Text as Data

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The Vector Space Model of Text

1) Task:
   - Numerous tasks will suppose that we can measure document similarity or dissimiliarity
2) Objective Function
   - For a variety of tasks, will impose some measure or definition of similarity, dissimilarity, or distance.

$$d(\boldsymbol{X}_i, \boldsymbol{X}_j) = \text{Dissimilarity(Distance)} \rightsquigarrow \text{Bigger implies further apart}$$
$$s(\boldsymbol{X}_i, \boldsymbol{X}_j) = \text{Similarity} \rightsquigarrow \text{Bigger implies closer together}$$

   - Objective functions$\rightsquigarrow$ determine which points we compare and aggregate similarity, dissimilarity, and distance
3) Optimization
   - Depends on the particular task, likely arranging/grouping objects to find similarity
4) Validation
   - Are the mathematical definitions of similarity actually similar for our particular purpose?

Texts and Geometry

Consider a document-term matrix

$$\boldsymbol{X} = \begin{pmatrix} 1 & 2 & 0 & \ldots & 0 \\ 0 & 0 & 3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \ldots & 3 \end{pmatrix}$$

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- Provides a geometry $\rightsquigarrow$ modify with word weighting
- Natural notions of distance
- Kernel Trick: richer comparisons of large feature spaces
- Building block for clustering, supervised learning, and scaling

Texts in Space

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$$\text{Doc1} = (1, 1, 3, \ldots, 5)$$

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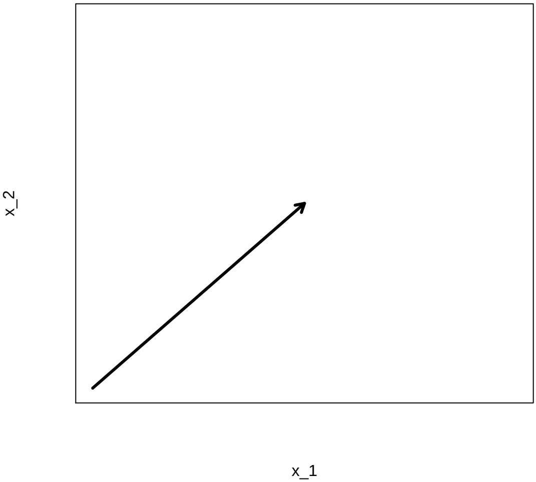
Vector Length



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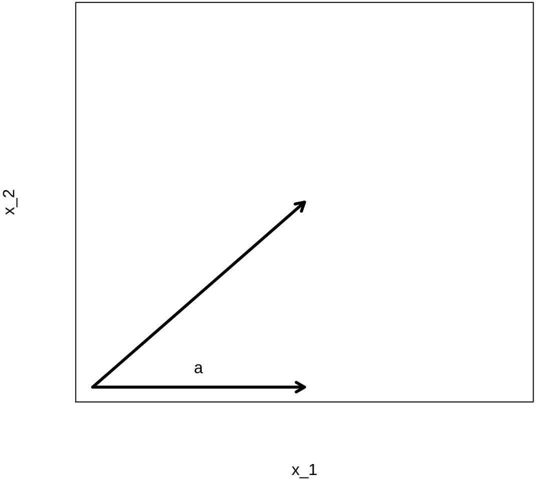
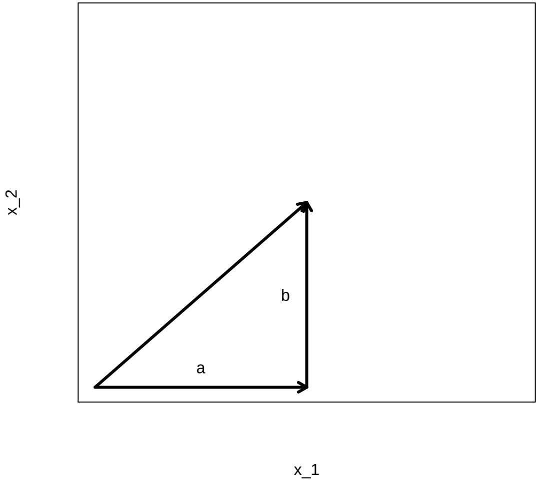


