signal of precision $\kappa_{si} = 0$ at price $p_i = 0$ to firm i, and firm i accepts the offer. The equilibrium acceptance/rejection decisions can be characterized as follows. Each firm $i \in [0,1]$ accepts the provider's offer if $\Delta_i = \mathbb{E}[\pi(a(\kappa_{si}, \kappa_{\mathbf{s-i}}))] - \mathbb{E}[\pi(a(0, \kappa_{\mathbf{s-i}}))] \geq p_i$, i.e., if price p_i is lower than the expected surplus of firm i. Thus, it is optimal for the provider to offer $p_i = \Delta_i$ for all $i \in [0,1]$.

Using the equilibrium characterization from Lemma 3, we can compute the expected surplus Δ_i of firm i, which, in turn, is equal to price p_i, i.e.,

$$p_i = \alpha^2 \left(\frac{\gamma_2}{2}\right) \left(\frac{\kappa_{si}}{\kappa_x + \kappa_{si}}\right) \left(\frac{1}{\kappa_x}\right) \Big/ \left[1 - \beta \frac{\kappa_x}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right]^2. \tag{30}$$

The provider's expected equilibrium profit is given by

$$\Pi(\kappa_{\mathbf{s}}, \beta) = \int_0^1 p_i di = \alpha^2 \left(\frac{\gamma_2}{2}\right) \left(\frac{1}{\kappa_x} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right) \Big/ \left[1 - \beta \frac{\kappa_x}{\kappa_z} \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di\right]^2, \tag{31}$$

and her problem can now be simply written as

$$\begin{aligned} \max_{\{\kappa_{si}\}_{i\in[0,1]}} \quad & \Pi(\kappa_{\mathbf{s}}, \beta) \\ s.t. \quad & 0 \le \kappa_{si} \le \kappa_z \quad \forall i \in [0,1]. \end{aligned} \tag{32}$$

The following lemma allows us to further simplify the optimization problem above.

Lemma 4. *The objective function of problem (32) depends on $\{\kappa_{si}\}_{i\in[0,1]}$ only through a constant*

$$D = \int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}} di. \tag{33}$$

Furthermore, for any optimal solution $\{\kappa^*_{si}\}_{i\in[0,1]}$ of problem (32), there exist a constant solution $\bar{\kappa}_s$ that is feasible and achieves the same objective value of $\{\kappa^*_{si}\}_{i\in[0,1]}$.

Proof: The first statement follows directly from expression (31). For the second statement, let $\{\kappa^*_{si}\}_{i\in[0,1]}$ be an optimal solution of problem (32), with corresponding $D^* = \displaystyle\int_0^1 \frac{\kappa^*_{si}}{\kappa_x + \kappa^*_{si}} di$. Define constant $\bar{\kappa}_s$ as $\bar{\kappa}_s := \dfrac{D^* \kappa_x}{1 - D^*}$. Note that $\dfrac{\bar{\kappa}_s}{\kappa_x + \bar{\kappa}_s} = D^*$, which implies that $\bar{\kappa}_s$ achieves the same objective value as $\{\kappa^*_{si}\}_{i\in[0,1]}$. Finally, we need to verify that $\bar{\kappa}_s$ is feasible. By the feasibility of $\{\kappa^*_{si}\}_{i\in[0,1]}$, i.e., $0 \le \kappa^*_{si} \le \kappa_z$ for all $i \in [0,1]$, it follows that $0 \le D^* \le \dfrac{\kappa_z}{\kappa_x + \kappa_z}$ and thus $0 \le \bar{\kappa}_s \le \kappa_z$. This implies that the constant $\bar{\kappa}_s$ is feasible and it achieves the maximum objective value, which completes the proof. Q.E.D.

Lemma 4 allows us to solve a simplified problem, in which the provider offers a signal of precision κ_s to all firms $i \in [0,1]$. Furthermore, using the optimal value for κ_s together with equation (33) allows us to characterize the set of optimal solutions for the original problem (32). In particular, replacing κ_s for κ_{si} in problem (32), the provider's problem simplifies to

$$\begin{aligned} \max_{\kappa_s} \quad & \Pi = \alpha^2 \left(\frac{\gamma_2}{2}\right) \left(\frac{\kappa_s}{\kappa_x}\right) \frac{\kappa_s + \kappa_x}{[(1 - \beta\kappa_s/\kappa_z)\kappa_x + \kappa_s]^2} \\ s.t. \quad & 0 \le \kappa_s \le \kappa_z. \end{aligned} \tag{34}$$

Proof of part (a): Let $\beta \ge -(1 + \kappa_z/\kappa_x)$ and consider the simplified problem (34). Differentiating the objective with respect to κ_s yields

$$\frac{\partial \Pi}{\partial \kappa_s} = \alpha^2 \left(\frac{\gamma_2}{2}\right) \frac{(1 + \beta\kappa_s/\kappa_z)\kappa_x + \kappa_s}{[(1 - \beta\kappa_s/\kappa_z)\kappa_x + \kappa_s]^3}. \tag{35}$$

The assumption on β implies that (35) is positive, which means that it is optimal to set $\kappa^*_s = \kappa_z$. By Lemma 4, this implies that any solution $\{\kappa^*_{si}\}_{i\in[0,1]}$ to problem (32) that is feasible and such that

$$\int_0^1 \frac{\kappa^*_{si}}{\kappa_x + \kappa^*_{si}} di = \frac{\kappa_z}{\kappa_x + \kappa_z},$$

is an optimal solution. Thus, problem (32) has a unique optimal solution in this case, i.e.,$\kappa_{si}^* = \kappa_z, \forall i \in [0,1]$. Replacing this solution into (30) we obtain

$$p_i^* = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z + \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2} = p^*.$$

Proof of part (b): Let $\beta < -(1+\kappa_z/\kappa_x)$ and consider problem (34). In this case, the partial derivative given in (35) evaluated at $\kappa_s^* = \kappa_z$ is negative, so the provider is better off by offering noisy signals to the firms. Solving for the optimal κ_s using a firm's first-order optimality condition yields

$$\kappa_s^* = \frac{\kappa_x}{-(1+\beta\kappa_x/\kappa_z)} < \kappa_z.$$

By Lemma 4, this implies that any solution $\{\kappa_{si}^*\}_{i\in[0,1]}$ to problem (32) that is feasible and such that

$$\int_0^1 \frac{\kappa_{si}^*}{\kappa_x + \kappa_{si}^*} di = -\frac{\kappa_z}{\beta\kappa_x}, \tag{36}$$

is an optimal solution. Finally, replacing (36) into (30) yields $p_i^* = \alpha^2\left(\frac{\gamma_2}{2}\right)\frac{\kappa_{si}^*}{4(\kappa_x+\kappa_{si}^*)\kappa_x}$. Q.E.D.

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EC.1. Additional Proofs

Proof of Proposition 6

Recall from our results in Section 3 that, without any loss of generality, we can restrict attention to equilibria in which the provider offers the market signal to all firms, i.e., $\lambda = 1$. Also recall that it is always optimal for the provider to offer market signals that are independent conditional on the realization of her signal, i.e., $\rho_\xi = 0$.

