Contracting and vertical control by a dominant platform*

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Abstract

We study a platform that sells productive inputs (such as e-commerce and distribution services) to a fringe of producers in an upstream market, while also selling its own output in the corresponding downstream market. The platform faces a tradeoff: any output that it sells downstream increases competition with the fringe of producers and lowers the downstream price, which in turn reduces demand for the platform's productive inputs and decreases upstream revenue. Adopting a mechanism design approach, we characterize the optimal menu of contracts the platform offers in the upstream market. These contracts involve price discrimination in the form of nonlinear pricing and quantity discounts. If the platform is a monopoly in the upstream market, then we show that the tradeoff always resolves in favor of consumers and at the expense of producers. However, if the platform faces competition in the upstream market, then it has an incentive to undermine this competition by engaging in activities, such as "killer" acquisitions and exclusive dealing, that harm both consumers and producers.

Keywords: Vertical intermediation, online platforms, antitrust policy, mechanism design, nonlinear pricing, killer acquisitions, exclusive dealing

JEL-Classification: D42, D82, L43

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1 Introduction

Online platforms increasingly act as gatekeepers that enable producers to access downstream markets, while also competing with producers in these downstream markets. A prominent example is Amazon, which sells e-commerce and distribution services to producers in an upstream market, while also selling AmazonBasics and other private-label products downstream. Should platforms be allowed to control whom they compete with in downstream markets through their upstream market interactions?

Many regulators have emphasized that the potential conflict of interest that such platforms face is a cause for antitrust concern. This was succinctly articulated by U.S. Senator Elizabeth Warren in a 2019 article titled "Here's How We Can Break Up Big Tech":

> "Many big tech companies own a marketplace, where buyers and sellers transact, while also participating on the marketplace. This can create a conflict of interest that undermines competition."

Platforms such as Amazon, Apple, Facebook, and Google face increasing regulatory scrutiny over this potential conflict of interest. For example, a 2020 U.S. House Majority Report found that

> "Amazon's dual role as an operator of its marketplace that hosts third-party sellers, and a seller in that same marketplace, creates an inherent conflict of interest."

The logic behind these regulatory concerns is that platforms can exploit their upstream market power to quash rival producers, thereby reducing competition in downstream markets and harming consumers. The "conflict of interest" that these platforms face can be quantified by the following tradeoff: Any output that the platform sells downstream might increase downstream revenue, but this also increases competition with rival producers and lowers the downstream price. In turn, this reduces the value of the platform to rival producers, which decreases upstream revenue.

In this paper we study the antitrust implications of a platform acting both as a producer in a downstream market and an upstream supplier to rival producers. We find that banning a monopolist platform from producing in downstream markets can only harm consumers because platforms that produce positive output in equilibrium always reduce downstream prices. Consequently, the claimed "conflict of interest," or tradeoff between the platform's upstream and downstream profits, always benefits the consumer, at the expense of producers. Intuitively, any output produced by the competitive fringe of producers is associated with a vertical externality that resembles double marginalization, while any output produced by the platform is only associated with a single marginalization effect. If the platform's own production costs are reduced, the corresponding substitution towards output produced by the platform results in higher overall production in the downstream market, which benefits consumers. However, when the platform is not a monopolist, meaning that producers can access downstream markets through alternative distribution channels, platforms may have an incentive to undermine this upstream market competition. For example, the platform may profitably engage in "killer" horizontal acquisitions (acquire and then shuttering smaller upstream competitors) or exclusive dealing (offer contracts that preclude producers from accessing alternative distribution channels).[1,2] These practices harm consumers by reducing overall output in the downstream market and would therefore warrant the scrutiny of antitrust authorities.

Our analysis introduces a general mechanism design framework for studying vertical market structures involving a dominant platform. In particular, we consider a model in which a platform sells a productive input to producers in an upstream market before competing with these producers in a downstream market. We characterize the optimal menu of contracts offered by the platform in the upstream market, assuming the platform seeks to maximize its total upstream and downstream profits. In our formulation, producers have private information about their costs, which gives rise to incentive and participation constraints. We first consider the case in which the platform monopolizes the upstream market and then add the possibility that producers have access to alternative distribution channels. In each case the optimal menu of upstream contracts involves a nonlinear pricing schedule that represents price discrimination in the form of quantity discounts.

The key technical difficulty that arises in solving for the optimal menu of upstream contracts is that producers' willingness to pay in the upstream market is endogenous to the downstream market outcome. We overcome this problem by rewriting the mechanism design problem as a nested optimization problem. First, the platform selects the optimal upstream selling mechanism and its own level of downstream output subject to a market-

[1] These results also resonate with ongoing antitrust concerns with regard to online platforms. Antitrust authorities have accused Amazon of limiting competition in upstream markets by acquiring smaller upstream competitors and engaging in exclusive dealing. For example, Amazon acquired Diapers.com in 2010 for $545 million but shut down the company in 2017, citing a lack of profitability. Analysts have speculated that Amazon always intended to eliminate Diapers.com following this acquisition. This example also resonates with some of the concerns raised by Khan (2017). With regard to exclusive dealing, the 2020 US House Majority Report quotes a former Amazon employee as stating that "It was not uncommon for Amazon to use its brand standards policy to shut down a brand's third-party seller account and force brands into an exclusive wholesaler relationship."

[2] The term *killer acquisitions* was original used in the context of the pharmaceutical industry (see Cunningham et al. (2021) for a recent study). Recently this term has been applied more broadly, including to acquisitions made by online platforms for the purpose of entrenching their market dominance.

clearing constraint that specifies the price induced in the downstream market. This problem resembles a nonlinear monopoly pricing involving a capacity constraint. Second, the platform optimizes over the price induced in the downstream market. The market-clearing constraint captures the impact of the upstream market selling mechanism on the downstream market price, which in turn impacts the producers' willingness to pay of producers in the upstream market, as well as the platform's own profits in the downstream market.

An implication of our consumer surplus analysis for antitrust policy is that banning platforms from producing in downstream markets can only harm consumers. A similar result holds if the platform is banned from selling downstream market access in the upstream market. This suggests that there is more to the "conflict of interest" identified by antitrust authorities than meets the eye. Naturally, consumers would be better off if the platform's upstream business interests were separated from its downstream business interests. However, this may be difficult to achieve in practice and our analysis shows that simple bans will only serve to make consumers worse off. This resonates with recent antitrust policies. For example, in 2019 India introduced new laws—intended to protect small local businesses—that prevented online retailers from selling products through vendors in which they hold an equity stake (BBC News, 2019). Amazon lobbied strongly against this new law, which prevented it from selling AmazonBasics products on its own platform. Our analysis suggests that while such laws should indeed protect the interests of producers, they may harm consumers.

The remainder of this paper is structured as follows. First, we discuss our modelling approach and results in the context of the related literature on vertical control in Section 1.1, before surveying some additional related literature in Section 1.2. Section 2 introduces our general setup. In Section 3 we characterize the optimal menu of contracts offered by the platform when it monopolizes the upstream market, and investigate the implications for consumers and producers. In Section 4 we extend this analysis by considering the case where the platform is a dominant firm in the upstream market, providing scope to discuss the implications of acquisitions and exclusive dealing in the upstream market. Section 5 concludes the paper.

1.1 Interpretation of the model and relation to vertical control

First and foremost, this paper contributes to the vast literature on vertical integration and foreclosure in vertical market structures (see Riordan (2005) and Rey and Tirole (2007) for comprehensive surveys). As mentioned, a vertical externality that resembles double marginalization plays an important role in our consumer surplus analysis. Most models of vertical market structures assume complete information, where double marginalization only arises when restrictions are imposed on the contracting space (for example, by restricting attention to posted-price mechanisms).[3] If one allows for more general contracts (for example, by considering two-part tariffs) the double marginalization effect vanishes. This result raises important questions concerning the robustness of double marginalization, particularly since the elimination of this effect is frequently cited as a defence for vertical mergers. In this paper we study an incomplete information setting, where a double marginalization effect arises without imposing any restrictions on the contracting space. Since producers have private information concerning their production costs, the platform must pay producers information rents in the upstream market. This prevents it from extracting the full monopoly profit.

More generally the “single monopoly profit theory” originating with the Chicago school (see Posner, 1976; Bork, 1978) contends that exclusionary conduct—such as vertical mergers and exclusive dealing—cannot profitably expand the market power of an upstream firm that already captures the full monopoly profit.[4] While there has been a recent upsurge of interest in the antitrust implications of exclusionary conduct, this neutrality result can make these practices difficult to study in formal models. In our model the “single monopoly profit theory” does not hold for two reasons. First, and as previously discussed, we assume that producers possess private information concerning their production costs. Moreover, in Section 4 we also assume that producers have access to alternative distribution channels and do not necessarily require the services of the platform to access the downstream market. These factors prevent the platform from capturing the full monopoly profit in the upstream market, and provide scope for studying practices such as exclusive dealing.

1.2 Related literature

This paper is closely related to recent work by Hagiu, Teh, and Wright (2020), Anderson and Bedre-Defolie (2021), and Madsen and Vellodi (2021), who are similarly motivated by anticompetitive allegations against platforms such as Amazon. However, there are several important differences. In particular, Madsen and Vellodi examine how platforms may use information about downstream demand to decide whether or not to launch their own version of the same product. By contrast, we focus on the platform's tradeoff between upstream and downstream revenue in a model with no aggregate uncertainty in the downstream product market. Equivalently, the platform may already have detailed information about the downstream product market, or it may simply be unable to commit to making its entry decision

[3] For two recent noteable exceptions, see Loertscher and Marx (2021a) and Choné et al. (2021). See also Loertscher and Marx (2021b) for a recent survey of incomplete information modelling in industrial organization.

[4] Our previous observation concerning double marginalization in complete information models is essentially a special case of this more general theory.

independently of marketplace data that it collects. While Hagiu et al. and Anderson and Bedre-Defolie also study the platform's tradeo between upstream and downstream revenue, their papers and ours di er in modeling approaches. Hagiu et al. and Anderson and Bedre-Defolie provide a detailed model of the downstream product market: Hagiu et al. consider a model involving vertically di erentiated goods, while Anderson and Bedre-Defolie study a market involving horizontally di erentiated goods. By contrast, we adopt a parsimonious model of the downstream product market, but enrich the platform's upstream contracting space. This allows us to study price the implications of discrimination in the upstream market involving the platform and rival producers, and analyze its e ects on downstream consumers.

Our paper also relates to a growing literature on platforms, including early contributions by Rochet and Tirole (2003, 2006), Armstrong (2006), and Weyl (2010). Many of these papers focus on \cross-side" externalities between di erent sides of the platform: more consumers on the platform increases its value to producers, and vice versa. While a similar e ect arises in our model, it is caused by a pecuniary externality: selling the upstream input to more producers increases downstream production, thereby lowering the price of the downstream good and bene ting consumers.