- Pythogorean Theorem: Side with length a

Vector Length



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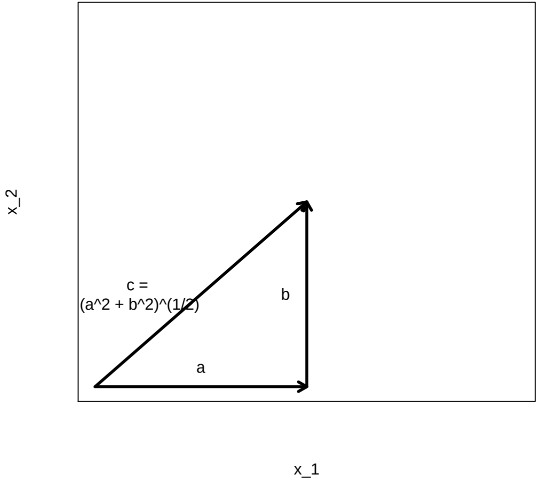

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- $c = \sqrt{a^2 + b^2}$

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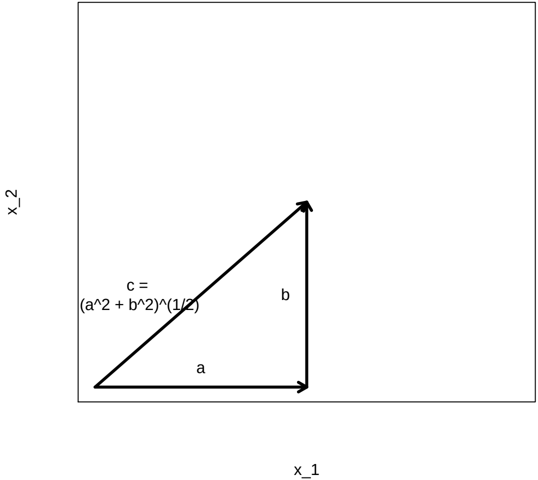

- Pythogorean Theorem: Side with length a
- Side with length b and right triangle
- $c = \sqrt{a^2 + b^2}$
- This is generally true

Vector (Euclidean) Length

Definition

Suppose $\boldsymbol{v} \in \Re^J$. *Then, we will define its length as*

$$
\begin{aligned}
||\boldsymbol{v}|| &= (\boldsymbol{v} \cdot \boldsymbol{v})^{1/2} \\
&= (v_1^2 + v_2^2 + v_3^2 + \ldots + v_J^2)^{1/2}
\end{aligned}
$$

Measures of Dissimilarity

Initial guess$\rightsquigarrow$ Distance metrics

Properties of a metric: (distance function) $d(\cdot, \cdot)$. Consider arbitrary documents $\boldsymbol{X}_i$, $\boldsymbol{X}_j$, $\boldsymbol{X}_k$

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Measures of Dissimilarity

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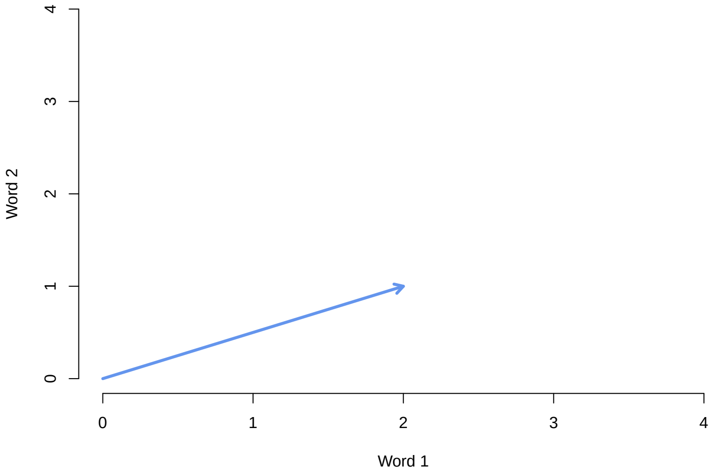
Explore distance functions to compare documents$\leadsto$Do we want additional assumptions/properties?