Throughout the proof we rescale firms' profit so that $\alpha = \gamma_0/(\gamma_2 - \gamma_1) = 1$ Note that this is without loss of generality and it considerably simplifies the exposition. We conjecture that equilibrium strategies are linear in x_i, s_i, and S_i, and then verify our hypothesis. In particular, we conjecture that

$$a(x_i, s_i, S_i) = b_1 x_i + b_2 S_i + b_3 s_i.$$

By definition $A = \int_0^1 a(x_i, s_i, S_i)\, di$, thus, we have $A = b_1\theta + b_2 A + b_3 z$. This further implies that

$$A = \frac{b_1\theta + b_3 z}{1 - b_2}.$$

Using the above equation for A, firm i's expected profit simplifies to

$$\begin{aligned}\mathbb{E}[\pi_i|x_i, s_i, S_i] &= \gamma_0 a_i \mathbb{E}[\theta|x_i, s_i, S_i] + \gamma_1 a_i \frac{b_1 \mathbb{E}[\theta|x_i, s_i, S_i] + b_3 \mathbb{E}[z|x_i, s_i, S_i]}{1 - b_2} - \frac{\gamma_2}{2} a_i^2 \\ &= a_i \mathbb{E}[\theta|x_i, s_i, S_i] \left(\gamma_0 + \frac{\gamma_1 b_1}{1 - b_2}\right) + a_i \mathbb{E}[z|x_i, s_i, S_i] \left(\frac{\gamma_1 b_3}{1 - b_2}\right) - \frac{\gamma_2}{2} a_i^2.\end{aligned}$$

Taking the first-order optimality condition with respect to a_i, $\frac{\partial}{\partial a_i}\mathbb{E}[\pi_i|x_i, s_i, S_i] = 0$, and recalling that $z = \zeta + \theta$, we can express the equilibrium action of firm i as

$$a_i = \left[(1 - \beta) + \beta \frac{b_1 + b_3}{1 - b_2}\right] E[\theta|x_i, s_i, S_i] + \beta \left(\frac{b_3}{1 - b_2}\right) E[\zeta|x_i, s_i, S_i]. \tag{EC.1}$$

Before proceeding, we make a change of variable, setting $\kappa_s = t\kappa_z$ with $t \in [0, 1]$. The conditional expectations in (EC.1) are given respectively by

$$\begin{aligned}E[\theta|x_i, s_i, S_i] \;=\; &\kappa_x \frac{(1 - b_2)^2 \kappa_z + (1 - t) b_3^2 \kappa_\nu}{D} x_i + \kappa_\nu \frac{(1 - b_2)\,[b_1 + (1 - t) b_3]\,\kappa_z}{D} S_i + \\ &+ t\kappa_z \frac{(1 - b_2)^2 \kappa_z - b_1 b_3 \kappa_\nu}{D} z_i,\end{aligned} \tag{EC.2}$$

and

$$\begin{aligned}E[\zeta|x_i, s_i, S_i] \;=\; &-\kappa_x \frac{(1 - b_2)^2 t\kappa_z + b_3\,(b_1 + b_3)\,(1 - t)\kappa_\nu}{D} x_i + \kappa_\nu \frac{(1 - b_2)\,[b_3 (1 - t)\kappa_x - b_1 t\kappa_z]}{D} S_i + \\ &+ t\kappa_z \frac{b_1\,(b_1 + b_3)\,\kappa_\nu + (1 - b_2)^2 \kappa_x}{D} z_i,\end{aligned} \tag{EC.3}$$

where

$$D = \kappa_\nu \left[b_1^2\kappa_z + b_3(1-t)\left(b_3\left(\kappa_x+\kappa_z\right)+2b_1\kappa_z\right)\right] + \kappa_z\left(1-b_2\right)^2\left(t\kappa_z+\kappa_x\right).$$

These expressions are obtained using the formula for the conditional expectation of Gaussian random vectors. Substituting (EC.2) and (EC.3) into (EC.1), and solving for b_1, b_2, b_3, yields the following system of equations, which the equilibrium coefficients must satisfy:

$$b_1 = \kappa_x \frac{(1-\beta)b_3^2(1-t)\kappa_\nu + (1-b_2)\left[\beta b_1 + (1-\beta)(1-b_2) + \beta b_3(1-t)\right]\kappa_z}{D}, \tag{EC.4}$$

$$b_2 = \kappa_\nu \frac{b_1\left\{\left[\beta b_1 + (1-\beta)(1-b_2)\right] + \beta b_3^2(1-t) + b_3(1-t)\left[2\beta b_1 + (1-\beta)(1-b_2)\right]\right\}\kappa_z + \beta b_3^2(1-t)\kappa_x}{D}, \tag{EC.5}$$

$$b_3 = t\kappa_z \frac{(1-b_2)\left[\beta b_1 + (1-\beta)(1-b_2)\right]\kappa_z + b_3\left[\beta(1-b_2)(\kappa_x+\kappa_z) - (1-\beta)b_1\kappa_\nu\right]}{D}. \tag{EC.6}$$

Let $a_i^1 := b_1 x_i + b_2 S_i + b_3 s_i$ denote the equilibrium action of firm i and let $a_i^0 := \tilde{b}_1 x_i + \tilde{b}_2 S_i$ denote the action that firm i would have taken if it deviated from the equilibrium path and did not purchase the market signal s_i. In that case, firm i's information set is (x_i, S_i) and the coefficients $\tilde{b}_1, \tilde{b}_2$ can be then characterized as above. We have

$$\tilde{b}_1 = \kappa_x \frac{(1-\beta)b_3^2\kappa_\nu + (1-b_2)\left[\beta b_1 + (1-\beta)(1-b_2) + \beta b_3\right]\kappa_z}{\kappa_x\left[b_3^2\kappa_\nu + (1-b_2)^2\kappa_z\right] + (b_1+b_3)^2\kappa_\nu\kappa_z}, \tag{EC.7}$$

$$\tilde{b}_2 = \kappa_\nu \frac{\beta b_3^2\kappa_x + (b_1+b_3)\left[\beta b_1 + (1-\beta)(1-b_2) + \beta b_3\right]\kappa_z}{\kappa_x\left[b_3^2\kappa_\nu + (1-b_2)^2\kappa_z\right] + (b_1+b_3)^2\kappa_\nu\kappa_z}. \tag{EC.8}$$

Note that these coefficients depend on b_1, b_2, b_3, since they are derived under the assumption that firm i deviates, while all other firms are playing according to their equilibrium strategies. Using a_i^1 and a_i^0 we can characterize the expected profits of a firm that observes (x_i, s_i, S_i) as

$$E[\pi_i(a_i^1, A)] = \gamma_2\left[\frac{1}{2\kappa_\theta} + (\beta - 1/2)\left(\frac{1}{\kappa_z}\right)\frac{b_3^2}{(b_1+b_3)^2} - \frac{b_1^2}{2\kappa_x} - \frac{b_2^2}{2\kappa_\nu} - \left(\frac{1-t}{t\kappa_z}\right)\frac{b_3^2}{2}\right],$$

and the expected profit of a firm that does not observe s_i as

$$E[\pi_i(a_i^0, A)] = \gamma_2\left[\frac{1}{2\kappa_\theta} + (\beta - \tilde{b}_2/2)\ \tilde{b}_2\left(\frac{1}{\kappa_z}\right)\frac{b_3^2}{(b_1+b_3)^2} - \frac{\tilde{b}_1^2}{2\kappa_x} - \frac{\tilde{b}_2^2}{2\kappa_\nu}\right].$$

The equilibrium surplus of a firm from purchasing the market signal is thus given by

$$\Delta := E[\pi_i(a_i^1, A)] - E[\pi_i(a_i^0, A)].$$

As we establish in Lemma 2, at equilibrium the monopolist offers to each firm i price $p_i = \Delta$, and all firms accept her offer. Thus, the expected equilibrium profit of the monopolist is $\Pi = \int_0^1 p_i\ di = \Delta$, i.e.,

$$\Pi(t, \kappa_\nu, \beta) = \gamma_2\left\{\left[(1-\tilde{b}_2)\beta - (1-\tilde{b}_2^2)/2\right]\left(\frac{1}{\kappa_z}\right)\frac{b_3^2}{(b_1+b_3)^2} - \frac{b_1^2-\tilde{b}_1^2}{2\kappa_x} - \frac{b_2^2-\tilde{b}_2^2}{2\kappa_\nu} - \left(\frac{1-t}{t\kappa_z}\right)\frac{b_3^2}{2}\right\}. \tag{EC.9}$$