Our paper is also connected to work by Martimort and Stole (2009), Calzolari and Denicob (2013), and Calzolari and Denicolo (2015) on exclusive contracts in environments with incomplete information. Like these papers, we study the anticompetitive implications of exclusive contracts and nd that the ability to write exclusive contracts generally enhances the platform's pro tability. We contribute to this literature by considering a setting in which the platform writes contracts in an upstream market and the structure of the optimal menu of contracts depends on the equilibrium price in a downstream market. This provides a tractable framework for investigating the implications of exclusive dealing for consumers in downstream markets.[5]

Finally, our work is related to the growing literature on partial mechanism design, or \mechanism design with a competitive fringe," which includes early contributions by Philippon and Skreta (2012), Tirole (2012), and Fuchs and Skrzypacz (2015). More recent work in this literature include Loertscher and Muir (2021) and Kang (2021b). Like these papers, we limit the ability of the platform to monopolize the upstream market by giving producers access to an outside option. This places additional constraints on the upstream contracts that the platform can write, which we view as realistically capturing upstream competitive

[5] Another di erence is that the platform in our model can also produce in the downstream market. The ability of the platform to directly capture a share of downstream revenue limits the pro tability of exclusive contracts, and provides an additional procompetitive e ect not present in these earlier papers.

pressures that the platform might face. To overcome the complications that arise due to these constraints, we adapt and extend the methodological approach of Kang (2021a) to obtain a tractable analysis.

2 Model

We study a vertical structure in which interactions between a platform, producers and consumers are divided into an upstream market and a downstream market, as illustrated in Figure 1. In the upstream market the platform sells downstream market access to producers; and in the downstream market both the platform and the producers sell a final good to consumers. The platform raises profits by selling access in the upstream market, as well as by selling the final good in the downstream market. Producers can bypass the platform entirely and sell through alternative, non-platform distribution channels for a separate fee. One could think of this fee as representing the cost associated with producers using in-house e-commerce, distribution and delivery services. Producers therefore determine the quantity of output that they sell through the platform, as well as the quantity of output that they sell through alternative channels. We assume that, from the perspective of consumers, final goods purchased on the platform and through alternative channels are perfect substitutes.[6]

Downstream market The downstream market is characterized by competition between a dominant firm (the platform) and a competitive fringe (producers). Let Q_1 denote the total quantity of output that producers sell to consumers through the platform and Q_2 denote the total quantity of output producers sell to consumers through alternative channels. Let y denote the total quantity of output that the platforms sells to consumers. As previously stated, we assume that consumers view goods sold through the platform and non-platform markets as perfect substitutes. We let $D : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ and $P : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ respectively denote the demand function and the inverse demand function associated with downstream consumers. The aggregate quantities $y + Q_1$ and Q_2 and the downstream price p then satisfies $p = P(y + Q_1 + Q_2)$ and $y + Q_1 + Q_2 = D(p)$. We assume that the functions P and D are decreasing and continuously differentiable functions.

[6] Long before the rise of e-commerce and large online platforms, it was already common for bricks-and-mortar retailers (such as Safeway or Macy's) to sell other companies' products, while simultaneously competing against them by selling store brands. The analysis of this paper could apply equally well to this situation. However, as highlighted in the introduction, the rise of large online platforms has resulted in renewed interest in the antitrust issues surrounding such market structures.

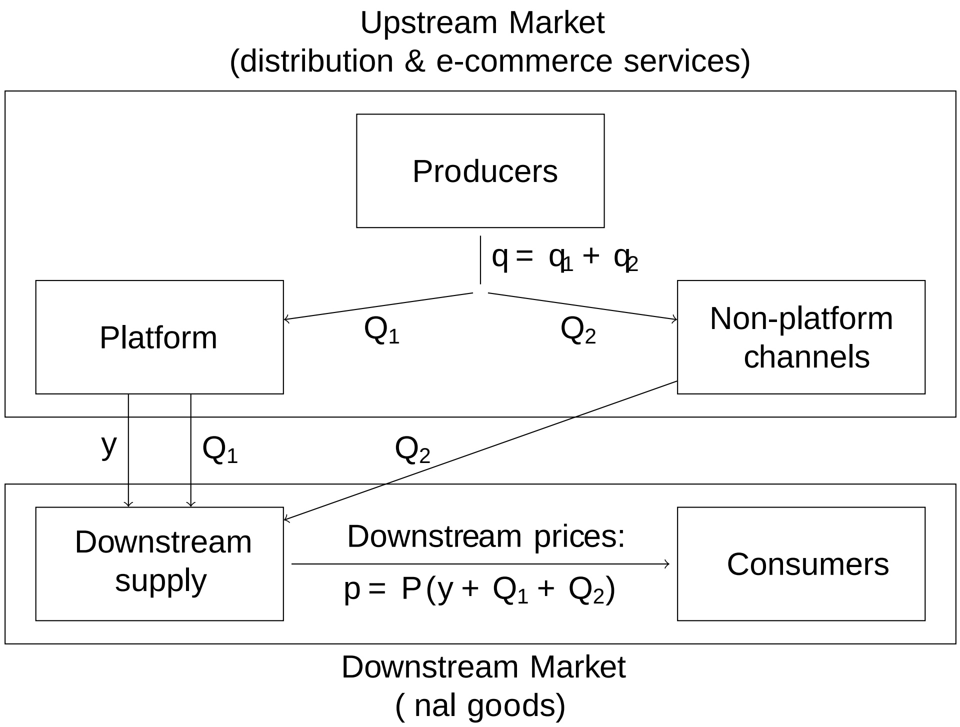

Figure 1: Illustration of vertical structure involving a platform, producers and consumers.

Producers We assume that there is a mass $m > 0$ of producers. Producers can produce any positive quantity $q \geq 0$ of total output and are distinguished by their linear production costs $c \in [\underline{c}, \overline{c}] \subseteq \mathbb{R}$. These costs are distributed according to an absolutely continuous distribution F whose density f has full support on $[\underline{c}, \overline{c}]$ and these costs are the private information of each producer. We assume that for a producer with a linear production cost of $c \in [\underline{c}, \overline{c}]$, the total cost of production $C(q)$ of producing the quantity $q \geq 0$ is given by

$$C(q) = cq + g(q);$$

where g is a strictly convex function that is common to all producers and satisfies the Inada conditions $g(0) = 0$, $g' > 0$ on $(0, \infty)$, $g'(0) = 0$ and $\lim_{q \to \infty} g'(q) = \infty$. We also assume that the per-unit cost of accessing a non-platform distribution channel is given by $t_2 > 0$ and suppose that the platform charges a per unit fee of t_1. The payoff of producer of type c that distributes the quantity $q_1 \geq 0$ using the platform and the quantity $q_2 \geq 0$ using a non-platform distribution channel is then given by

$$(p - c)(q_1 + q_2) - g(q_1 + q_2) - t_1 q_1 - t_2 q_2.$$

Here, we have constructed the cost functions of producers in such a way that we can adopt a first-order approach when we characterize the set of upstream contracts offered by the platform. If we were to set $g(q) \equiv 0$ and consider a setting with linear production costs, then we could not adopt a first-order approach and would need to introduce capacity constraints in order to have a well-defined problem. We discuss this generalization in Section 4.4. Aside from restricting attention to strictly convex production costs, the other substantive restriction we have imposed here is that producers only have private information concerning a linear component of their production costs. This simplifies the analysis considerably as the downstream price p, which is endogenous to the set of contracts offered in the upstream market, also linearly enters producers' payoffs.

Platform The platform designs the selling mechanism in the upstream market. By the revelation principle, without loss of generality we can restrict attention to direct, incentive compatible mechanisms $\langle q_1, t_1 \rangle$. The allocation rule $q_1 : [\underline{c}, \bar{c}] \to \mathbb{R}_{\geq 0}$ maps the report $\hat{c}$ of each producer to the quantity of output $q_1(\hat{c})$ sold by that producer through the platform channel. The payment rule $t_1 : [\underline{c}, \bar{c}] \to \mathbb{R}_{\geq 0}$ maps the report $\hat{c}$ of each producer to the per-unit payment made by that producer to the platform. One can equivalently think of the platform as posting the menu of contracts $\langle q_1, t_1 \rangle$ and then allowing each producer to select their desired contract from this menu. We let c_P denote the linear production cot of the platform and assume that the cost to the platform of producing $y \geq 0$ units of output is given by

$$C_P(y) = c_P y + g(y).$$

That is, we assume the platform has access to a production technology that is similar to that of the producers.[7] We also assume that the platform has a cost advantage over its upstream rivals (and, consequently, is also a dominant firm in the upstream market) and can provide producers with access to the downstream market at zero marginal cost. Finally, to ensure that we have a non-trivial problem where it's possible for the platform to extract a strictly positive level of profit from the upstream market, we assume that $P(0) - \underline{c} > 0$.[8]

Timing The timing of the game is as follows. First, the platform announces the upstream market selling mechanism $\langle q_1, t_1 \rangle$ for the upstream market and commits to a level of production $y \geq 0$ in the downstream market. Given y and $\langle q_1, t_1 \rangle$, each producer then decides

[7] Note that one important difference here is that the output produced by the platform and the output produced by an individual producer are measured in different units: the platform produces units of aggregate output, whereas the output produced by an individual producer is infinitesimal.

[8] By continuity, this is a sufficient condition for a positive mass of producers to enter the downstream market when the platform provides its services at zero cost.

how much output q to produce in total, the quantity of output q_1 that they sell through the platform and the quantity of output q_2 that they sell through non-platform channels. The platform, producers and consumers then participate in the downstream market.

3 Monopoly platform in upstream market

In this section, we consider a simple application of our general framework to build intuition that will be useful in the full treatment that we provide in Section 4. To best illustrate the economic insights, we make the major simplifying assumption that producers do not have access to alternative distribution channels other than the platform. Thus the platform exerts monopoly power in the upstream market. Formally, this assumption is equivalent to setting $q_2 = 0$. For notational simplicity, we omit variable subscripts throughout this section.

3.1 Optimal menu of upstream contracts

We now derive the optimal upstream selling mechanism absent downstream production on the part of the platform. What distinguishes the problem faced by the platform from a standard mechanism design problem is that the valuations of producers (which depend on the price p in the downstream market) are *endogenous* to the upstream market mechanism $\langle q, t\rangle$. We solve this fixed point problem by rewriting to platform's problem as a nested optimization problem. First, we fix the price p in the downstream market and determine the optimal upstream selling mechanism, subject to the producers' incentive compatibility and individual rationality constraints, we well as the constraint that this mechanism induces the price p in the downstream market. We then maximize over the price p that is induced in the downstream market. Formally, this approach is justified by the following lemma.