Measuring the Distance Between Documents

Euclidean Distance



Measuring the Distance Between Documents

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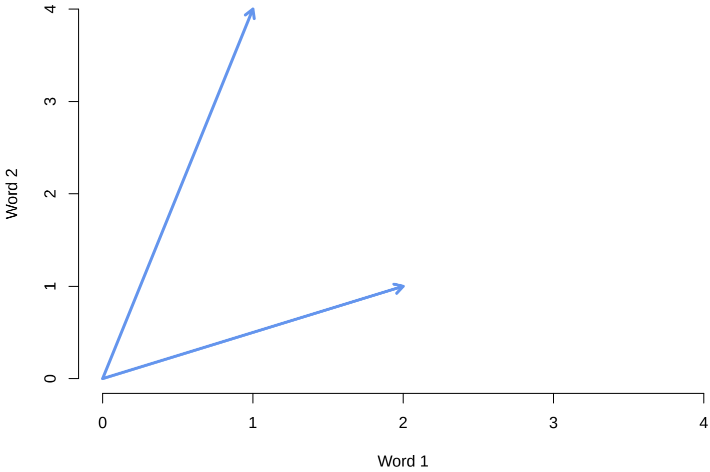

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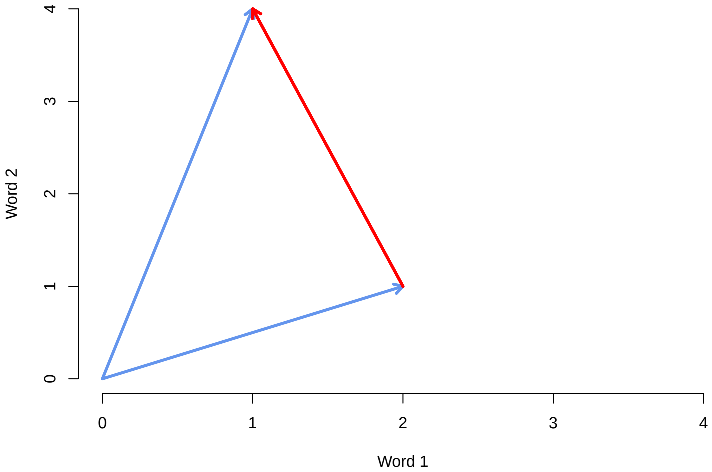

Measuring the Distance Between Documents

Definition

The Euclidean distance between documents $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ as

$$||\boldsymbol{X}_i - \boldsymbol{X}_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

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$$||\boldsymbol{X}_i - \boldsymbol{X}_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Suppose $\boldsymbol{X}_i = (1, 4)$ and $\boldsymbol{X}_j = (2, 1)$. The distance between the documents is:

$$\begin{aligned} ||(1,4) - (2,1)|| &= \sqrt{(1-2)^2 + (4-1)^2} \\ &= \sqrt{10} \end{aligned}$$

Measuring the Distance Between Documents

Many distance metrics

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Many distance metrics Consider the Minkowski family

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Many distance metrics Consider the Minkowski family

Definition

The Minkowski Distance between documents $\boldsymbol{X}_i$ *and* $\boldsymbol{X}_j$ *for value p is*

$$d_p(\mathbf{X}_i, \mathbf{X}_j) = \left(\sum_{m=1}^{J} |x_{im} - x_{jm}|^p \right)^{1/p}$$

Members of the Minkowski Family

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Manhattan metric

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$$d_1(\mathbf{X}_i, \mathbf{X}_j) = \sum_{m=1}^{J} |x_{im} - x_{jm}|$$

Members of the Minkowski Family

Manhattan metric

$$
\begin{aligned}
d_1(\mathbf{X}_i, \mathbf{X}_j) &= \sum_{m=1}^{J} |x_{im} - x_{jm}| \\
d_1((1,4),(2,1)) &= |1| + |3| = 4
\end{aligned}
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Minkowski (p) metric

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Increasing $p \rightsquigarrow$ greater importance of coordinates with largest differences

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In words: distance between documents only the biggest difference

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All other differences do not contribute to distance measure

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Decreasing $p \rightsquigarrow$ greater importance of coordinates with smallest differences

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If we let $p \rightarrow \infty$ Obtain maximum-metric (Chebyshev's Metric)

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In words: distance between documents only the biggest difference
All other differences do not contribute to distance measure
Decreasing $p \rightsquigarrow$ greater importance of coordinates with smallest differences

$$\lim_{p \rightarrow -\infty} d_p(\mathbf{X}_i, \mathbf{X}_j) = \min_{m=1}^{J} |x_{im} - x_{jm}|$$