Proof of part (a): Given the change of variable $\kappa_s = t\kappa_z$, the provider's decision variable is $t \in [0,1]$. Clearly it is never optimal to set $t \le 0$ and, thus, the provider's problem simplifies to:

$$\max \quad \Pi(t, \kappa_\nu, \beta), \quad s.t. \quad t \le 1.$$

Let

$$\Pi^*(\kappa_\nu) := \max_t \ \Pi(t, \kappa_\nu, \beta) \quad s.t. \quad t \le 1.$$

Our goal is to characterize how the maximum profit changes in the presence of some information leakage, i.e., we are interested in determining the sign of $\partial\Pi^*/\partial\kappa_\nu$ evaluated at $\kappa_\nu = 0$. To this end, we use the envelope theorem for constrained optimization problems. The Lagrangian associated with the provider's problem is given as

$$L(t, \mu, \kappa_\nu, \beta) = \Pi(t, \kappa_\nu, \beta) + \mu(1 - t).$$

By the envelope theorem we have that

$$\frac{\partial \Pi^*}{\partial \kappa_\nu}(\kappa_\nu) = \frac{\partial L}{\partial \kappa_\nu}(t^*, \mu^*, \kappa_\nu, \beta)\bigg|_{t^*=t^*(\kappa_\nu), \mu^*=\mu^*(\kappa_\nu)} = \frac{\partial \Pi}{\partial \kappa_\nu}(t^*, \kappa_\nu, \beta)\bigg|_{t^*=t^*(\kappa_\nu)}, \tag{EC.10}$$

where the second equality holds since the constraint itself does not depend on κ_ν. Differentiating (EC.9) with respect to κ_ν yields

$$\frac{\partial \Pi}{\partial \kappa_\nu}(t, \kappa_\nu, \beta) = \gamma_2\Big(K_0 + K_1 + K_2 + K_3 + K_4 + K_5 + K_6\Big), \tag{EC.11}$$

where

$$\begin{aligned}
K_0 &= \frac{\tilde{b}_2 \tilde{b}_2^{\kappa_\nu} - \tilde{b}_2^{\kappa_\nu}\beta}{(b_1 + b_3)^2 \kappa_z} b_3^2 \\
K_1 &= \frac{2(1 - \tilde{b}_2)\beta - (1 - \tilde{b}_2^2)}{(b_1 + b_3)^2 \kappa_z} b_3 b_3^{\kappa_\nu} \\
K_2 &= \frac{2(1 - \tilde{b}_2)\beta - (1 - \tilde{b}_2^2)}{(b_1 + b_3)^3 \kappa_z} b_3^2 (b_1^{\kappa_\nu} + b_3^{\kappa_\nu}) \\
K_3 &= -\frac{b_1 b_1^{\kappa_\nu} - \tilde{b}_1 \tilde{b}_1^{\kappa_\nu}}{\kappa_x} \\
K_4 &= -\frac{b_2 b_2^{\kappa_\nu} - \tilde{b}_2 \tilde{b}_2^{\kappa_\nu}}{\kappa_\nu} \\
K_5 &= -\frac{1-t}{t\kappa_z} b_3 b_3^{\kappa_\nu} \\
K_6 &= \frac{b_2^2 - \tilde{b}_2^2}{2\kappa_\nu^2},
\end{aligned}$$

and

$$b_j^{\kappa_\nu} := \frac{\partial}{\partial \kappa_\nu} b_j(t, \kappa_\nu), \quad j = 1,2,3 \qquad \text{and} \qquad \tilde{b}_k^{\kappa_\nu} := \frac{\partial}{\partial \kappa_\nu} \tilde{b}_k(t, \kappa_\nu), \quad k = 1,2. \tag{EC.12}$$

Next, we evaluate (EC.11) at $\kappa_\nu = 0$. To do so, we compute and evaluate all the coefficients at $\kappa_\nu = 0$ and then replace them in (EC.11). In particular, evaluating equations (EC.4)-(EC.8) at $\kappa_\nu = 0$ yields

$$b_1(t,0) = \frac{(1-\beta t)\kappa_x}{t\kappa_z + (1-\beta t)\kappa_x}, \qquad b_2(t,0) = 0, \qquad b_3(t,0) = \frac{t\kappa_z}{t\kappa_z + (1-\beta t)\kappa_x},$$
$$\tilde{b}_1(t,0) = 1, \qquad \tilde{b}_2(t,0) = 0.$$

Differentiating (EC.4)-(EC.8) with respect to κ_ν, and then evaluating at $\kappa_\nu = 0$ yields

$$\begin{aligned}
b_1^{\kappa_\nu}(t,0) &= -\frac{\kappa_x\left[(1-\beta t)^2\kappa_x + (1-t)t\kappa_z\right]^2}{\left[(1-\beta t)\kappa_x + t\kappa_z\right]^4}, \\
b_2^{\kappa_\nu}(t,0) &= \frac{(1-\beta t)^2\kappa_x + (1-t)t\kappa_z}{\left[(1-\beta t)\kappa_x + t\kappa_z\right]^2}, \\
b_3^{\kappa_\nu}(t,0) &= -\frac{t\kappa_z\left[\kappa_x + (1-2\beta)t\kappa_x + t\kappa_z\right]\left[(1-t)t\kappa_z + (1-\beta t)^2\kappa_x\right]}{\left[(1-\beta t)\kappa_x + t\kappa_z\right]^4}, \\
\tilde{b}_1^{\kappa_\nu}(t,0) &= -\frac{t^2\kappa_z^2 + (1-\beta t)^2\kappa_x^2 + t(2-\beta t)\kappa_x\kappa_z}{\kappa_x\left[(1-\beta t)\kappa_x + t\kappa_z\right]^2}, \\
\tilde{b}_2^{\kappa_\nu}(t,0) &= \frac{t^2\kappa_z^2 + (1-\beta t)^2\kappa_x^2 + t(2-\beta t)\kappa_x\kappa_z}{\kappa_x\left[(1-\beta t)\kappa_x + t\kappa_z\right]^2}.
\end{aligned}$$

Replacing the above sets of coefficients into Equation (EC.11) and simplifying, yields the following

$$\left.\frac{\partial \Pi}{\partial \kappa_\nu}(t,\kappa_\nu,\beta)\right|_{\kappa_\nu=0} = \gamma_2 \frac{c_5 t^5\kappa_z^5 + c_4 t^4\kappa_z^4\kappa_x + c_3 t^3\kappa_z^3\kappa_x^2 + c_2 t^2\kappa_z^2\kappa_x^3 + c_1 t\kappa_z\kappa_x^4}{2\kappa_x^2\left[t\kappa_z + (1-\beta t)\kappa_x\right]^5}, \tag{EC.13}$$

where

$$c_5 = -1, \qquad c_4 = 3\beta t - 5, \qquad c_3 = -[(5\beta^2-1)t - 14\beta + 2]t - 9,$$
$$c_2 = -7 + t\{-4 + 19\beta + t[1 + (4+t)\beta - (15+4t)\beta^2 + 5t\beta^3]\},$$
$$c_1 = 2(\beta t - 1)^2((2\beta - 1)t - 1).$$

Finally, to determine the sign of $\left.\frac{\partial \Pi^*}{\partial \kappa_\nu}(\kappa_\nu)\right|_{\kappa_\nu=0}$, we consider the following two cases:

(i) Case 1: $\beta \in (-\infty, 0)$. Recall that $t \in [0,1]$, and consider Equation (EC.13). It is straightforward to see that the denominator is always positive. Next, we show that the numerator is always negative. Note that coefficients c_5, c_4 and c_1 are always negative. Moreover, we have that $(5\beta^2-1)t - 14\beta + 2 > -14\beta + 1 > 0$, which implies that c_3 is negative for all $\beta < 0$ and $t \in [0,1]$. Finally, noting that $1 + (4+t)\beta - (15+4t)\beta^2 + 5t\beta^3 < 1$ it is immediate to verify that c_2 is negative. This establishes that $\left.\frac{\partial \Pi}{\partial \kappa_\nu}(t,\kappa_\nu,\beta)\right|_{\kappa_\nu=0} < 0$ for all $\beta < 0$ and $t \in [0,1]$. In particular,

$$\left.\frac{\partial \Pi}{\partial \kappa_\nu}(t^*,\kappa_\nu,\beta)\right|_{t^*=t^*(\kappa_\nu),\kappa_\nu=0} < 0 \qquad \forall \beta < 0.$$

(ii) Case 2: $\beta \in [0, 1/2)$. Recall that when $\kappa_\nu = 0$ we have $t^* = 1$ for all β in this interval. We next substitute $t^* = 1$ in Equation (EC.13) and show that it is negative. Note that the denominator simplifies to $2\kappa_x^2 [\kappa_z + (1-\beta)\kappa_x]^5 < 0$, and the numerator simplifies to

$$-\kappa_z \left[\kappa_z^4 + (5-3\beta)\kappa_x\kappa_z^3 + (10-14\beta+5\beta^2)\kappa_x^2\kappa_z^2 + (1-\beta)(10-14\beta+5\beta^2)\kappa_x^3\kappa_z + 4(1-\beta)^3\kappa_x^4\right].$$

Noting that $10 - 14\beta + 5\beta^2$ is positive (as a quadratic expression with negative determinant), we conclude that the numerator is always negative. This establishes that

$$\frac{\partial \Pi}{\partial \kappa_\nu}(t^*, \kappa_\nu, \beta)\bigg|_{t^*=t^*(\kappa_\nu), \kappa_\nu=0} < 0 \qquad \forall \beta \in [0, 1/2).$$

Combining Cases 1 and 2 discussed above, applying the envelope theorem, and using Equation (EC.10), we conclude that

$$\frac{\partial \Pi^*}{\partial \kappa_\nu}(\kappa_\nu)\bigg|_{\kappa_\nu=0} < 0 \qquad \forall \beta \in (-\infty, 1/2),$$

which completes the proof of part (a).

Proof of part (b): Let $\hat{\beta} = -(1 + \kappa_z/\kappa_x)$. When $\kappa_\nu = 0$, Proposition 3 implies that $\hat{\beta}$ is such that $\kappa_s^* = \kappa_z$ is the unconstrained optimum at $\beta = \hat{\beta}$. In other words,

$$\arg\max_t \; \Pi(t, 0, \hat{\beta}) = 1,$$

and

$$\frac{\partial}{\partial t}\Pi(t, 0, \hat{\beta})\bigg|_{t=1} = 0. \tag{EC.14}$$

The proof follows by showing that there exists constant $\bar{K}$ such that for all $\kappa_\nu \leq \bar{K}$, there exists $\bar{T}$ such that $\forall\ t \in [1-\bar{T}, 1)$ the following holds

$$\Pi(t, \kappa_\nu, \hat{\beta}) > \Pi(1, \kappa_\nu, \hat{\beta}).$$

In particular, this would imply that in the presence of leakage it is optimal for the provider to distort her signal at $\beta = \hat{\beta}$ (continuity of Π with respect to β would then imply the statement of the proposition).

In order to prove the above, let us define

$$g(\kappa_\nu, \beta) = \frac{\partial}{\partial t}\Pi(t, \kappa_\nu, \beta)\bigg|_{t=1},$$

and show that $\dfrac{\partial g}{\partial \kappa_\nu}(0, \hat{\beta}) < 0$. For any β, we have

$$\frac{\partial g}{\partial \kappa_\nu}(0, \beta) = \gamma_2\Big(G_0 + G_1 + G_2 + G_3 + G_4 + G_5 + G_6 + G_7 + G_8 + G_9\Big), \tag{EC.15}$$

where

$$G_0 = \frac{\tilde{b}_2 \tilde{b}_2^{\kappa_\nu} - \beta \tilde{b}_2^{\kappa_\nu}}{\kappa_z (b_1 + b_3)^2} 2 b_3^t b_3 \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_1 = \frac{\tilde{b}_2 \tilde{b}_2^{t} - \beta \tilde{b}_2^{t}}{\kappa_z (b_1 + b_3)^2} 2 b_3^{\kappa_\nu} b_3 \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_2 = \frac{\tilde{b}_2^{\kappa_\nu} + \tilde{b}_2^t - \beta \tilde{b}_2^{t,\kappa_\nu} + \tilde{b}_2 \tilde{b}_2^{t,\kappa_\nu}}{\kappa_z (b_1 + b_3)^2} b_3^2 \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_3 = -\frac{(b_1^{\kappa_\nu} + b_3^{\kappa_\nu})(\tilde{b}_2 \tilde{b}_2^t - \beta \tilde{b}_2^t)}{\kappa_z (b_1 + b_3)^3} 2 b_3^2 \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_4 = \frac{2\beta(1-\tilde{b}_2) + (\tilde{b}_2^2 - 1)}{\kappa_z (b_1 + b_3)^2} \left(b_3^{\kappa_\nu} b_3^t + b_3^{t,\kappa_\nu} b_3 \right) \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_5 = -\frac{2\beta(1-\tilde{b}_2) + (\tilde{b}_2^2 - 1)}{\kappa_z (b_1 + b_3)^3} \left[\left(b_1^{t,\kappa_\nu} + b_3^{t,\kappa_\nu} \right) b_3^2 + 2 \left(b_1^{\kappa_\nu} + b_3^{\kappa_\nu} \right) b_3^t b_3 \right] \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_6 = -\frac{b_1^{\kappa_\nu} b_1^t - \tilde{b}_1^{\kappa_\nu} \tilde{b}_1^t + b_1 b_1^{t,\kappa_\nu} - \tilde{b}_1 \tilde{b}_1^{t,\kappa_\nu}}{\kappa_x} \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_7 = -\frac{b_2^{\kappa_\nu} b_2^t - \tilde{b}_2^{\kappa_\nu} \tilde{b}_2^t + b_2 b_2^{t,\kappa_\nu} - \tilde{b}_2 \tilde{b}_2^{t,\kappa_\nu}}{\kappa_\nu} \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_8 = \frac{b_2 b_2^t - \tilde{b}_2 \tilde{b}_2^t}{\kappa_\nu^2} \Bigg|_{t=1,\kappa_\nu=0}$$

$$G_9 = \frac{b_3^{\kappa_\nu} b_3}{\kappa_z} \Bigg|_{t=1,\kappa_\nu=0},$$

and

$$b_j^t := \frac{\partial}{\partial t} b_j(t,\kappa_\nu), \qquad b_j^{t,\kappa_\nu} := \frac{\partial^2}{\partial \kappa_\nu \partial t} b_j(t,\kappa_\nu), \qquad \text{for} \quad j = 1,2,3\ ,$$
$$\tilde{b}_k^t := \frac{\partial}{\partial t} \tilde{b}_k(t,\kappa_\nu), \qquad \tilde{b}_k^{t,\kappa_\nu} := \frac{\partial^2}{\partial \kappa_\nu \partial t} \tilde{b}_k(t,\kappa_\nu), \qquad \text{for} \quad k = 1,2.$$

To compute the second derivative above, we evaluate the coefficients at $t = 1$ and $\kappa_\nu = 0$. In particular, evaluating (EC.4)-(EC.8) at $t = 1, \kappa_\nu = 0$, and solving the resulting system yields

$$b_1(1,0) = \frac{(1-\beta)\kappa_x}{\kappa_z + (1-\beta)\kappa_x}, \qquad b_2(1,0) = 0, \qquad b_3(1,0) = \frac{\kappa_z}{\kappa_z + (1-\beta)\kappa_x}, \qquad \tilde{b}_1(1,0) = 0, \qquad \tilde{b}_2(1,0) = 0.$$