Lemma 1. *An incentive compatible and individually rational upstream selling mechanism $\langle q, t\rangle$ induces the price p in the downstream market if and only if the market-clearing constraint*

$$D(p) = m \int_{\underline{c}}^{\bar{c}} q(c)\, dF(c) \tag{MC}$$

is satisfied.

Given any direct selling mechanism $\langle q, t\rangle$ downstream market price p, the payoff for a producer of type c that reports to be of type $\hat{c}$ is given by

$$(p - c)q(\hat{c}) - g(q(\hat{c})) - t(\hat{c})q(\hat{c}).$$

Incentive compatibility then requires

$$(p - c)q(c) - g(q(c)) - t(c)q(c) \geq (p - c)q(\hat{c}) - g(q(\hat{c})) - t(\hat{c})q(\hat{c}) \quad \forall c, \hat{c} \in [\underline{c}, \bar{c}] \tag{IC}$$

and individual rationality requires

$$(p - c)q(c) - g(q(c)) - t(c)q(c) \geq 0 \quad \forall c \in [\underline{c}, \bar{c}]. \tag{IR}$$

Putting all of this together, the mechanism design problem faced by the platform is given by

$$\max_{p \geq 0} \max_{\substack{q:[\underline{c},\bar{c}] \to \mathbb{R}_{\geq 0}, \\ t:[\underline{c},\bar{c}] \to \mathbb{R}}} m \int_{\underline{c}}^{\bar{c}} t(c)q(c)\, dF(c)$$

$$\text{s.t.} \quad (p - c)q(\hat{c}) - g(q(\hat{c})) - t(c)q(c) \geq (p - c)q(\hat{c}) - g(q(\hat{c})) - t(\hat{c})q(\hat{c}) \quad \forall c, \hat{c} \in [\underline{c}, \bar{c}]; \tag{IC}$$

$$(p - c)q(c) - g(q(c)) - t(c)q(c) \geq 0 \quad \forall c \in [\underline{c}, \bar{c}]; \tag{IR}$$

$$D(p) = m \int_{\underline{c}}^{\bar{c}} q(c)\, dF(c). \tag{MC}$$

We now focus on the inner optimization problem and start by using the incentive compatibility constraints to characterize the transfer rule t in terms of the allocation rule q. To that end, we have the following useful lemma.

Lemma 2. *The function $v(q; c) := (p - c)q - g(q)$ exhibits strict increasing differences in $(q; -c)$.*

Lemma 2 implies that any allocation rule q can be implemented by an incentive compatible direct mechanism if and only if the allocation rule q is decreasing in $c \in [\underline{c}, \bar{c}]$ (see, for example, Proposition 1 in Rochet (1987)). Given any decreasing allocation rule q, applying the Envelope Theorem (see Milgrom and Segal, 2002) then yields

$$(p - c)q(c) - g(q(c)) - t(c)q(c) = \int_{c}^{\bar{c}} q(x)\, dx + k,$$

where k is an arbitrary constant. This last result, which pins down the corresponding transfer rule t up to an arbitrary constant, is known as *payoff equivalence*. The individual rationality constraint associated with producers of type $\underline{c}$ implies that $k \geq 0$. Rearranging this expression we have

$$t(c)q(c) = (p - c)q(c) - g(q(c)) - \int_{c}^{\bar{c}} q(x)\, dx - k.$$

The platform's upstream revenue can therefore be written

$$R^{\text{Upstream}} = m\int_{\underline{c}}^{\bar{c}} t(c)\,dF(c) = \int_{\underline{c}}^{\bar{c}} \left((p - c)q(c) - g(q(c)) - \int_{c}^{\bar{c}} q(x)\,dx \right) dF(c) - k.$$

Using

$$\int_{\underline{c}}^{\bar{c}}\int_{c}^{\bar{c}} q(x)\,dx\,dF(c) = \int_{\underline{c}}^{\bar{c}} q(x)\int_{\underline{c}}^{x} dF(c)\,dx = \int_{\underline{c}}^{\bar{c}} q(x)\frac{F(x)}{f(x)}\,dF(x)$$

as well as the fact that we must have $k = 0$ under any optimal mechanism, we can rewrite this as

$$R^{\text{Upstream}} = m\int_{\underline{c}}^{\bar{c}} \left(\left(p - c - \frac{F(c)}{f(c)} \right) q(c) - g(q(c)) \right) dF(c).$$

The optimization problem faced by the platform is therefore given by

$$\begin{gathered}\max_{p \geq 0}\ \max_{q:[\underline{c},\bar{c}]\to\mathbb{R}_{\geq 0}}\ m\int_{\underline{c}}^{\bar{c}} \left(\left(p - c - \frac{F(c)}{f(c)} \right) q(c) - g(q(c)) \right) dF(c)\\ \text{s.t.}\quad D(p) = m\int_{\underline{c}}^{\bar{c}} q(c)\,dF(c); \quad q(\cdot)\ \text{decreasing}\end{gathered}$$

To complete solving the inner problem, we consider its dual. Letting λ denote the Lagrange multiplier (or shadow price) associated with the market-clearing constraint (which is essentially a quantity constraint) and introducing the virtual type function $\phi(c) = c + \frac{F(c)}{f(c)}$, the Lagrange dual function (see, for example, Chapter 5 in Boyd and Vandenberghe, 2004) associated with the inner problem is given by

$$\begin{gathered}L(p;\lambda) := \max_{q:[\underline{c},\bar{c}]\to\mathbb{R}_{\geq 0}}\ m\int_{\underline{c}}^{\bar{c}} \left[(p - \phi(c) - \lambda)\,q(c) - g(q(c)) \right] dF(c) + \lambda D(p) \\ \text{s.t.}\quad q(\cdot)\ \text{decreasing}\end{gathered} \tag{1}$$

The dual problem is therefore given by

$$\min_{\lambda\in\mathbb{R}} L(p;\lambda).$$

and the designer's full optimization problem becomes

$$\max_{p\geq 0}\min_{\lambda\in\mathbb{R}} L(p;\lambda). \tag{2}$$

Since strong duality holds, solving the dual problem yields the value of the primal problem (the inner problem under the optimal mechanism). Moreover, since the primal problem is convex, the provided the solution to the dual problem is feasible for the primal problem, solving the dual problem also yields the optimal solution to the primal problem.

We now complete our characterization of the optimal menu of upstream contracts o ered by the platform. First, since g is a strictly convex function, the solution q^M to the relaxed version of the maximization problem given in (1) satis es the rst-order condition

$$g^0(q^M(c)) = (p \quad (c) \quad)_+ :$$

Provided we have a problem that is regular in the sense of Myerson (1981) and is increasing in c, this solution also satis es the constraint that q is decreasing in c. Note that $p > (\underline{c}) = \underline{c}$ must hold under any candidate solution. Otherwise no producers would enter the downstream market and the platform makes zero pro t, which cannot be optimal since by assumption we have $\underline{c} \quad P(0) > 0$. We can then rewrite (2) as

$$\max_{p \; 0} \min_{\; 2R} \left(m \int_{\underline{c}}^{\min f \quad ^1(p \quad);\bar{c}g} (p \quad (c) \quad) \, q^M(c) \quad g(q^M(c)) \; dF(c) + \; D(p) \right) ;$$

where, for $c \; 2 \; [\underline{c}; \min f \quad ^1(p \quad); \bar{c}g]$, q^M is characterized by

$$g^0(q^M(c)) = p \quad (c) \quad :$$

The optimal value ^{M} of the Lagrange multiplier is in turn pinned down by the market-clearing constraint

$$m \int_{\underline{c}}^{\min f \quad ^1(p \quad ^M);\bar{c}g} q^M(c) \, dF(c) = D(p): \qquad (3)$$

Finally, we can determine the optimal price p^M that the monopoly induces in the downstream market by solving

$$\max_{p \; 0} \left(m \int_{\underline{c}}^{\min f \quad ^1(p \quad ^M);\bar{c}g} p \quad (c) \quad ^M \; q^M(c) \quad g(q^M(c)) \; dF(c) + \; ^M D(p) \right) :$$

The corresponding first-order condition is given by

$$m\int_{\underline{c}}^{\phi^{-1}(p^M-\tau^M)}\left[\left(1-\left.\frac{d\tau^M}{dp}\right|_{p=p^M}\right)q^M(c)+\left.\frac{dq^M(c)}{dp}\right|_{\tau=\tau^M;\,p=p^M}\left(p-\phi(c)-\tau^M-g'(q^M(c))\right)\right]dF(c)$$
$$+\tau^M D'(p^M)+\left.\frac{d\tau^M}{dp}\right|_{p=p^M}D(p^M)=0.$$

Exploiting the first-order condition that pins down q^M, this simplifies to

$$m\left(1-\left.\frac{d\tau^M}{dp}\right|_{p=p^M}\right)\int_{\underline{c}}^{\phi^{-1}(p^M-\tau^M)}q^M(c)\,dF(c)+\tau^M D'(p^M)+\left.\frac{d\tau^M}{dp}\right|_{p=p^M}D(p^M)=0.$$

Using the market-clearing constraint that pins down τ^M then yields

$$\left(1-\left.\frac{d\tau^M}{dp}\right|_{p=p^M}\right)D(p^M)+\tau^M(p^M)D'(p^M)+\left.\frac{d\tau^M}{dp}\right|_{p=p^M}D(p^M)=0$$

and simplifying this last expression we have

$$D(p^M)=-\tau^M(p^M)D'(p^M). \tag{4}$$

Summarizing all of these calculations, we have the following proposition.

Proposition 1. *Suppose that the distribution F is regular in the sense that $\phi(c)=c+\frac{F(c)}{f(c)}$ is increasing in $c\in[\underline{c},\bar{c}]$. Then for all $c\in[\underline{c},\bar{c}]$, the optimal upstream allocation rule q^M and the optimal price p^M that the platform induces in the downstream market are characterized by the equations*

$$g'(q^M(c))=\left[p^M-\phi(c)-\tau^M\right]_+;\quad m\int_{\underline{c}}^{\bar{c}}q^M(c)\,dF(c)=D(p^M)\quad\text{and}\quad D(p^M)=-\tau^M D'(p^M).$$

Moreover, for all $c\in[\underline{c},\bar{c}]$, the optimal transfer rule t^M is in turn pinned down by

$$t^M(c)q^M(c)=(p-\phi(c)-\tau^M)q^M(c)-g(q^M(c)).$$

For the special case involving a quadratic cost function $g(q)=\gamma q^2/2$ with $\gamma>0$ and producer types that are uniformly distributed on the unit interval $[0;1]$ we have

$$q^M(c) = \left[\frac{p^M \quad 2c \quad {}^M}{}\right]_+ ; \qquad {}^M(p^M) = \begin{cases} p^M \quad 2\sqrt{\frac{D\ (p^M)}{m}}; & D\ (p^M) \quad m; \\ p^M \quad \frac{D\ (p^M)}{m} \quad 1; & D\ (p^M) > m \end{cases}$$

$$\text{and} \quad D(p^M) = \quad {}^M D^0(p^M):$$

Moreover, if $D\ (p^M) \quad 1$ then the corresponding optimal transfer rule t^M is given by

$$t^M(c) = \frac{p^M + 2c + 3 \quad {}^M}{4} 1 \quad c \; 2 \quad \underline{c}; \frac{p^M \quad {}^M}{2} \tag{5}$$

and if $D\ (p^M) > 1$ then this transfer rule is

$$t^M(c) = \frac{p^M + 2c + 3 \quad {}^M}{4} + \frac{(p^M \quad 2 \quad {}^M)^2}{4(p^M \quad 2c \quad {}^M)}: \tag{6}$$

As proposition 1 shows, the allocation rule q^M translates to a nonlinear pricing schedule t^M and represents a form of second-degree price discrimination that minimizes the information rents of the producers. Moreover, we have the following corollary, which shows that the optimal nonlinear pricing schedule exhibits quantity discounts.

