The Georeg Regularizer
Guillermo Angeris Shane Barratt Jonathan Tuck∗†
April 1, 2019

Abstract
In this paper, we introduce the georeg regularizer as a natural generalization of the elastic net regularizer by combining all the norms. We present a simple example showing some interesting properties of the regularizer and introduce a new cvxpy atom, geo_reg, which implements the georeg regularizer.

1 Introduction
We define the $p$-norm of a vector $x \in \mathbb{R}^n$ to be
$$\|x\|_p = \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p}.$$ Note that any $p$-norm (and its $p$th power) is always convex [1]. We will define the $m$th georeg regularizer as the following convex combination of powers of norms:
$$g_m(x, \alpha) = \left(\sum_{k=1}^{m} \alpha^{-k}\right)^{-1} \sum_{k=1}^{m} \frac{\|x\|^k_{\alpha^k}}{\alpha^k} = \left(1 - \frac{\alpha^m + 1}{1 - \alpha^{-1}} - 1\right)^{-1} \left(1 - \frac{\frac{|x_i|}{\alpha}}{1 - \frac{|x_i|}{\alpha}} - 1\right), \quad \alpha \neq 1.$$

We note (a) that $g_m(\cdot, \alpha)$ is convex in its domain for any $\alpha > 0$ as it is a sum of convex functions and (b) that the georeg regularizer reduces to the elastic net regularizer if we choose $m = 2$. Additionally, the series is absolutely convergent as $m \uparrow \infty$ whenever $|x_i| < \alpha$ and $\alpha > 1$ for all $i = 1, \ldots, m$.

In this paper, we focus on the $m = \infty$ case which reduces to
$$g_\infty(x, \alpha) = (\alpha - 1) \left(1 - \frac{1}{1 - \frac{|x_i|}{\alpha}} - 1\right).$$

∗Authors listed in alphabetical order.
†{angeris, sbarratt, jonathantuck}@stanford.edu
1This is well known, but we needed to make the paper longer.
and is easily representable by a set of second-order cone constraints (SOCs). Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a loss function, then the learning problem for $x$ can be written as the following problem with SOCs and variables $x, r, t \in \mathbb{R}^n$:

$$
\begin{align*}
\text{minimize} \quad & f(x) + 1^T r - n, \\
\text{subject to} \quad & \|(2, r_i - 1 + t_i/\alpha)\|_2 \leq r_i + 1 - t_i/\alpha, \quad i = 1, \ldots, n \\
& -t \leq x \leq t.
\end{align*}
$$

If $f$ is convex and easily representable in conic form, the problem can be written as a conic program, which is often efficiently solvable [1].

## 2 Example

As an example, we solve the problem

$$
\begin{align*}
\text{minimize} \quad & \|Ax - b\|^2_2 + \lambda g_{\infty}(x, \alpha), \\
\end{align*}
$$

where $x \in \mathbb{R}^4$ is the optimization variable, and $A \in \mathbb{R}^{100 \times 4}$ and $b \in \mathbb{R}^{100}$ are problem data. We randomly generate $A$ and $b$ and plot the values of the optimal $x$ for varying values of $\lambda > 0$ in figure 1.

## Acknowledgements

The authors would like to thank A. Agrawal in advance for accepting the cvxpy pull request.

## References