Furthermore, differentiating (EC.4)-(EC.8) with respect to κ_ν, evaluating at $t = 1, \kappa_\nu = 0$, and solving the resulting system yields

$$b_1^{\kappa_\nu}(1,0) = -\frac{(1-\beta)^4 \kappa_x^3}{[(1-\beta)\kappa_x + \kappa_z]^4},$$
$$b_2^{\kappa_\nu}(1,0) = \frac{(1-\beta)^2 \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2},$$

$$b_3^{\kappa_\nu}(1,0) = -\frac{(1-\beta)^2\,[2(1-\beta)\kappa_x + \kappa_z]\,\kappa_x\kappa_z}{[(1-\beta)\kappa_x + \kappa_z]^4},$$
$$\tilde{b}_1^{\kappa_\nu}(1,0) = -\frac{(1-\beta)^2\kappa_x^2 + (2-\beta)\kappa_x\kappa_z + \kappa_z^2}{\kappa_x\,[(1-\beta)\kappa_x + \kappa_z]^2},$$
$$\tilde{b}_2^{\kappa_\nu}(1,0) = \frac{(1-\beta)^2\kappa_x^2 + (2-\beta)\kappa_x\kappa_z + \kappa_z^2}{\kappa_x\,[(1-\beta)\kappa_x + \kappa_z]^2}.$$

Next, differentiating (EC.4)-(EC.8) with respect to t, evaluating at $t=1, \kappa_\nu = 0$, and solving the resulting system yields

$$b_1^t(1,0) = -\frac{\kappa_x\kappa_z}{[(1-\beta)\kappa_x + \kappa_z]^2}, \qquad b_2^t(1,0) = 0, \qquad b_3^t(1,0) = \frac{\kappa_x\kappa_z}{[(1-\beta)\kappa_x + \kappa_z]^2},$$
$$\tilde{b}_1^t(1,0) = 0, \qquad \tilde{b}_2^t(1,0) = 0.$$

Finally, differentiating (EC.4)-(EC.8) with respect to both t and κ_ν, evaluating at $t=1, \kappa_\nu = 0$, and solving the resulting system yields

$$b_1^{t,\kappa_\nu}(1,0) = \frac{2(1-\beta)^2\,[3(1-\beta)\kappa_x + \kappa_z]}{[(1-\beta)\kappa_x + \kappa_z]^5}\kappa_x^2\kappa_z,$$
$$b_2^{t,\kappa_\nu}(1,0) = -\frac{3(1-\beta)\kappa_x + \kappa_z}{[(1-\beta)\kappa_x + \kappa_z]^3}\kappa_z,$$
$$b_3^{t,\kappa_\nu}(1,0) = \frac{-3(1-\beta)^3\kappa_x^3 + 5(1-\beta)^2\kappa_x^2\kappa_z + 5(1-\beta)\kappa_x\kappa_z^2 + \kappa_z^3}{[(1-\beta)\kappa_x + \kappa_z]^5}\kappa_z,$$
$$\tilde{b}_1^{t,\kappa_\nu}(1,0) = -\frac{2\beta\kappa_x\kappa_z}{[(1-\beta)\kappa_x + \kappa_z]^3},$$
$$\tilde{b}_2^{t,\kappa_\nu}(1,0) = \frac{2\beta\kappa_x\kappa_z}{[(1-\beta)\kappa_x + \kappa_z]^3}.$$

Finally, we substitute the coefficients in Equation (EC.15) and obtain that

$$\frac{\partial g}{\partial \kappa_\nu}(0,\beta) = -\gamma_2\frac{\kappa_z\,[(1-\beta)^2\,(\beta^2+\beta+2)\,\kappa_x^3 + 5(1-\beta)^2\kappa_x^2\kappa_z + (4\beta^2-9\beta+4)\kappa_x\kappa_z^2 + (1-2\beta)\kappa_z^3]}{[(1-\beta)\kappa_x + \kappa_z]^6}.$$

It is immediate to verify that the above equation is strictly negative for all $\beta \in (-\infty, 1/2)$, and in particular it is strictly negative at $\hat{\beta} = -(1+\kappa_z/\kappa_x)$. Moreover, noting that $g(0,\hat{\beta}) = 0$, we conclude that there exists $\bar{K}$ such that for all $\kappa_\nu < \bar{K}$ we have $g(\kappa_\nu, \hat{\beta}) < 0$. This in turn implies that there exists $\bar{T}$ such that $\forall\ t \in [1-\bar{T}, 1)$ we have

$$\Pi(t,\kappa_\nu,\hat{\beta}) > \Pi(1,\kappa_\nu,\hat{\beta}).$$

Thus, at $\beta = \hat{\beta}$ it is optimal for the seller to set $t^* < 1$, i.e., distort her information and set $\kappa_s^* < \kappa_z$.

To complete the proof, note that by continuity of $\Pi(t,\kappa_\nu,\beta)$ with respect to β, there exists $\bar{\beta} \in (0,\hat{\beta})$ such that $\Pi(t,\kappa_\nu,\hat{\beta}) - \Pi(1,\kappa_\nu,\hat{\beta}) > 0$ for all $\beta \in [\hat{\beta}, \bar{\beta})$, and thus $\kappa_s^* < \kappa_z$ for all $\beta \in [\hat{\beta}, \bar{\beta})$. Q.E.D.

Proof of Proposition 7

Following the same steps as in the proof of Proposition 5 we can derive the equilibrium quantity decisions of each firm i in the competition subgame. These, in turn, allow us to compute the equilibrium surplus of a firm i that observes a signal of precision κ_{si} as

$$\Delta_i = \mathbb{E}[\pi(q(\kappa_{si},\kappa_{\mathbf{s-i}}))] - \mathbb{E}[\pi(q(0,\kappa_{\mathbf{s-i}}))] = K\left(\frac{\kappa_{si}/c_i}{\kappa_x+\kappa_{si}}\right)\Big/\kappa_x(1-\gamma_1 D\kappa_x/\kappa_z)^2,$$

where $K = \dfrac{\gamma_0^2/2}{\left(1-\gamma_1\int_0^1 \frac{1}{c_i}di\right)^2}$ and $D = \displaystyle\int_0^1 \frac{\kappa_{si}/c_i}{\kappa_x+\kappa_{si}}di.$

Moreover, firm $i \in [0,1]$ accepts the provider's offer $\{p_i,\kappa_{si}\}$ if and only if $\Delta_i \geq p_i$. Thus, it is optimal for the provider to offer $p_i = \Delta_i$ for all $i\in[0,1]$ and leave no surplus to the firms.

The provider's equilibrium profit is then given by

$$\Pi(\kappa_{\mathbf{s}},D) = \int_0^1 (p_i - v\kappa_{si})di = K\frac{D}{\kappa_x(1-\gamma_1 D\kappa_x/\kappa_z)^2} - v\int_0^1 \kappa_{si}di,$$

and her problem can be written as

$$\begin{aligned}
\max_{\{\kappa_{si}\}_{i\in[0,1]}} \quad & \Pi(\kappa_{\mathbf{s}},D)\\
s.t. \quad & D = \int_0^1 \frac{\kappa_{si}/c_i}{\kappa_x+\kappa_{si}}di\\
& 0\leq \kappa_{si}\leq \kappa_z \quad \forall i\in[0,1].
\end{aligned}$$

Differentiating the objective with respect to κ_{si} yields

$$\frac{d\Pi}{d\kappa_{si}} = \frac{\partial \Pi}{\partial D}\cdot\frac{\partial D}{\partial \kappa_{si}} + \frac{\partial \Pi}{\partial \kappa_{si}} = K\cdot\frac{1+\gamma_1 D\kappa_x/\kappa_z}{c_i(\kappa_x+\kappa_{si})^2(1-\gamma_1 D\kappa_x/\kappa_z)^3} - v, \tag{EC.16}$$

for all $i\in[0,1]$.