Corollary 1. The schedule $T^M(c) = (p \quad (c) \quad {}^M)q^M(c) \quad g(q^M(c))$ of total payments made by producers to the platform is concave in the quantity $q^M(c)$.

From the general characterization

$$g^0(q^M(c)) = \left[p^M \quad (c) \quad {}^M\right]_+$$

we can see that, relative to e ciency, q^M exhibits two distortions. First, the platform restricts the quantity that is sold in the upstream market in order to lower the information rents that it pays to producers. This distortion corresponds to the fact that producers' virtual costs $(c) > c$ rather than actual costs c appear in the characterization of q^M. Second, the platform restricts the quantity sold upstream in order to raise the downstream price and increase producers' upstream valuations. This distortion corresponds to that fact that ${}^M > 0$ appears in the characterization of q^M. We therefore see that this problem gives rise to a vertical externality that is similar to double marginalization.

3.2 Implications of downstream platform production

We now extend the simple model considered in Section 3.1 by introducing the possibility that the platform also produces output in the downstream market. Intuitively, the platform now faces a tradeoff between its upstream and downstream profits. Any additional profit that the platform makes in the upstream market is associated with a lower equilibrium price in the downstream market and a negative impact on the platform's own downstream profits. Consequently, when the platform becomes more efficient at producing in the downstream market (captured by a lower platform cost c_P) and its downstream profits become relatively more important, it will reduce the quantity that it sells in the upstream market. This is captured by the following proposition, which we prove by adopting a monotone comparative statics approach.

Proposition 2. Let $R(Q) = QP(Q)$ denote the revenue associated with selling the total quantity Q in the downstream market and suppose that R is a concave function. Then as the platform's cost c_P decreases, the platform increases its own downstream production y^M and decreases the total quantity $\int_{\underline{c}}^{\bar{c}} q^M(c)\, dF(c)$ that it sells in the upstream market.

Unsurprisingly, producers are adversely affected when the platform also sells output in the downstream market and producer surplus is increasing in the platform's linear cost c_P. The implications of platform production for consumer surplus in the downstream market is more subtle and cannot be determined using a simple monotone comparative statics approach. We now characterize the optimal menu of upstream contracts $\langle q^M; t^M \rangle$, level of platform output y^M and the downstream market price p^M and investigate the implications of platform production for consumer surplus.

When the platform also produces in the downstream market, its nested optimization problem becomes

$$\max_{p \geq 0} \max_{\substack{y \geq 0; \\ q:[\underline{c};\bar{c}] \to \mathbb{R}_{\geq 0}}} m \int_{\underline{c}}^{\bar{c}} [(p - \psi(c))\, q(c) - g(q(c))]\, dF(c) + y(p - c_P) - g(y)$$

$$\text{s.t.} \quad D(p) = m \int_{\underline{c}}^{\bar{c}} q(c)\, dF(c) + y; \quad q(\cdot) \text{ decreasing}$$

Letting λ denote the Lagrange multiplier associated with the market-clearing constraint, the

Lagrange dual function is now given by

$$L(p;\lambda) := \max_{\substack{y \geq 0;\\ q:[\underline{c},\overline{c}] \to \mathbb{R}_{\geq 0}}} m \int_{\underline{c}}^{\overline{c}} [(p - \phi(c) - \lambda)\, q(c) - g(q(c))]\, dF(c) + y(p - c_P - \lambda) - g(y)$$
$$+ \lambda D(p)$$
$$\text{s.t.} \quad q(\cdot) \text{ decreasing}$$

Maintaining our assumption that we have a regular mechanism design problem and the function ϕ is increasing, the optimal allocation rule is still characterized by

$$g'(q^M(c)) = (p - \phi(c) - \lambda)_+ :$$

Similarly, the platform's optimal level of production is characterized by

$$g'(y^M) = (p - c_P - \lambda)_+ : \tag{7}$$

Note that any candidate solution must be such that $p - \lambda > 0$ because otherwise the platform would make zero profit in both the upstream and the downstream markets, which cannot be optimal. The designer's full optimization problem then becomes

$$\max_{p \geq 0} \min_{\lambda \in \mathbb{R}} \left\{ m \int_{\underline{c}}^{\min\{\phi^{-1}(p-\lambda);\overline{c}\}} \left[(p - \phi(c) - \lambda)\, q^M(c) - g(q^M(c)) \right] dF(c) + y^M(p - c_P - \lambda) \right.$$
$$\left. - g(y^M) + \lambda D(p) \right\};$$

where y^M is characterized by (7) and, for $c \in [\underline{c}, \min\{\phi^{-1}(p-\lambda);\overline{c}\}]$, q^M is characterized by

$$g'(q^M(c)) = (p - \phi(c) - \lambda) :$$

The optimal value λ^M of the Lagrange multiplier is pinned down by the first-order condition

$$m \int_{\underline{c}}^{\min\{\phi^{-1}(p-\lambda^M);\overline{c}\}} q^M(c)\, dF(c) + y^M = D(p): \tag{8}$$

This first-order condition shows that for a given value of p, a decrease in c_P must lead to an increase in λ^M so that the left-hand-side of (8) remains constant. Intuitively, if c_P decreases then the platform's equilibrium level of output increases and the price in the downstream market will only remain constant if this increase is offset by a corresponding decrease in the output produced by the competitive fringe. Finally, we determine the optimal price p^M by solving

$$\max_{p \quad 0} \left(m \int_{\underline{c}}^{\min f \quad ^1(p \quad ^M);\bar{c}g} \quad p \quad (c) \quad ^M \quad q^M(c) \quad g(q^M(c)) \ \ dF(c) \right.$$

$$\left. + \ y^M(p \quad c_P \quad ^M) \quad g(y^M) \quad \right) ;$$

The corresponding rst-order condition is given by

$$m \quad 1 \quad \frac{d \ ^M}{dp}\bigg|_{p=p^M} \, ! \int_{\underline{c}}^{\min f \quad ^1(p^M \quad ^M);\bar{c}g} q^M(c)\,dF(c) + \ ^M(p^M)D^0(p^M) + \frac{d \ ^M}{dp}\bigg|_{p=p^M} D(p^M)$$

$$+ \quad 1 \quad \frac{d \ ^M}{dp}\bigg|_{p=p^M} \, ! \quad y^M = 0:$$

Using the rst-order condition given in (8) we have

$$1 \quad \frac{d \ ^M}{dp}\bigg|_{p=p^M} \, ! \quad D(p^M) \quad y^M \ + \ ^M(p^M)D^0(p^M) + \frac{d \ ^M}{dp}\bigg|_{p=p^M} D(p^M)$$

$$+ \quad 1 \quad \frac{d \ ^M}{dp}\bigg|_{p=p^M} \, ! \quad y^M = 0$$

and simplifying this last expression again yields

$$D(p^M) = \quad ^M(p^M)D^0(p^M):$$

Summarizing all of this, we have the following proposition.

Proposition 3. Suppose that the distribution F is regular in the sense that $(c) = c + \frac{F(c)}{f(c)}$ is increasing in $c \, 2 \, [\underline{c}; \bar{c}]$. Then for all $c \, 2 \, [\underline{c}; \bar{c}]$, the optimal upstream allocation rule q^M, the optimal level of downstream platform production y^M and the optimal price p^M that the platform induces in the downstream market are characterized by the equations

$$g^0(q^M(c)) = \quad p^M \quad (c) \quad ^M \ _+ ; \quad g^0(y^M) = \quad p^M \quad c_P \quad ^M \ _+ ;$$

$$m \int_{\underline{c}}^{\bar{c}} q^M(c)\,dF(c) + y^M = D(p^M); \quad \text{and} \quad D(p^M) = \quad ^M D^0(p^M):$$

Moreover, for all $c \in [\underline{c}, \bar{c}]$, the optimal transfer rule t^M is in turn pinned down by

$$t^M(c)q^M(c) = (\ p - c)q^M(c) - g(q^M(c)) - \int_c^{\bar{c}} q^M(x)\,dx.$$

Next, consider the special case involving a quadratic cost function $g(q) = \ \frac{q^2}{2}$ with $\ > 0$ and producer types that are uniformly distributed on the unit interval $[0, 1]$. Then we have $y^M = 0$ and the solution from Proposition 2 applies if $c_P \geq \bar{c}_P$, where the cutoff $\bar{c}_P$ is given by

$$\bar{c}_P = \begin{cases} 2\sqrt{\frac{\ D(p^M)}{m}}; & \ D(p^M) \leq m \\ 1 + \frac{\ D(p^M)}{m}; & \ D(p^M) > m. \end{cases}$$

If $c_P < \bar{c}_P$ then we have $y^M > 0$ and the solution to the platform's problem is given by

$$q^M(c) = \left[\frac{p^M - 2c\ \ {}^M}{\ }\right]_+; \quad y^M = \frac{p^M - c_P\ \ {}^M}{\ };$$

$${}^M = \begin{cases} p + \frac{2 - 2\sqrt{1 + mc_P + \ mD(p)}}{m}; & c_P + \ D(p) \leq 2 + m \\ p - \frac{m + c_P + \ D(p)}{1 + m}; & c_P + \ D(p) > 2 + m \end{cases} \quad \text{and} \quad D(p^M) = \ {}^M D'(p^M).$$

Moreover, if $c_P + \ D(p^M) \leq 2 + m$ then the corresponding optimal transfer rule is given by (5) and if $c_P + \ D(p^M) > 2 + m$ then the corresponding transfers are given by (6).

Proposition 2 shows that the platform's equilibrium output y^M decreases in c_P, while the total equilibrium quantity $m\int_{\underline{c}}^{\bar{c}} q^M(c)\,dF(c)$ that it sells in the upstream market increases in c_P. This proposition, which adopted a simple monotone comparative statics approach, was silent with regard to how the total quantity $y^M + m\int_{\underline{c}}^{\bar{c}} q^M(c)\,dF(c)$ supplied in the downstream market varies with c_P. However, we are now in a position to prove that this aggregate quantity in fact decreases in c_P.