Comparing the Metrics

Suppose $\boldsymbol{X}_i = (10, 4, 3)$, $\boldsymbol{X}_j = (0, 4, 3)$, and $\boldsymbol{X}_k = (0, 0, 0)$

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$$\begin{aligned} d_1(\boldsymbol{X}_i, \boldsymbol{X}_j) &= 10 \\ d_1(\boldsymbol{X}_i, \boldsymbol{X}_k) &= 10 + 4 + 3 = 17 \end{aligned}$$

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d_2(\boldsymbol{X}_i, \boldsymbol{X}_j) &= 10 \\
d_2(\boldsymbol{X}_i, \boldsymbol{X}_k) &= \sqrt{10^2 + 4^2 + 3^2} = \sqrt{125} = 11.18
\end{aligned}
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d_4(\boldsymbol{X}_i, \boldsymbol{X}_k) &= \sqrt{10^4 + 4^4 + 3^4} = (10337)^{1/4} = 10.08
\end{aligned}$$

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d_\infty(\boldsymbol{X}_i, \boldsymbol{X}_j) &= 10
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More generally: $\boldsymbol{\Sigma}$ could be symmetric and positive-definite

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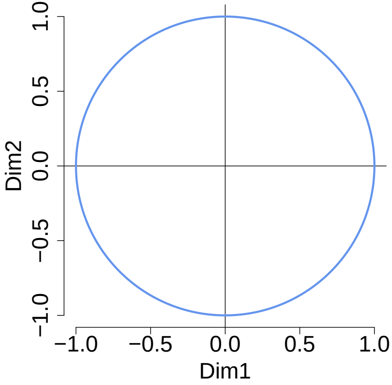
More generally: $\boldsymbol{\Sigma}$ could be symmetric and positive-definite
What does $\boldsymbol{\Sigma}$ do?

Some Intuition: The Unit Circle



$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

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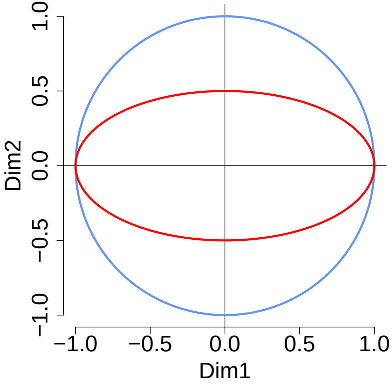

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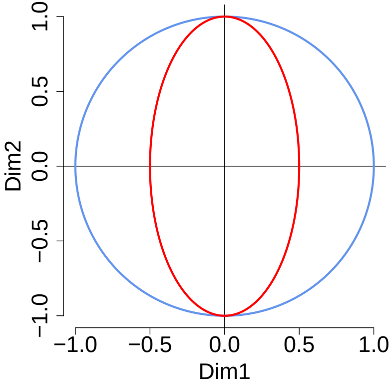

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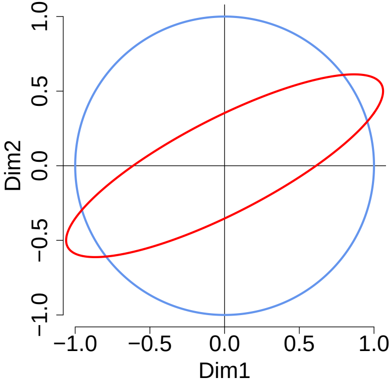

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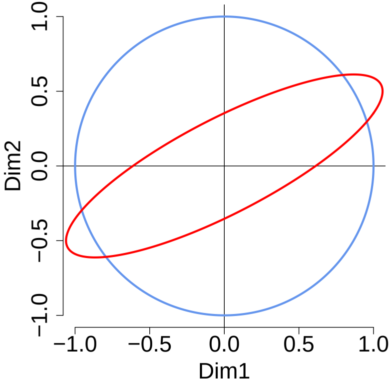

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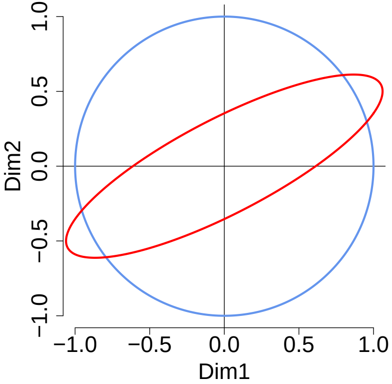