Proof of Part (a): Let $\overline{v} = K/(c_{\min}\kappa_x^2)$. Suppose that $v>\overline{v}$ and consider the profile of precisions $\{\kappa_{si}=0\}_{i\in[0,1]}$, with corresponding $D=0$. Evaluating (EC.16) at the profile above yields

$$\frac{d\Pi}{d\kappa_{si}} = \frac{K}{c_i\kappa_x^2} - v,$$

for all $i\in[0,1]$. Our assumption on v then implies that, at $\{\kappa_{si}=0\}_{i\in[0,1]}$, we have $\dfrac{d\Pi}{d\kappa_{si}}<0$ for all $i\in[0,1]$, thus the provider can only increase her profit by decreasing κ_{si} for some i. Then, the non-negativity constraints on the κ_{si}'s imply that the optimal solution is to set $\kappa_{si}^*=0$ for all $i\in[0,1]$.

Proof of Part (b): Let $\underline{v} = K\dfrac{\kappa_z + \left(1+\gamma_1 \int_0^1 \frac{1}{c_i} di\right)\kappa_x}{c_{\max}\left[\kappa_z + \left(1-\gamma_1 \int_0^1 \frac{1}{c_i} di\right)\kappa_x\right]^3}$. Suppose that $v < \underline{v}$ and consider the profile of precisions $\{\kappa_{si} = \kappa_z\}_{i\in[0,1]}$, with corresponding $D = \dfrac{\kappa_z}{\kappa_x+\kappa_z}\displaystyle\int_0^1 \frac{1}{c_i} di$. Evaluating (EC.16) at the profile above yields

$$\frac{d\Pi}{d\kappa_{si}} = K\frac{\kappa_z + \left(1+\gamma_1 \int_0^1 \frac{1}{c_i} di\right)\kappa_x}{c_i\left[\kappa_z + \left(1-\gamma_1 \int_0^1 \frac{1}{c_i} di\right)\kappa_x\right]^3} - v,$$

for all $i \in [0,1]$. Our assumption on v then implies that, at $\{\kappa_{si} = \kappa_z\}_{i\in[0,1]}$, we have $\dfrac{d\Pi}{d\kappa_{si}} > 0$ for all $i \in [0,1]$, thus the provider can only increase her profit by increasing κ_{si} for some i. This, in turn, implies that the optimal solution is to set $\kappa_{si}^* = \kappa_z$ for all $i \in [0,1]$.

Proof of Part (c): Consider now the remaining case in which v takes an intermediate value. Let κ_{si}^* be the optimal solution, with corresponding $D^* = \displaystyle\int_0^1 \frac{\kappa_{si}^*/c_i}{\kappa_x+\kappa_{si}^*} di$. Substituting the optimal solution into equation (EC.16) yields

$$\frac{d\Pi}{d\kappa_{si}} = K\frac{1+\gamma_1 D^*\kappa_x/\kappa_z}{c_i(\kappa_x+\kappa_{si}^*)^2(1-\gamma_1 D^*\kappa_x/\kappa_z)^3} - v, \tag{EC.17}$$

for all $i \in [0,1]$. Let $c^* = \dfrac{K(1+\gamma_1 D^*\kappa_x/\kappa_z)}{v\ \kappa_x^2(1-\gamma_1 D^*\kappa_x/\kappa_z)^3}$. Then, if $c_i > c^*$ we have

$$\frac{d\Pi}{d\kappa_{si}} < \left(\frac{\kappa_x^2}{(\kappa_x+\kappa_{si}^*)^2} - 1\right) v.$$

This implies that $d\Pi/d\kappa_{si} < 0$ for all $\kappa_{si}^* \geq 0$, thus it is optimal to set $\kappa_{si}^* = 0$.
On the other hand, if $c_i < \dfrac{\kappa_x^2}{(\kappa_x+\kappa_z)^2}c^*$ we have

$$\frac{d\Pi}{d\kappa_{si}} > \left[\left(\frac{\kappa_x+\kappa_z}{\kappa_x+\kappa_{si}^*}\right)^2 - 1\right] v,$$

which implies that $d\Pi/d\kappa_{si} > 0$ for all $\kappa_{si}^* \in [0,\kappa_z]$, thus it is optimal to set $\kappa_{si}^* = \kappa_z$ in this case. Finally, for intermediate values of c_i, i.e., $c_i \in \left[c^*, \dfrac{\kappa_x^2}{(\kappa_x+\kappa_z)^2}c^*\right]$, the optimal κ_{si}^* is a solution to $d\Pi/d\kappa_{si} = 0$, i.e.,

$$\kappa_{si}^* = \sqrt{\frac{K(1+\gamma_1 D^*\kappa_x/\kappa_z)}{c_i v(1-\gamma_1 D^*\kappa_x/\kappa_z)^3}} - \kappa_x = \kappa_x\left(\sqrt{c^*/c_i} - 1\right).$$

Q.E.D.

Proof of Corollary 2

Consider the optimal thresholds $\overline{v}$ and $\underline{v}$ characterized in the proof of Proposition 7. For the case of two types we have

$$\overline{v} = \left(\frac{1}{c} + \delta\right) \frac{K}{\kappa_x^2} \qquad \text{and} \qquad \underline{v} = \left(\frac{1}{c} - \delta\right) K \frac{\kappa_z + (1 + \gamma_1/c)\,\kappa_x}{[\kappa_z + (1 - \gamma_1/c)\,\kappa_x]^3},$$

where $K = \dfrac{\gamma_0^2/2}{(1-\gamma_1/c)^2}$. It is easy to see that when $v > \overline{v}$ or $v < \underline{v}$ the optimal precisions are not affected by a marginal increase in dispersion. For intermediate values of v there are four cases to consider. Before considering them in turn, recall from Proposition 7 that the optimal selling strategy has the following structure

$$\kappa_{sk}^* = \begin{cases} 0 & \text{if } \frac{1}{c_k} < \frac{1}{c^*} \\ \kappa_z & \text{if } \frac{(\kappa_x+\kappa_z)^2}{\kappa_x^2 c^*} < \frac{1}{c_k} \\ \kappa_x\left(\sqrt{c^*/c_k} - 1\right) & \text{otherwise} \end{cases}, \qquad k = 1, 2\ , \tag{EC.18}$$

where

$$c^* = \frac{K(1 + \gamma_1 D^* \kappa_x/\kappa_z)}{v\ \kappa_x^2 (1 - \gamma_1 D^* \kappa_x/\kappa_z)^3},$$

which is always positive when $v \in (\underline{v}, \overline{v})$, and

$$D^* = \frac{1}{2}\left(\frac{1}{c} + \delta\right) \frac{\kappa_{s1}^*}{\kappa_x + \kappa_{s1}^*} + \frac{1}{2}\left(\frac{1}{c} - \delta\right) \frac{\kappa_{s2}^*}{\kappa_x + \kappa_{s2}^*}. \tag{EC.19}$$

Moreover, differentiating c^* with respect to δ yields $\dfrac{\partial c^*}{\partial \delta} = \dfrac{\partial c^*}{\partial D^*}\left(\dfrac{\partial D^*}{\partial \delta} + \dfrac{\partial D^*}{\partial c^*}\dfrac{\partial c^*}{\partial \delta}\right)$, which implies that

$$\frac{\partial c^*}{\partial \delta} = \frac{\partial c^*}{\partial D^*}\frac{\partial D^*}{\partial \delta} \Big/ \left(1 - \frac{\partial c^*}{\partial D^*}\frac{\partial D^*}{\partial c^*}\right). \tag{EC.20}$$