Proposition 4. The equilibrium quantity $y^M + m\int_{\underline{c}}^{\bar{c}} q^M(c)\,dF(c)$ supplied in the downstream market decreases in c_P. Consequently, the downstream market price p^M increases in c_P and consumer surplus decreases in c_P.

The previous proposition shows that the platform's tradeoff between upstream and downstream profits protects consumers. However, producers are harmed because they each producer a lower level of output, which reduces their total information rents.[9] Intuitively, we

[9]Note that producers are not directly harmed by the reduction in the downstream equilibrium price. Since this component of producers valuation is not private information, the platform can extract it in the upstream market.

can think of the platform as a downstream monopoly that has access to two production technologies: it can use the competitive fringe to produce output and it can produce output itself. Since the platform must pay information rents to producers, the output it produces using the competitive fringe exhibits a vertical externality akin to double marginalization. However, the output that the platform produces itself does not. Consequently, as c_s decreases and the platform substitutes away from downstream production that exhibits this double marginalization effect, consumers benefit.

4 Dominant platform in upstream market

We now generalize our analysis by returning to the full model introduced in Section 2. That is, we consider the possibility that sellers have access to non-platform distribution channels. Following our approach in the previous section we first characterize the optimal menu of upstream selling mechanisms absent any downstream production on the part of the platform. We then consider several extensions of this baseline case.

4.1 Optimal menu of upstream contracts

We now derive the optimal menu of contracts the platform offers in the upstream market, assuming that the platform does not produce its own output in the downstream market. For now, we also assume that the platform cannot engage in exclusive dealing. Specifically, we assume that the platform cannot monitor producers' use of non-platform distribution channels and after selecting the desired platform contract $\langle q_1; t_1 \rangle$, producers are free to sell any quantity $q_2 \geq 0$ through non-platform distribution channels. Given a feasible downstream market price p and an incentive compatible and individually rational direct mechanism $\langle q_1; t_1 \rangle$, the optimal level of output $q_2(q_1(c); c)$ that a producers of type c then sells through non-platform distribution channels is given by

$$q_2(q_1(c); c) = \arg\max_{q_2 \geq 0} \{(p - c)(q_1(c) + q_2) - g(q_1(c) + q_2) - t_1(c) q_1(c) - t_2 q_2\}:$$

The quantity $q_2(q_1(c); c)$ therefore satisfies the first-order condition

$$g'(q_1(c) + q_2(q_1(c); c)) = (p - c - t_2)_+ :$$

Letting $T_1(c) = t_1(c) q_1(c)$ denote the total payment that a producer of type c makes to the platform under a given direct mechanism $\langle q_1; t_1 \rangle$, the platform now solves the nested optimization problem given by

$$\max_{p \geq 0} \max_{\substack{q_1:[\underline{c},\overline{c}]\to\mathbb{R}_{\geq 0};\\ T_1:[\underline{c},\overline{c}]\to\mathbb{R}}} m \int_{\underline{c}}^{\overline{c}} T_1(c)\, dF(c)$$

$$\begin{aligned}
\text{s.t.} \quad & (p-c)(q_1(c)+q_2(q_1(c);c) - q_1(\hat{c}) - q_2(q_1(\hat{c});c)) - g(q_1(c)+q_2(q_1(c);c)) \\
& + g(q_1(\hat{c})+q_2(q_1(\hat{c});c)) - T_1(c) \geq -T_1(\hat{c}) + t_2(q_2(q_1(c);c) - q_2(q_1(\hat{c});c)) \;\; \forall c,\hat{c} \in [\underline{c},\overline{c}] && \text{(IC)} \\
& (p-c)(q_1(c)+q_2(q_1(c);c)) - g(q_1(c)+q_2(q_1(c);c)) - T_1(c) - t_2 q_2(q_1(c);c) \\
& \geq \max_{q_2 \geq 0}\{(p-c-t_2)q_2 - g(q_2)\} \;\; \forall c \in [\underline{c},\overline{c}]; && \text{(IR)} \\
& g'(q_1(c)+q_2(q_1(c);c)) = (p-c-t_2)_+ \;\; \forall c \in [\underline{c},\overline{c}]; && \text{(FOC)} \\
& D(p) = m \int_{\underline{c}}^{\overline{c}} (q_1(c)+q_2(q_1(c);c))\, dF(c). && \text{(MC)}
\end{aligned}$$

However, we can substantially simplify this problem by exploiting the following observation: without loss of generality we can focus on equilibria such that $q_2(q_1(c);c) = 0$ for all $c \in [\underline{c},\overline{c}]$. From the perspective of producers, the productive inputs offered by both the platform and the non-platform channels are perfect substitutes. Consequently, the platform can always offer contracts that essentially replicate the outside option and ensure that all producers set $q_2(q_1(c);c) = 0$ in equilibrium. From the perspective of the platform, it provides productive inputs in the upstream market at zero marginal cost. Consequently, it is never optimal for it to offer a set of upstream contracts that induces producers to sell a positive mass of output through non-platform channels. Formally, we have the following lemma.

Lemma 3. *Without loss of generality we can restrict attention to incentive compatible and individually rational direct mechanisms $\langle q_1; t_1 \rangle$ such that $g'(q_1(c)) \geq (p-c-t_2)_+$ and $q_2(q_1(c);c) = 0$ for all $c \in [\underline{c},\overline{c}]$.*

Lemma 3 shows that when producers can access non-platform distribution channels, this constrains the platform's ability to restrict the total quantity sold in the upstream market. As we saw in the previous section, by restricting the quantity it sells upstream, the platform can extract higher rents from producers and put upward pressure on the downstream market price. Using Lemma 3 we can now rewrite the platform's problem as

$$\max_{p \geq 0} \max_{\substack{q_1:[\underline{c},\bar{c}] \to \mathbb{R}_{\geq 0};\\ T_1:[\underline{c},\bar{c}] \to \mathbb{R}}} m \int_{\underline{c}}^{\bar{c}} T_1(c)\, dF(c) \tag{9}$$

$$\text{s.t.} \quad (p-c)(q_1(c)-q_1(\hat{c})) \geq g(q_1(c)) - g(q_1(\hat{c})) + T_1(c) - T_1(\hat{c}) \;\; \forall c, \hat{c} \in [\underline{c}, \bar{c}]; \tag{IC}$$

$$(p-c)q_1(c) - g(q_1(c)) - t_1(c)q_1(c) \geq \max_{q_2 \geq 0} \{(p-c-t_2)q_2 - g(q_2)\} \;\; \forall c \in [\underline{c}, \bar{c}]; \tag{IR}$$

$$g'(q_1(c)) \geq (p-c-t_2)_+ \;\; \forall c \in [\underline{c}, \bar{c}]; \tag{LP}$$

$$D(p) = m \int_{\underline{c}}^{\bar{c}} (q_1(c) + q_2(q_1(c); c))\, dF(c). \tag{MC}$$

This is similar to the problem that we solved in Section 3.1. However, we now have a set of quantity constraints which insure that no producers sell a positive quantity of output through non-platform distributions channels after signing a contract with the platform. Adapting the terminology and notation of Calzolari and Denicolo (2015), we refer to these as *limit pricing* constraints and let q_1^{lim} denote the allocation rule that satisfies $g'(q_1^{lim}(c)) = (p-c-t_2)_+$.[10] We also have a set of type-dependent individual rationality constraints which ensure that the producers don't bypass the platform altogether and only utilize non-platform distribution channels. Since we now have a mechanism design problem involving a type-dependent outside option, we solve this problem by adopting the approach of Jullien (2000). Specifically, we again consider the indirect payoff function

$$v(q_1; c) = (p-c)q_1 - g(q_1)$$

but we now make the change of variables

$$U(c) = v(q_1(c); c) - T_1(c).$$

This allows us to eliminate the transfer rule from the platform's objective function by rewriting it as

$$m \int_{\underline{c}}^{\bar{c}} (v(q_1(c); c) - U(c))\, dF(c).$$

[10] As we shall see shortly, whenever the quantity constraint is binding for a producer of type c, the platform sells this type a quantity that is higher than the corresponding monopoly quantity in order to foreclose its upstream competition and ensure that this type does not sell a positive level of output through non-platform distribution channels. Calzolari and Denicolo (2015) use the term *limit pricing* to refer to transfer rules that implement the allocation q_1 satisfying $g'(q_1(c)) = (p-c-t_2)_+$ for all $c \in [\underline{c}, \bar{c}]$.

By Lemma 2 the indirect payoff function v exhibits strict increasing differences in $(q_1; -c)$. Combining this single-crossing property with the Envelope Theorem then shows that incentive compatibility is equivalent to requiring that $U'(c) = -q_1(c)$ holds for all $c \in [\underline{c}; \bar{c}]$ and q_1 is decreasing in c. The platform's problem thus becomes

$$\max_{p \geq 0} \max_{q_1:[\underline{c};\bar{c}] \to \mathbb{R}_{\geq 0}} m \int_{\underline{c}}^{\bar{c}} (v(q_1(c); c; p) - U(c))\, dF(c) + D(p)$$

$$\text{s.t.} \quad q_1(\cdot) \text{ decreasing} \quad \text{and} \quad \frac{dU(c)}{dc} = -q_1(c) \ \forall c \in [\underline{c}; \bar{c}]; \tag{IC}$$

$$U(c) \geq \max_{q_2 \geq 0} \{(p - c - t_2)q_2 - g(q_2)\} \ \forall c \in [\underline{c}; \bar{c}]; \tag{IR}$$

$$q_1(c) \leq q_1^{lim}(c) \ \forall c \in [\underline{c}; \bar{c}]; \tag{LP}$$

$$D(p) = m \int_{\underline{c}}^{\bar{c}} q_1(c)\, dF(c): \tag{MC}$$

This is an optimal control program where q_1 is the state variable and U is the control variable. Besides the market-clearing constraint, this control problem is similar to that studied by Calzolari and Denicolo (2015), who derive the optimal nonlinear pricing strategy of a dominant firm that faces a competitive fringe. Following our approach from Section 3.1 and letting λ denote the Lagrange multiplier associated with the market-clearing constraint, the Lagrange dual function is now given by

$$L(p; \lambda) := \max_{q_1:[\underline{c};\bar{c}] \to \mathbb{R}_{\geq 0}} m \int_{\underline{c}}^{\bar{c}} [(p - c - \lambda)q(c) - g(q(c)) - U(c)]\, dF(c) + \lambda D(p)$$

$$\text{s.t.} \quad (IC); \quad (IR) \quad \text{and} \quad (LP): \tag{IC}$$

The single-crossing condition guarantees that the individual rationality constraint is only binding for a single marginal type e.[11] Using this fact, the platform's objective function can be written

$$m \int_{\underline{c}}^{e} [(p - c - \lambda)q(c) - g(q(c)) - U(c)]\, dF(c) + m t_2 \int_{e}^{\bar{c}} q^{lim}(c)\, dF(c):$$

Integrating by parts and using $U'(c) = -q_1(c)$ we have

$$\int_{\underline{c}}^{e} U(c) f(c)\, dc = U(e)F(e) - \int_{\underline{c}}^{e} U'(c)F(c)\, dc = U(e)F(e) + \int_{\underline{c}}^{\bar{c}} q_1(c)F(c)\, dc:$$

[11] Note that if t_2 is sufficiently small then we will have $e = \underline{c}$. If t_2 is sufficiently large then the limit pricing constraint will not bind for any type and e will be such that both $q^M(e) = 0$ and $q^{lim}(e) = 0$.