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Measuring Distance with Mahalanobis

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Then

$$d_{\text{Mah}}(\boldsymbol{X}_i, \boldsymbol{X}_j) = \sqrt{\sum_{m=1}^{J} \frac{(x_{im} - x_{jm})^2}{\sigma_m^2}}$$

Measuring Similarity

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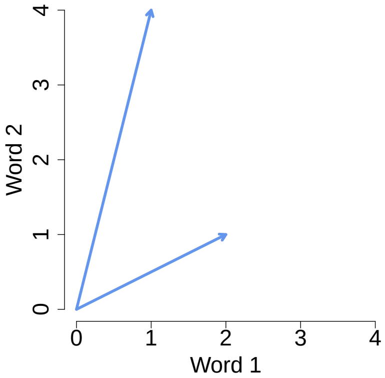
How should additional words be treated?

Measuring Similarity



Measure 1: Inner product

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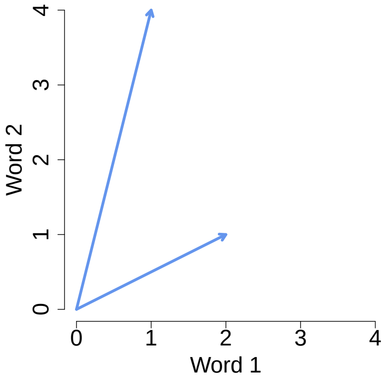
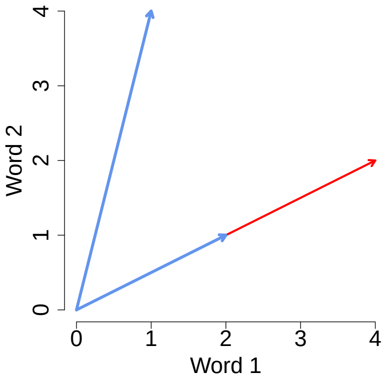
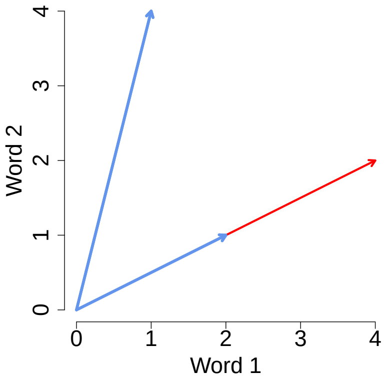


Measure 1: Inner product

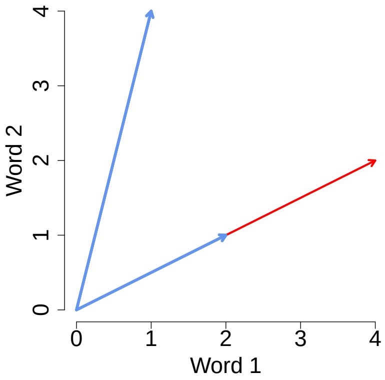
$$(2,1)^{'} \cdot (1,4) \quad = \quad 6$$



Word 2
0
1
2
3
4
0
1
2
3
4
Word 1



Problem(?): length dependent



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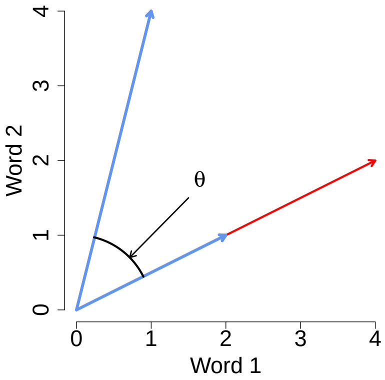
$$(4,2)^{'}(1,4) = 12$$



Problem(?): length dependent

$$
\begin{aligned}
(4,2)^{'}(1,4) &= 12 \\
a \cdot b &= ||a|| \times ||b|| \times \cos\theta
\end{aligned}
$$

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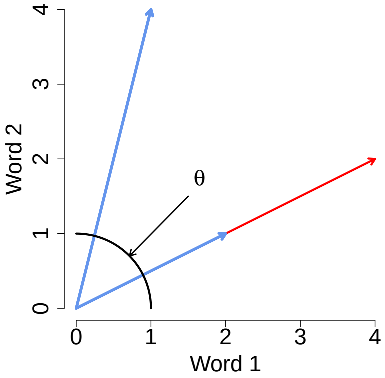