Note that

$$\frac{\partial c^*}{\partial D^*} = \frac{2\gamma_1 K(2 + \gamma_1 D^* \kappa_x/\kappa_z)}{v\ \kappa_x \kappa_z (1 - \gamma_1 D^* \kappa_x/\kappa_z)^4},$$

is always negative since $\gamma_1 < 0$ and $2 + \gamma_1 D^* \kappa_x/\kappa_z > 0$. We proceed with considering the four cases in turn:

Case 1: $(1/c + \delta) \in \left[\dfrac{1}{c^*}, \dfrac{(\kappa_x+\kappa_z)^2}{\kappa_x^2 c^*}\right]$ and $(1/c - \delta) \in \left[\dfrac{1}{c^*}, \dfrac{(\kappa_x+\kappa_z)^2}{\kappa_x^2 c^*}\right]$. The optimal precision takes intermediate values for both firm types. Thus, replacing the optimal precisions from (EC.18) into (EC.19) yields

$$D^* = \frac{1}{c} - \frac{1}{\sqrt{c^*}}\left(\frac{1}{2}\sqrt{\frac{1}{c} + \delta} + \frac{1}{2}\sqrt{\frac{1}{c} - \delta}\right).$$

Note that $\frac{\partial D^*}{\partial \delta} > 0$ (since the square root is a concave function), therefore we can verify from (EC.20) that $\frac{\partial c^*}{\partial \delta} < 0$. Differentiating the optimal precision of type 2 firms with respect to δ yields

$$\frac{\partial \kappa_{s2}^*}{\partial \delta} = \frac{\partial}{\partial \delta}\left[\kappa_x \sqrt{c^*\left(\frac{1}{c} - \delta\right)} - \kappa_x\right] = \frac{\kappa_x}{2\sqrt{c^*\left(\frac{1}{c} - \delta\right)}}\left[-c^* + \frac{\partial c^*}{\partial \delta}\left(\frac{1}{c} - \delta\right)\right],$$

which is always negative since $\frac{\partial c^*}{\partial \delta} < 0$. For type 1 firms we have

$$\frac{\partial \kappa_{s1}^*}{\partial \delta} = \frac{\kappa_x}{2\sqrt{c^*\left(\frac{1}{c} + \delta\right)}}\left[c^* + \frac{\partial c^*}{\partial \delta}\left(\frac{1}{c} + \delta\right)\right] = \frac{\kappa_x c^*}{2\sqrt{c^*\left(\frac{1}{c} + \delta\right)}}\left(1 + \frac{\frac{\partial c^*}{\partial D^*}\sqrt{\frac{1}{c} + \delta}\left(\sqrt{\frac{1}{c} + \delta} - \sqrt{\frac{1}{c} - \delta}\right)}{\sqrt{\frac{1}{c} - \delta}\left[4c^{*3/2} - \frac{\partial c^*}{\partial D^*}\left(\sqrt{\frac{1}{c} + \delta} + \sqrt{\frac{1}{c} - \delta}\right)\right]}\right),$$

which is positive if and only if

$$\left(\delta - \sqrt{\frac{1}{c} + \delta}\,\sqrt{\frac{1}{c} - \delta}\right)\frac{\partial c^*}{\partial D^*} + 2\sqrt{\frac{1}{c} + \delta}\; c^{*3/2} > 0.$$

The latter always holds if $\delta < 1/(c\sqrt{2})$.

Case 2: $(1/c + \delta) \in \left[\frac{1}{c^*}, \frac{(\kappa_x + \kappa_z)^2}{\kappa_x^2 c^*}\right]$ and $(1/c - \delta) < \frac{1}{c^*}$. In this case, κ_{s1}^* takes intermediate values and $\kappa_{s2}^* = 0$. Replacing the optimal precisions into (EC.19) yields

$$D^* = \frac{1}{c} - \frac{\sqrt{\frac{1}{c} + \delta}}{\sqrt{c^*}}.$$

Note that $\frac{\partial D^*}{\partial \delta} < 0$, which implies that $\frac{\partial c^*}{\partial \delta} > 0$ and therefore

$$\frac{\partial \kappa_{s1}^*}{\partial \delta} = \frac{\kappa_x}{2\sqrt{c^*\left(\frac{1}{c} + \delta\right)}}\left[c^* + \frac{\partial c^*}{\partial \delta}\left(\frac{1}{c} + \delta\right)\right] > 0.$$

For type 2 firms we have $\frac{\partial \kappa_{s2}^*}{\partial \delta} = 0$, which follows from $\kappa_{s2}^* = 0$ and continuity of the optimal threshold with respect to δ.

Case 3: $(1/c - \delta) \in \left[\frac{1}{c^*}, \frac{(\kappa_x + \kappa_z)^2}{\kappa_x^2 c^*}\right]$ and $(1/c + \delta) > \frac{(\kappa_x + \kappa_z)^2}{\kappa_x^2 c^*}$. In this case, κ_{s2}^* takes intermediate values and $\kappa_{s1}^* = \kappa_z$. Replacing the optimal precisions into (EC.19) yields

$$D^* = \frac{1}{2}\left(\frac{1}{c} - \frac{\sqrt{\frac{1}{c} - \delta}}{\sqrt{c^*}}\right) + \frac{1}{2}\left(\frac{\kappa_z}{\kappa_x + \kappa_z}\right)\left(\frac{1}{c} + \delta\right).$$

Note that $\frac{\partial D^*}{\partial \delta} < 0$, which implies that $\frac{\partial c^*}{\partial \delta} > 0$ and therefore

$$\frac{\partial \kappa_{s2}^*}{\partial \delta} = \frac{\kappa_x}{2\sqrt{c^*\left(\frac{1}{c} - \delta\right)}}\left[-c^* + \frac{\partial c^*}{\partial \delta}\left(\frac{1}{c} - \delta\right)\right] < 0.$$

For type 1 firms we have $\frac{\partial \kappa_{s1}^*}{\partial \delta} = 0$.

Case (iv): $(1/c-\delta) < \frac{1}{c^*}$ and $(1/c+\delta) > \frac{(\kappa_x+\kappa_z)^2}{\kappa_x^2 c^*}$. In this case, $\kappa_{s2}^* = 0$, $\kappa_{s1}^* = \kappa_z$, and $\frac{\partial \kappa_{s1}^*}{\partial \delta} = \frac{\partial \kappa_{s2}^*}{\partial \delta} = 0$.

Combining the cases above, we have that $\delta < 1/(c\sqrt{2})$ implies $\frac{\partial \kappa_{s1}^*}{\partial \delta} \geq 0$ and $\frac{\partial \kappa_{s2}^*}{\partial \delta} \leq 0$, thus completing the proof of the corollary. Q.E.D.

Proof of Proposition 8

We begin by stating a lemma which is the discrete analogue of Proposition 1 presented in Section 3. The proof of the lemma follows similar arguments and is therefore omitted.

LEMMA EC.1. *The competition subgame between the firms has a unique Bayes-Nash equilibrium in linear strategies. Furthermore, the equilibrium quantities of the firms are given by*

$$q_i = \begin{cases} \alpha_n[(1-\omega_{k,n})x_i + \omega_{k,n}s_i] & \text{if } i \in K \\ \alpha_n x_i & \text{if } i \in N \setminus K \end{cases},$$

where

$$\omega_{k,n} = \frac{\kappa_s}{\left(1-\beta_n \frac{k-1}{n-1}\rho\right)\kappa_x + \kappa_s},$$

and $\alpha_n = \gamma_0 / \left(\gamma_2 - \frac{n+1}{n}\gamma_1\right)$.