Using this to rewrite the Lagrange dual function we have

$$\max_{q_1:[\underline{c},\bar{c}]\to\mathbb{R}_{0}} m \int_{\underline{c}}^{e} [(p \quad (c) \quad) q_1(c) \quad g(q_1(c))]\, dF(c) \quad U(e)F(e) + t_2 \int_{e}^{\bar{c}} q^{lim}(c)\, dF(c)$$

$$\text{s.t.} \quad q_1(\,) \text{ decreasing} \quad \text{and } q_1(c) \quad q_1^{lim}(c)\ \forall c \in [\underline{c},\bar{c}]:$$

Ignoring the nal remaining constraints, the solution q_1^M to the relaxed version of this optimization problem satis es the rst-order condition

$$g^0\ q_1^M(c) = (p \quad (c) \quad)_+ :$$

Since g is a strictly convex function we can directly impose the constraint

$$g^0(q_1(c)) \quad (p \quad c \quad t_2)_+$$

on this solution, which yields

$$g^0(q_1(c)) = \max \quad (p \quad (c) \quad)_+ ; (p \quad c \quad t_2)_+ \quad :$$

Equivalently, we can write this as $q_1(c) = \max\{q_1^M(c); q_1^{lim}(c)\}$. If is an increasing in c then both the functions $(p \quad (c) \quad)_+$ and $(p \quad c \quad t_2)_+$ are decreasing in c. Since the minimum of two decreasing functions is itself a decreasing function, this implies that q_1 is decreasing in c. Consequently, q_1 solves the optimization problem associated with the Lagrange dual function. The Lagrange dual function can therefore be written

$$m \int_{\underline{c}}^{\bar{c}} [(p \quad (c) \quad) q_1(c) \quad g(q_1(c))]\, dF(c) \quad t_2F(e)q^{lim}(e) + t_2 \int_{e}^{\bar{c}} q^{lim}(c)\, dF(c) \quad :$$

Two cases are now possible. If t_2 is su ciently small and $q^M(c) \quad q^{lim}(c)$ holds for all $c \in [\underline{c};\bar{c}]$), then we have $e = \underline{c}$.[12] Otherwise, we have $e = \sup_{c \in [\underline{c};\bar{c}]} \{c : q^M(e) > q^{lim}(e)\}$.[13] Our assumption that is increasing in c does not preclude the possibility that the quantity schedules $q_1^M(c)$ and $q_1^{lim}(c)$ intersect at multiple points. However, if the function $F(\,)/f(\,)$ happens to be increasing in c then the quantity schedules q^M and q^{lim} cross at most once, regardless of the value of the Lagrange multiplier.[14]

[12] Since $(c) \quad c$ and $(\underline{c}) = \underline{c}$, if $q^M(\underline{c}) \quad q^{lim}(\underline{c})$, this implies that $q^M(c) \quad q^{lim}(c)$ holds for all $c \in [\underline{c};\bar{c}]$.

[13] Note that we may have $q^{lim}(e) = 0$ in this case. Moreover, since $(c) \quad c$ and $(\underline{c}) = \underline{c}$, if there exists some value of c such that $q^M(c) > q^{lim}(c)$, then we must have $q^M(\underline{c}) > q^{lim}(\underline{c})$.

[14] This property holds for our leading example in which producer costs are uniformly distributed on the unit interval. More generally, assuming that $F(c)/f(c)$ is increasing in c is equivalent to assuming that F is a log-concave function. Although this is stronger than the more common assumption of Myersonian regularity

The solution to the platform's problem is now characterized by the saddle point problem

$$\max_{p \geq 0} \min_{\in R} \quad m \int_{\underline{c}}^{e} [(p \quad (c) \quad) q_1(c) \quad g(q_1(c))] \, dF(c) \quad t_2 F(e) q^{lim}(e)$$
$$+ \; t_2 \int_{e}^{\bar{c}} q^{lim}(c) \, dF(c) \quad + \quad D(p) \quad ;$$

The optimal value is therefore pinned down by the market-clearing constraint

$$D(p) = m \int_{\underline{c}}^{\bar{c}} q_1(c) \, dF(c);$$

while the optimal downstream market price p is characterized by

$$p = \arg\max_{p \geq 0} \quad m \int_{\underline{c}}^{e} [(p \quad (c) \quad) q_1(c) \quad g(q_1(c))] \, dF(c) \quad t_2 F(e) q^{lim}(e)$$
$$+ \; t_2 \int_{e}^{\bar{c}} q^{lim}(c) \, dF(c) \quad + \quad D(p) \quad :$$

Summarizing all of this, we have the following proposition.

Proposition 5. Suppose that the distribution F is regular in the sense that $(c) = c + \frac{F(c)}{f(c)}$ is increasing in $c \in [\underline{c}; \bar{c}]$. Then for all $c \in [\underline{c}; \bar{c}]$, the optimal upstream allocation rule q and the optimal price p that the platform induces in the downstream market are characterized by the equations

$$q_1(c) = \max \quad q_1^M(c); q_1^{lim}(c) \quad ; \quad m \int_{\underline{c}}^{\bar{c}} q_1(c) \, dF(c) = D(p);$$

$$p = \arg\max_{p \geq 0} \quad m \int_{\underline{c}}^{e} [(p \quad (c) \quad) q_1(c) \quad g(q_1(c))] \, dF(c) \quad t_2 F(e) q^{lim}(e)$$
$$+ \; t_2 \int_{e}^{\bar{c}} q^{lim}(c) \, dF(c) \quad + \quad D(p) \quad ;$$

where $q_1^M(c)$ and $q_1^{lim}(c)$ are such that $g^0 \; q_1^M(c) \; = (p \quad (c) \quad)_+$ and $g^0 \; q_1^{lim}(c) \; = (p \quad c \quad t_2)_+$, respectively, and e is given by $e = \underline{c}$ if the set $\{c : q_1^M(c) > q_1^{lim}(c)\}$ is empty and $e = \sup_{c \in [\underline{c}; \bar{c}]} \{c : q_1^M(c) > q_1^{lim}(c)\}$.

If the platform faces a non-trivial problem and the limit pricing constraint is binding for some type $c \in [\underline{c}; \bar{c}]$ then the availability of non-platform distribution channels reduces the

(which merely requires that the function is increasing), many standard demand curves satisfy this property (see, for example, An (1998) and Bagnoli and Bergstrom (2005)).

platform's upstream market power and overall profits. If t_2 is sufficiently small, we can also immediately see that the availability of these non-platform distribution channels increases the total quantity sold in the upstream market and decreases the downstream equilibrium price.

Proposition 6. The platform's total upstream profits are increasing in t_2. Moreover, the optimal downstream price p^* is decreasing in t_2. Consequently, downstream consumer surplus increases in equilibrium as t_2 decreases.

4.2 Implications of downstream platform production

Extending our analysis from the previous subsection to allow for downstream production on the part of the platform now proceeds in precisely the same manner as in Section 3.2. Specifically, the platform's problem is now given by

$$\max_{p \geq 0} \max_{\substack{q_1:[\underline{c},\bar{c}]\to \mathbb{R}_{\geq 0};\\ T_1:[\underline{c},\bar{c}]\to \mathbb{R}}} m \int_{\underline{c}}^{\bar{c}} T_1(c)\, dF(c) + (p - c_P)y - g(y)$$

$$\text{s.t.} \quad (p - c)(q_1(c) + q_2(q_1(c);c) - q_1(\hat{c}) - q_2(q_1(\hat{c});c)) - g(q_1(c) + q_2(q_1(c);c))$$

$$+ g(q_1(\hat{c}) + q_2(q_1(\hat{c});c)) \geq T_1(c) - T_1(\hat{c}) + t_2(q_2(q_1(c);c) - q_2(q_1(\hat{c});c)) \;\; \forall c, \hat{c} \in [\underline{c},\bar{c}] \quad \text{(IC)}$$

$$(p - c)(q_1(c) + q_2(q_1(c);c)) - g(q_1(c) + q_2(q_1(c);c)) - T_1(c) - t_2 q_2(q_1(c);c)$$

$$\geq \max_{q_2 \geq 0} \{(p - c - t_2)q_2 - g(q_2)\} \;\; \forall c \in [\underline{c},\bar{c}]; \quad \text{(IR)}$$

$$g'(q_1(c) + q_2(q_1(c);c)) = (p - c - t_2)_+ \;\; \forall c \in [\underline{c},\bar{c}]; \quad \text{(FOC)}$$

$$D(p) = m \int_{\underline{c}}^{\bar{c}} (q_1(c) + q_2(q_1(c);c))\, dF(c) + y. \quad \text{(MC)}$$

Letting λ denote the Lagrange multiplier associated with the market-clearing constraint, the optimal level y^* of downstream platform production satisfies

$$g'(y^*) = (p - c_P - \lambda)_+$$

and the solution to the platform's problem is therefore characterized by the saddle point problem

$$\max_{p \geq 0} \min_{\lambda \in \mathbb{R}} m \left[\int_{\underline{c}}^{e} [(p - \phi(c) - \lambda)q_1(c) - g(q_1(c))]\, dF(c) - t_2 F(e) q^{\lim}(e) \right.$$

$$\left. + t_2 \int_{e}^{\bar{c}} q^{\lim}(c)\, dF(c) \right] + (p - c_P - \lambda)y^* - g(y^*) + \lambda D(p).$$

Consequently, we have the following proposition.