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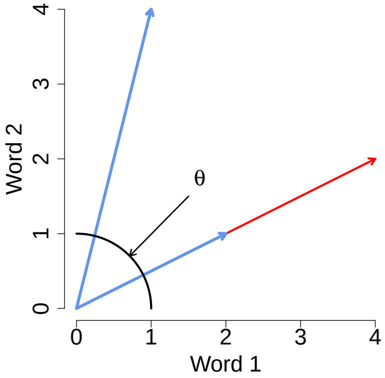

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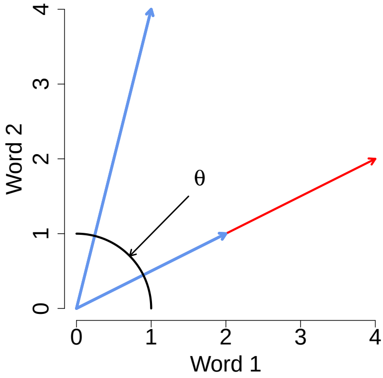

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- Useful for clustering, hierarchies of topics

Kernel Similarity

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Suppose we have documents $\boldsymbol{X}_i$ *and* $\boldsymbol{X}_j$*. Define the Gaussian kernel as*

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- Result $\rightsquigarrow$ often justify setting some kernel weights to zero

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- $\rightsquigarrow$ Kernels provide methods for capture wide array of transformations.
- Kernel Trick$\rightsquigarrow$ calculate inner products on untransformed data (Gaussian Kernel), implicitly use wide array of ϕ's.

Weighting Words

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- Use training set to identify separating words (Monroe, Ideology measurement)

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- Decreases at rate $\frac{1}{n_j} \Rightarrow$ diminishing "penalty" for more common use
- Other functional forms are fine, embed assumptions about penalization of common use

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Weighting Words: TF-IDF

$$\mathbf{X}_{i,\text{idf}} \equiv \underbrace{\mathbf{X}_i}_{\text{tf}} \times \textbf{idf} = (X_{i1} \times \text{idf}_1, X_{i2} \times \text{idf}_2, \ldots, X_{iJ} \times \text{idf}_J)$$

$$\mathbf{X}_{j,\text{idf}} \equiv \mathbf{X}_j \times \textbf{idf} = (X_{j1} \times \text{idf}_1, X_{j2} \times \text{idf}_2, \ldots, X_{jJ} \times \text{idf}_J)$$

How Does This Matter For Measuring Similarity/Dissimilarity?

Inner Product

$$\begin{aligned}\mathbf{X}_{i,\text{idf}} \cdot \mathbf{X}_{j,\text{idf}} &= (\mathbf{X}_i \times \textbf{idf})'(\mathbf{X}_j \times \textbf{idf}) \\ &= (\text{idf}_1^2 \times X_{i1} \times X_{j1}) + (\text{idf}_2^2 \times X_{i2} \times X_{j2}) + \\ &\quad \ldots + (\text{idf}_J^2 \times X_{iJ} \times X_{jJ})\end{aligned}$$

Weighting Words: Inner Product

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$$\boldsymbol{\Sigma} = \begin{pmatrix} \text{idf}_1^2 & 0 & 0 & \ldots & 0 \\ 0 & \text{idf}_2^2 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & \text{idf}_J^2 \end{pmatrix}$$

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If we use tf-idf for our documents, then

$$\begin{aligned} d_2(\boldsymbol{X}_i, \boldsymbol{X}_j) &= \sqrt{\sum_{m=1}^{J} (x_{im,\text{idf}} - x_{jm,\text{idf}})^2} \\ &= \sqrt{(\boldsymbol{X}_i - \boldsymbol{X}_j)' \boldsymbol{\Sigma} (\boldsymbol{X}_i - \boldsymbol{X}_j)} \end{aligned}$$

Final Product

Applying some measure of distance, similarity (if symmetric) yields:

$$\mathbf{D} = \begin{pmatrix} 0 & d(1,2) & d(1,3) & \ldots & d(1,N) \\ d(2,1) & 0 & d(2,3) & \ldots & d(2,N) \\ d(3,1) & d(3,2) & 0 & \ldots & d(3,N) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d(N,1) & d(N,2) & d(N,3) & \ldots & 0 \end{pmatrix}$$