Using the above lemma we can characterize the expected equilibrium profits of an uninformed and of an informed firm, respectively as

$$\mathbb{E}\left[\pi^1|\theta\right] = \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right)\left[\theta^2 + \frac{2\beta_n\rho\kappa_s}{[(1-\beta_n\rho)\kappa_x+\kappa_s]^2} - \frac{(1-\beta_n\rho)^2\kappa_x+\kappa_s}{[(1-\beta_n\rho)\kappa_x+\kappa_s]^2}\right],$$

and

$$\mathbb{E}\left[\pi^0|\theta\right] = \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right)\left[\theta^2 - \frac{1}{\kappa_x}\right],$$

where we used the assumption that $k = n$. The provider extracts all surplus generated by the information she sells. Thus, we can use the law of total expectation to characterize the provider's expected profit as

$$\Pi(\kappa_s, \rho, \kappa_x) = n \left(\mathbb{E}\left[\pi^1\right] - \mathbb{E}\left[\pi^0\right]\right) = n\ \alpha_n^2\left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{\kappa_s+\kappa_x}{[(1-\beta_n\rho)\kappa_x+\kappa_s]^2}.$$

Thus, the provider's optimization problem is

$$\begin{aligned} \max_{\rho,\kappa_s} \quad & \Pi(\kappa_s,\rho,\kappa_x) \\ s.t. \quad & \frac{\kappa_s}{\kappa_z} \leq \rho \leq 1 \\ & \kappa_s \leq \kappa_z. \end{aligned}$$

Differentiating the objective with respect to ρ yields

$$\frac{\partial \Pi}{\partial \rho} = 2n\ \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) \frac{\beta_n \kappa_s(\kappa_x + \kappa_s)}{[(1-\beta_n\rho)\kappa_x + \kappa_s]^3}, \tag{EC.21}$$

which is always negative since $\beta_n < 0$. Thus, it is optimal for the provider to set $\rho^* = \kappa_s/\kappa_z$. Replacing ρ^* into the objective and then differentiating with respect to κ_s yields

$$\frac{\partial \Pi}{\partial \kappa_s} = n\ \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) \frac{(1+\beta_n\kappa_s/\kappa_z)\kappa_x + \kappa_s}{[(1-\beta_n\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s]^3}.$$

When $\beta_n \geq -(1+\kappa_z/\kappa_x)$, the above expression is always positive and it is optimal to set $\kappa_s^* = \kappa_z$. Otherwise, the optimal precision is given by the solution to

$$(1+\beta_n\kappa_s/\kappa_z)\kappa_x + \kappa_s = 0,$$

which implies that $\kappa_s^* = -\kappa_z/(\beta_n + \kappa_z/\kappa_x)$. Substituting the optimal strategy into the objective, we obtain the following characterization of the optimal profits:

$$\Pi^* = \begin{cases} n\ \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right)\left(\frac{\kappa_z}{\kappa_x}\right) \frac{\kappa_z + \kappa_x}{[(1-\beta_n)\kappa_x + \kappa_z]^2} & \text{if } \beta_n \geq -(1+\kappa_z/\kappa_x) \\ n\ \alpha_n^2 \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) \frac{\kappa_z}{-4\beta_n\kappa_x^2} & \text{otherwise} \end{cases},$$

thus completing the proof. Q.E.D.

EC.2. Measure Theoretic Framework

Throughout the paper, we relied on a variant of an "exact law of large numbers," according to which the cross-sectional averages of firm-level variables coincide, almost surely, with the corresponding variables' expectations. In this appendix, we provide the appropriate formalism and conditions for such a result to hold. This framework, which builds on the construction by Sun (2006), also provides us with a Fubini-type property that allows us to exchange the order of integration (over the set of firms and the set of signal realizations) throughout the derivations.

Consider the probability space $([0,1], \mathcal{I}, M)$, where the unit interval $[0,1]$ denotes the set of firms in the economy, $\mathcal{I}$ is the σ-algebra of Lebesgue measurable sets, and M is the Lebesgue measure. Also consider the probability space $(\Omega, \mathcal{F}, P)$ capturing the uncertainty in the model, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of events, and P is a probability measure. Let $([0,1]\times\Omega, \mathcal{I}\otimes\mathcal{F}, \lambda\otimes P)$ denote the product probability space, where $\mathcal{I}\otimes\mathcal{F}$ is the product σ-algebra generated by the class of measurable rectangles $\{I\times F \mid I\in\mathcal{I} \text{ and } F\in\mathcal{F}\}$ and the product measure $M\otimes P$ has the property that

$$M\otimes P(I\times F) = M(I)\cdot P(F),$$

for all $I \in \mathcal{I}$ and $F \in \mathcal{F}$. Note that since M and P are probability measures and thus σ-finite, the desired property is straightforward from the product measure theorem (e.g., see Billingsley (2008)). Throughout, for a process $f:[0,1]\times\Omega\to\mathbb{R}$, we let f_i denote the random variable $f(i,\cdot):\Omega\to\mathbb{R}$ and f_ω denote the random variable $f(\cdot,\omega):[0,1]\to\mathbb{R}$.

Following Sun (2006), we define $([0,1]\times\Omega, \mathcal{I}\boxtimes\mathcal{F}, M\boxtimes P)$ to be a *Fubini extension* of the product space $([0,1]\times\Omega, \mathcal{I}\otimes\mathcal{F}, M\otimes P)$ if

(i) for any $M\boxtimes P$-integrable process f on $([0,1]\times\Omega, \mathcal{I}\boxtimes\mathcal{F})$, f_i is integrable on $(\Omega,\mathcal{F},P)$ for M-almost all $i\in I$ and f_ω is integrable on $([0,1],\mathcal{I},M)$ for P-almost all $\omega\in\Omega$;

(ii) $\int_\Omega f_i \, dP$ and $\int_{[0,1]} f_\omega \, dM$ are, respectively, M-integrable and P-integrable, and satisfy

$$\int_{[0,1]\times\Omega} f(i,\omega)\, d(M\boxtimes P)(i,\omega) = \int_{[0,1]}\left(\int_\Omega f_i(\omega)\, dP(\omega)\right) dM(i) = \int_\Omega\left(\int_{[0,1]} f_\omega(i)\, dM(i)\right) dP(\omega).$$

Throughout the paper, we assume that all relevant quantities lie in the Fubini extension of the product probability space constructed above. For additional details we refer the interested reader to Sun (2006), who provides a detailed treatment of the construction and establishes that there exist Fubini extensions in which one can construct measurable processes with essentially pairwise independent random variables taking any given distribution.[10] Moreover, we require any process f on $([0,1]\times\Omega, \mathcal{I}\boxtimes\mathcal{F})$ to be measurable, which is consistent with the information structure specified throughout the paper. This assumption, coupled with the fact that $M\boxtimes P$ is a probability measure, implies that f is $M\boxtimes P$-integrable. Thus, the Fubini-type property above holds for any random variable in the paper.

We now present the so called exact law of large numbers in the framework of the Fubini extension above:

Proposition EC.1 (**Sun (2006)**). *Consider a process $f:[0,1]\times\Omega\to\mathbb{R}$ that is square integrable with respect to $M\boxtimes P$. If random variables $\{f_i\}_{i\in[0,1]}$ are uncorrelated, then for any set $I\in\mathcal{I}$ satisfying $M(I)>0$,*

$$\int_I f(i,\omega)\, dM(i) = \int_{I\times\Omega} f(i,\omega)\, d(M\boxtimes P)(i,\omega) \quad \text{for } P\text{-almost all } \omega\in\Omega.$$

This result states that, under fairly general conditions, the sample average of a random variable over any set of firms I with positive measure is equal to that random variable's expectation. We use this result in our analysis and, specifically, to claim that $\int_{[0,1]} \epsilon(i,\omega)\, dM(i) = \int_{[0,1]} \xi(i,\omega)\, dM(i) = 0$. Note that we rely on a slight abuse of notation throughout the paper to simplify the exposition of our analysis and results. In particular, we write $\int_0^1 f_i\, di$ instead of $\int_{[0,1]} f_i\, dM(i)$.

[10] Also see Sun and Zhang (2009), who show that this result holds for the case in which the index space is an extension of the Lebesgue unit interval.