Proposition 7. Suppose that the distribution F is regular in the sense that $(c) = c + \frac{F(c)}{f(c)}$ is increasing in c 2 $[\underline{c}; \overline{c}]$. Then for all c 2 $[\underline{c}; \overline{c}]$, the optimal upstream allocation rule q, the optimal level of downstream platform production y and the optimal price p that the platform induces in the downstream market are characterized by the equations

$$q_1(c) = \max \; q_1^M(c); q_1^{lim}(c) \; ; \quad g^0(y) = (p \quad c_P \quad)_+ ; \quad m \int_{\underline{c}}^{\overline{c}} q_1(c)\, dF(c) + y = D(p);$$

$$p = \arg\max_{p \; 0} \; m \int_{\underline{c}}^{e} [(p \quad (c) \quad) q_1(c) \quad g(q_1(c))]\, dF(c) \quad t_2 F(e) q^{lim}(e)$$
$$+ t_2 \int_{e}^{\overline{c}} q^{lim}(c)\, dF(c) \; + \; D(p) \; ;$$

where $q_1^M(c)$ and $q_1^{lim}(c)$ are such that $g^0 \; q_1^M(c) = (p \quad (c) \quad)_+$ and $g^0 \; q_1^{lim}(c) = (p \quad c \quad t_2)_+$, respectively, and e is given by $e = \underline{c}$ if the set $f\, c : q_1^M(c) > q_1^{lim}(c) g$ is empty and $e = \sup_{c2[\underline{c};\overline{c}]} f\, c : q_1^M(c) > q_1^{lim}(c) g$.

Similarly to the case in which the platform monopolizes the upstream market, we have the following comparative statics.

Proposition 8. Let $R(Q) = QP(Q)$ denote the revenue associated with selling the total quantity Q in the downstream market and suppose that R is a concave function. The equilibrium quantity $y + m \int_{\underline{c}}^{\overline{c}} q(c)\, dF(c)$ supplied in the downstream market and, consequently, consumer surplus are decreasing in c_P.

4.3 \Killer" acquisitions and exclusive dealing

Since the platform's pro t is increasing in t_2, the platform now has an incentive to take measures that undermine this upstream market competition. Such practices can harm consumers by decreasing downstream entry and increasing the equilibrium price in the downstream market.

For example, the platform may have an incentive to increase the price o ered by the competitive fringe by acquiring and shuttering its upstream market rivals. We can determine the pro tability of a given \killer" acquisition by using the model introduced in this section to compare the cost of a given acquisition to the increase in the platform's pro ts associated with the corresponding change in the price t_2. There is now also scope for the platform to increase its pro ts by o ering exclusive contracts that prohibit some producers from making use of non-platform distribution channels whenever they are served by the platform. Such contracts benefit the platform by removing the limit pricing constraints from (9), meaning that the presence of the competitive fringe only imposes the type-dependent individual rationality constraints on the platform. In particular, the platform's problem becomes

$$\max_{p \geq 0} \max_{\substack{q_1:[\underline{c};\bar{c}]\to \mathbb{R}_{\geq 0};\\ T_1:[\underline{c};\bar{c}]\to \mathbb{R}}} m \int_{\underline{c}}^{\bar{c}} T_1(c)\, dF(c)$$

$$\text{s.t.} \quad (p-c)(q_1(c)-q_1(\hat{c})) \geq g(q_1(c)) - g(q_1(\hat{c})) + T_1(c) - T_1(\hat{c}) \;\; \forall c;\hat{c} \in [\underline{c};\bar{c}]; \qquad \text{(IC)}$$

$$(p-c)q_1(c) - g(q_1(c)) - t_1(c)q_1(c) \geq \max_{q_2 \geq 0} \{(p-c-t_2)q_2 - g(q_2)\} \;\; \forall c \in [\underline{c};\bar{c}]; \qquad \text{(IR)}$$

$$D(p) = m \int_{\underline{c}}^{\bar{c}} (q_1(c) + q_2(q_1(c);c))\, dF(c): \qquad \text{(MC)}$$

From our analysis in the previous section, we know that the individual rationality constraint binds for the marginal type $\tilde{c}$ when the platform cannot offer exclusive contracts. Producers with a cost exceeding $\tilde{c}$ receive their outside option. For producers with a cost less than $\tilde{c}$, the individual rationality constraint (which are restraints in utility space) are never binding but the limit pricing constraints (which are constraints in quantity space) may bind for these types. Consequently, types with a cost exceeding $\tilde{c}$ are unaffected when the platform engages in exclusive dealing. However, the platform strictly benefits from exclusive dealing when there are types less than $\tilde{c}$ for which the limit pricing constraints are binding. In particular, we can characterize the solution for the case where the platform can engages in exclusive dealing in a similar manner to Proposition 5. However, for all types with $c \in [\underline{c};\tilde{c}]$, the optimal allocation rule q_1^e is now given by $q_1^e(c) = q_1^M(c)$ (rather than $q_1(c) = \max\{q_1^M(c); q_1^{lim}(c)\}$, which applies for these types when the platform does not engage in exclusive dealing).[15]

4.4 Linear production costs with capacity constraints

Throughout this paper we studied in setting in which producers and the platform have convex production costs. This allowed us to adopt a tractable, first-order approach. However, if we set $g(q) \equiv 0$ then we obtain a linear model in which the payoff functions of producers are given by

$$(p-c)(q_1+q_2) - t_1 q_1 - t_2 q_2:$$

In order to analyse this linear model we need to introduce capacity constraints so that we have a well-defined problem that doesn't involve producers that only want to produce

[15] Note that each of these quantity schedules depends on the value of the Lagrange multiplier.

no output or an infinite level of output. In Appendix B we study such a linear problem, assuming that each producer has unit capacity. In this linear setting it is also tractable to consider the case where producers' outside options are heterogeneous and represent private information. While the analysis of this linear model is somewhat technical and involved, as the optimal selling mechanism may now involve rationing (see Theorem B.1). However, we can still show that our main result continues to hold and that consumers benefit from downstream production on the part of the platform (see Theorem B.2).

5 Conclusions

In this paper we develop a general mechanism design framework for studying vertical market structures involving a dominant firm. While this paper focused on the scope for the participation of platforms in downstream markets to harm consumers, there are many other antitrust concerns relating to platforms that have been raised by regulators. This, combined with the flexibility of our modelling approach, suggests many avenues for further research. For example, a key concern frequently cited by antitrust authorities is the extent to which exclusive access to consumer data expands the market power of online platforms. Another concern among regulations is whether platforms profitably steer consumers towards their own downstream products by strategically ordering search results. For example, in a related paper we study how product salience can expand the market power of dominant firms in downstream markets, which leaves open the question of what the economic implications of this is in vertical structures.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Proof. First, suppose that the allocation rule q induces a price p in the downstream market. Given that p is a market-clearing price it cannot induce excess demand or supply and we must have $D(p) = \int_{\underline{c}}^{\bar{c}} q(c)\,dF(c)$. Second, suppose p satisfies $D(p) = \int_{\underline{c}}^{\bar{c}} q(c)\,dF(c)$ and let p' denote the price induced in the downstream market. Then by assumption we also have $D(p') = \int_{\underline{c}}^{\bar{c}} q(c)\,dF(c)$ and hence $D(p) = D(p')$. Since the demand function D is continuous and strictly decreasing in p we can conclude that $p' = p$ and p is the price induced in the downstream market as required. □

A.2 Proof of Lemma 2

Proof. We have

$$\frac{\partial u}{\partial c} = -q \quad \text{and} \quad \frac{\partial^2 u}{\partial q \partial c} = -1 < 0;$$

which immediately shows that u exhibits strict increasing differences in $(-c; q)$. □

A.3 Proof of Proposition 1

Proof. It is well known that strong duality holds (see, for example, Theorem 2.165 in Bonnans and Shapiro, 2000). It now only remains to derive the expressions from Proposition 1 for the special case where $g(q) = -q^2/2$ and producers costs are uniformly distributed on the interval $[0; 1]$. This yields $\phi(c) = 2c$ and, for all $c \in [\underline{c}; \bar{c}]$, we have

$$q^M(c) = \left[p^M - 2c - \lambda^M\right]_+ :$$

Assuming that $\phi^{-1}(p^M - \lambda^M) = \frac{p^M - \lambda^M}{2} \le 1$, we have

$$\int_0^1 q^M(c)\,dF(c) = \int_0^{\frac{p^M-\lambda^M}{2}} \left[p^M - 2c - \lambda^M\right] dc = \left[-\frac{\left(p^M - 2c - \lambda^M\right)^2}{4}\right]_0^{\frac{p^M-\lambda^M}{2}} = \frac{\left(p^M - \lambda^M\right)^2}{4}:$$

Substituting this into (3), which is the first-order condition that pins down θ^M, we have

$$\theta^M = p^M - \sqrt{4D(p^M)}.$$

Of course, this solution is only valid if $\phi^{-1}(p^M - \theta^M) = \sqrt{D(p^M)} \le 1$ or, equivalently, if $D(p^M) \le 1$. If $\phi^{-1}(p^M - \theta^M) = \frac{p^M - \theta^M}{2} > 1$ then we have

$$\int_0^1 q^M(c)\,dc = \int_0^1 \frac{p^M - 2c - \theta^M}{\gamma}\,dc = \frac{\left(p^M - \theta^M\right)^2}{4} - \frac{\left(p^M - 2 - \theta^M\right)^2}{4} = \frac{p^M - \theta^M - 1}{\gamma}.$$

Substituting this into (3) then yields

$$\theta^M = p^M - D(p^M) - 1.$$

This solution is only valid if $\phi^{-1}(p - \theta^M(p)) = \frac{D(p)+1}{2} \ge 1$ or, equivalently, if $D(p) \ge 1$. Putting all of this together we have

$$\theta^M = \begin{cases} p^M - \sqrt{4D(p^M)}; & D(p^M) \le 1 \\ p^M - D(p^M) - 1; & D(p^M) > 1 \end{cases}$$

as required. We conclude by computing the transfer rule t^M. First, suppose that we have $D(p^M) \le 1$. Then for $c \in \left[\underline{c}, \frac{p^M - \theta^M}{2}\right]$ we have per-unit transfers of

$$\begin{aligned} t^M(c) &= (p^M - c) - \frac{\gamma}{2} q^M(c) - \frac{\int_c^{\frac{p^M - c^M}{2}} q^M(x)\,dx}{q^M(c)} \\ &= (p^M - c) - \frac{p^M - 2c - \theta^M}{2} - \frac{\int_c^{\frac{p^M - \theta^M}{2}} p^M - 2c - \theta^M\,dx}{p^M - 2c - \theta^M} \\ &= (p^M - c) - \frac{p^M - 2c - \theta^M}{2} - \frac{1}{4}\,\frac{p^M - 2c - \theta^M}{\gamma} \\ &= (p^M - c) - \frac{3(p^M - 2c - \theta^M)}{4} \\ &= \frac{p^M + 2c + 3\theta^M}{4}. \end{aligned}$$