Lower Triangle contains unique information $N(N-1)/2$

Spirling and Indian Treaties

Spirling (2013): model Treaties between US and Native Americans

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- Today: Text representation and similarity calculation
- Tuesday: Projecting to low dimensional space

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How do we preserve word order and semantic language?
After stemming, stopping, bag of wording:

```
- Peace Between Us
- No Peace Between Us
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are identical.
Spirling uses complicated representation of texts to preserve word order $\rightsquigarrow$ broad application

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Similarity and Dissimilarity of Many Things

Throughout the course we'll measure similarity between documents
We'll also (implicitly) study similarity of probability distributions
Develop a measure of distribution dissimilarity

Similarity of Probability Distributions

Definition

Suppose P is a continuous random variable with density $p : \Re \rightarrow \Re$ *and Q is a continuous random variable with density* $q : \Re \rightarrow q$.
We can define the KL-Divergence between P and Q as

$$KL(P||Q) = \int_{-\infty}^{\infty} p(x) \log \frac{p(x)}{q(x)} dx$$

Assessing Similarity of Other Things

KL-divergence measures dissimilarity between two distributions.

Consider a function. $f(x) = -x^2$.

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Maps numbers to other numbers.

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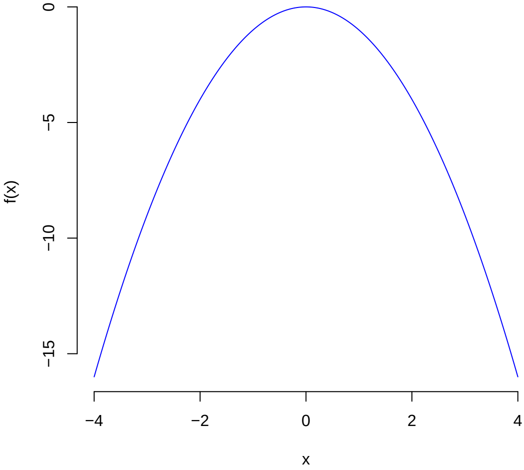
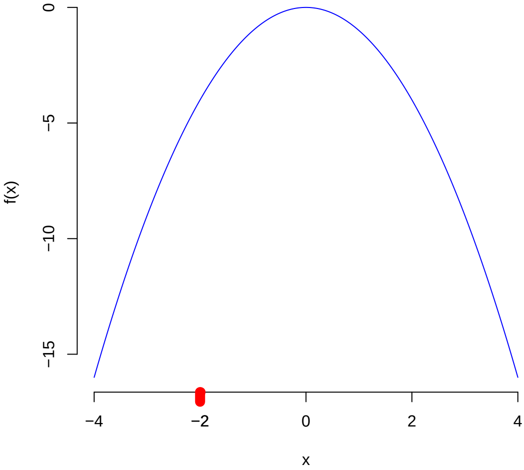
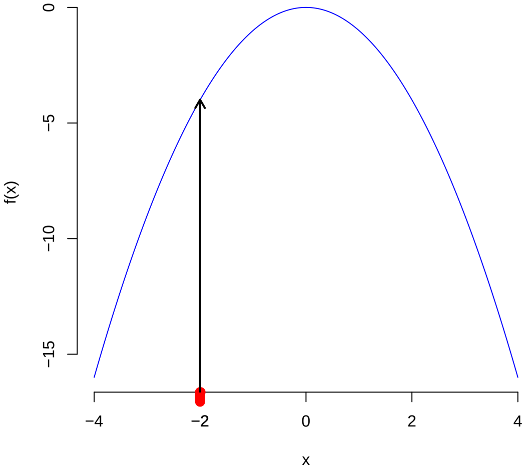


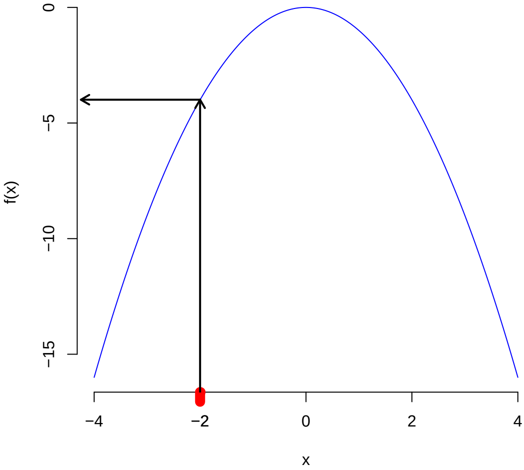
Take some input (-2 here)



Then obtain the value of $f(-2)$



Then obtain the value of $f(-2) = -4$



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KL($q||p$) is a functional. A functional takes functions as inputs, returns a real number.

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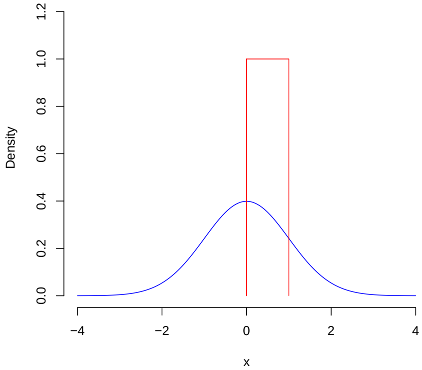
For example, we could set q = Uniform(0,1) and p = Normal(0, 1)



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KL($q||p$) maps from sets of distributions $q \in \mathcal{Q}$ and $p \in \mathcal{P}$ to positive real numbers.
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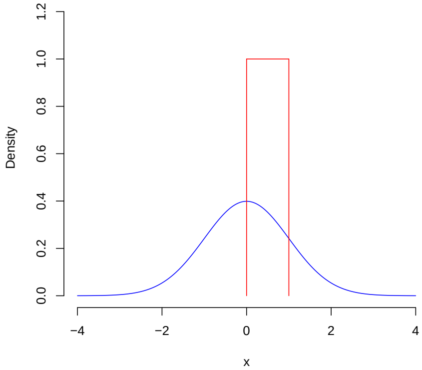
KL(Uniform(0,1)||Normal(0,1)) = 1.09



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Variational Approximation (topic models!): approximate one distribution p, with another, simpler distribution q.

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Variational Approximation (topic models!): approximate one distribution p, with another, simpler distribution q.
Then make this approximation the best possible–minimize the KL-divergence.

A simple example.

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Approximate a Normal(0,1) with symmetric Uniform distribution, Uniform(-b, b).

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Choose b to min. KL(Uniform(-b, b)|| Normal(0,1))

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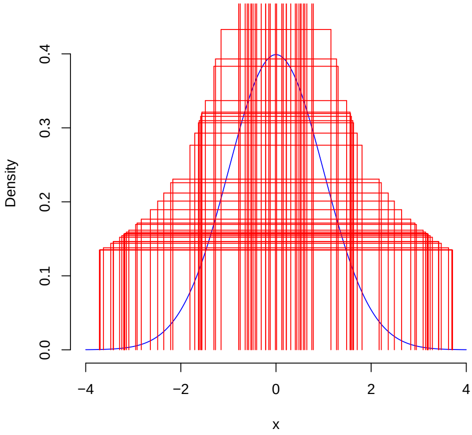

Answer:

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$b = \sqrt{3}$

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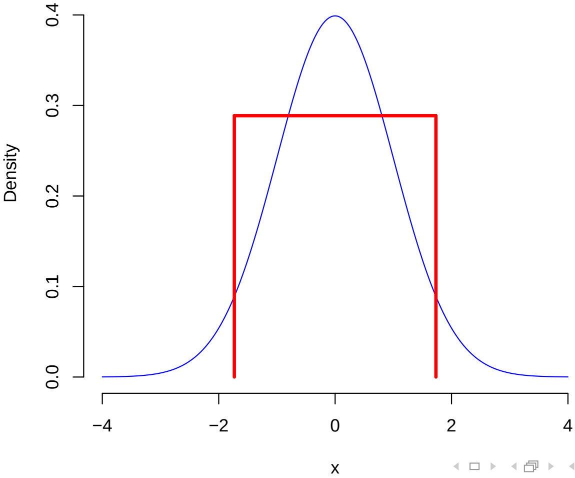

1) Documents in vector space $\rightsquigarrow$ geometry of texts
2) Many methods to measure similarity and dissimilarity