Next, suppose that $D(p^M) > 1$. Then we have per-unit transfers of

$$
\begin{aligned}
t^M(c) &= (p^M - c) - \frac{p^M - 2c - {}^M}{2} - \frac{\int_c^1 \frac{p^M - 2c - {}^M}{} dx}{p^M - 2c - {}^M} \\
&= (p^M - c) - \frac{p^M - 2c - {}^M}{2} - \frac{}{4} \frac{p^M - 2c - {}^M}{} + \frac{(p^M - 2 - {}^M)^2}{4(p^M - 2c - {}^M)} \\
&= \frac{p^M + 2c + 3 {}^M}{4} + \frac{(p^M - 2 - {}^M)^2}{4(p^M - 2c - {}^M)}.
\end{aligned}
$$

□

A.4 Proof of Proposition 2

Proof. When the platform also produces in the downstream market, its optimization problem becomes

$$
\begin{aligned}
\max_{\substack{p \geq 0;\, y \geq 0; \\ q:[\underline{c};\overline{c}] \to \mathbb{R}_{\geq 0}}} \; & m \int_{\underline{c}}^{\overline{c}} [(p - (\,c))\, q(c) - g(q(c))]\, dF(c) + y(p - c_P) - g(y) \\
\text{s.t.} \quad & D(p) = m \int_{\underline{c}}^{\overline{c}} q(c)\, dF(c) + y; \quad q(\,) \text{ decreasing}
\end{aligned}
$$

Rewriting the market-clearing constraint as $p = P\left(y + \int_{\underline{c}}^{\overline{c}} q(c)\, dF(c)\right)$ and directly substituting this constraint into the objective function we obtain

$$
\begin{aligned}
\max_{\substack{y \geq 0; \\ q:[\underline{c};\overline{c}] \to \mathbb{R}_{\geq 0}}} \; & m \int_{\underline{c}}^{\overline{c}} \left[\left(P\left(y + m \int_{\underline{c}}^{\overline{c}} q(x)\, dF(x)\right) - (\,c) \right) q(c) - g(q(c)) \right] dF(c) \\
& + y \left(P\left(y + m \int_{\underline{c}}^{\overline{c}} q(c)\, dF(c)\right) - c_P \right) - g(y) \\
\text{s.t.} \quad & q(\,) \text{ decreasing}
\end{aligned}
$$

We are interested in investigating how the selling mechanism used in the upstream market impacts the platform's profit in the downstream market. However, the platform's downstream profits are only impacted by the upstream market selling mechanism through the aggregate quantity that it sells in the upstream market. Motivated by this, we utilize the aggregation principal of Milgrom and Shannon (1994). Specifically, we let $Q = \int_{\underline{c}}^{\overline{c}} q(c)\, dF(c)$ denote the aggregate quantity that the platform sells in the upstream market with a unit mass of producers and consider the function $\Pi : \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ with

$$\Pi(y; Q; c_P) = (y + mQ)\,P(y + mQ) - c_P y - g(y)$$
$$- \min_{q:[\underline{c};\bar{c}]\to\mathbb{R}_{\geq 0}} \left\{ m \int_{\underline{c}}^{\bar{c}} [\psi(c)q(c) + g(q(c))]\, dF(c) : \int_{\underline{c}}^{\bar{c}} q(c)\, dF(c) = Q;\ q(\cdot) \text{ decreasing} \right\}.$$

Observe that the platform's optimization problem can now be rewritten

$$\max_{y \geq 0;\, Q \geq 0} \Pi(y; Q; c_P).$$

Moreover, Π satisfies

$$\frac{\partial^2 \Pi}{\partial Q \partial c_P} = 0 \quad \text{and} \quad \frac{\partial^2 \Pi}{\partial y \partial c_P} = -1.$$

and, consequently, has increasing difference in $(y; -Q; -c_P)$. Next, we verify that Π is supermodular in $(y; -Q)$. First, we consider the function

$$\Phi_1 : (y; Q) \mapsto (y + mQ)\,P(y + mQ) - c_P y - g(y).$$

Recall that by assumption the revenue function R associated with downstream demand is concave. This in turn implies that $R''(Q) = 2P' + QP''(Q) \leq 0$ holds $\forall Q \geq 0$ and, consequently, we have

$$\frac{\partial^2 \Phi_1}{\partial y\, \partial Q} = 2mP'(y + mQ) + m(y + mQ)\,P''(y + mQ) \leq 0.$$

Next, consider the function

$$\Phi_2 : (y; Q) \mapsto - \min_{q:[\underline{c};\bar{c}]\to\mathbb{R}_{\geq 0}} \left\{ m \int_{\underline{c}}^{\bar{c}} [cq(c) + g(q(c))]\, dF(c) : \int_{\underline{c}}^{\bar{c}} q(c)\, dF(c) = Q \right\}.$$

Since Φ_2 is independent of y, it is trivially supermodular in $(y; -Q)$. Since the sum of supermodular functions is supermodular, we conclude that Π is supermodular in $(y; -Q)$. Applying the monotone selection theorem of Topkis (1978) then yields the desired result. □

A.5 Proof of Proposition 3

Proof. For the first part of the proposition statement that concerns a general convex cost function, the details of the proof proceed in precisely the same manner as in the proof of Proposition 1. So we focus on the special case where $g(q) = \gamma q^2/2$ and producer costs are uniformly distributed on the unit interval $[0;1]$. In this case we have

$$q^M(c) = \left[\frac{p \quad (c)}{\quad}\right]_+ \quad \text{and} \quad y^M = \left[\frac{p \quad c_P}{\quad}\right]_+ :$$

From Proposition 2 we know that y^M is decreasing in c_P and consequently there exists a cutoff $\bar{c}_P > 0$ such that $y^M > 0$ for all $c_P < \bar{c}_P$. For now we assume that $y^M > 0$ and $c_P < \bar{c}_P$. The first-order condition that pins down ^{M} then becomes

$$m \int_0^{\min\{\frac{p \quad ^M}{2};1\}} \frac{p \quad 2c \qquad ^M}{\quad} dF(c) + \frac{p \quad c_P \qquad ^M}{\quad} = D(p): \tag{10}$$

If $p \quad ^M \quad 2$ then this becomes

$$\frac{m(p \quad ^M)^2}{4} + \frac{p \quad c_P \qquad ^M}{\quad} = D(p);$$

which can be rewritten as

$$(\ ^M)^2 \quad 2\ ^M \quad p + \frac{2}{m} \quad + p^2 + \frac{4}{m}(p \quad c_P \quad D(p)) = 0:$$

Factoring the left-hand-side of this last equation yields

$$^M \quad p \quad \frac{2 + 2\sqrt{1 + mc_P + \ mD(p)}}{m} \quad ^M \quad p \quad \frac{2 \quad 2\sqrt{1 + mc_P + \ mD(p)}}{m} = 0:$$

Solving this quadratic equation for ^{M} we have

$$^M = p + \frac{2 \quad 2\sqrt{1 + mc_P + \ mD(p)}}{m}:$$

Since we require $p \quad ^M$ we take the negative root which yields

$$^M = p + \frac{2 \quad 2\sqrt{1 + mc_P + \ mD(p)}}{m}:$$

This solution is only valid if $p \quad ^M \quad 2$ or, equivalently, if $c_P + \ D(p) \quad 2 + m$. If $p \quad ^M > 2$ then (10) becomes

$$\frac{m(p \quad ^M)^2}{4} \quad \frac{m(p \quad 2 \quad ^M)^2}{4} + \frac{p \quad c_P \qquad ^M}{\quad} = D(p):$$

which simpli es to

$$\frac{m(p \quad 1 \qquad ^{M})}{\qquad} + \frac{p \quad c_P \qquad ^{M}}{\qquad} = D(p):$$

Solving for ^{M} then yields

$$^{M} = p \qquad \frac{m + c_P + \ \ D\ (p)}{1 + m}:$$

This solution is only valid if $p \qquad ^{M} > 2$ or, equivalently, if $c_P + \ \ D\ (p) > 2 + m$. Summarizing, we have

$$^{M} = \begin{cases} p + \frac{2 \ \ 2 \sqrt{1 + mc_P + \ \ mD\ \ (p)}}{m}; & c_P + \ \ D\ (p) \qquad 2 + m \\ p \qquad \frac{m + c_P + \ \ D\ \ (p)}{1 + m}; & c_P + \ \ D\ (p) > 2 + m: \end{cases}$$

To complete the solution, it only remains to characterize the cuto $\bar{c}_P$. In particular, this cuto satis es

$$y^{M} = \frac{p^{M} \qquad \bar{c}_P \qquad ^{M}}{\qquad} = 0:$$

If $\bar{c}_P + \ \ D\ (p) \qquad 2 + m$ we have

$$\bar{c}_P = \frac{2 \sqrt{1 + m\bar{c}_P + \ \ mD\ (p)} \qquad 2}{m} \qquad) \qquad \bar{c}_P = 2 \sqrt{\frac{D\ (p)}{m}}$$

and this solution is only valid if

$$\bar{c}_P + \ \ D\ (p) = 2 \sqrt{\frac{D\ (p)}{m}} + \ \ D\ (p) \qquad 2 + m \qquad) \qquad D\ (p) \qquad m:$$

If $\bar{c}_P + \ \ D\ (p) > 2 + m$ we have

$$\bar{c}_P = \frac{m + \bar{c}_P + \ \ D\ (p)}{1 + m} \qquad) \qquad \bar{c}_P = 1 + \frac{D\ (p)}{m}$$

and this solution is only valid if

$$\bar{c}_P + \ \ D\ (p) = 1 + \frac{D\ (p)}{m} + \ \ D\ (p) > 2 + m \qquad) \qquad = \ \ D\ (p) > m:$$

Putting all of this together we have

$$\bar{c}_P = p^M \qquad {}^M = \begin{cases} 2\sqrt{\frac{D(p)}{m}}; & D(p) \quad m \\ 1+\frac{D(p)}{m}; & D(p) > m. \end{cases}$$

□

Proof of Proposition 4

Proof. To prove the proposition statement, we return to the variant of the platform's optimization problem that was introduced in the proof of Proposition 2. In particular, we let $Q = \int_{\underline{c}}^{\bar{c}} q(c)\,dF(c)$ denote the aggregate quantity that the platform sells in the upstream market with a unit mass of producers and consider the function $\;: \mathbb{R}_+ \quad \mathbb{R}_+ \quad \mathbb{R}_+ \to \mathbb{R}$ with

$$(\,y; Q; c_P) = (\,y + mQ)\,P(y + mQ) \quad c_P y \quad g(y)$$
$$\min_{q:[\underline{c};\bar{c}]\to \mathbb{R}\ 0} \int_{\underline{c}}^{\bar{c}} [\,(\,c)q(c) + g(q(c))]\ dF(c) : \int_{\underline{c}}^{\bar{c}} q(c)\ dF(c) = Q;\ q(\,)\text{ decreasing}\ .$$

The platform's optimization problem can now be rewritten

$$\max_{y\ 0;\,Q\ 0} (\,y; Q; c_P).$$

Letting R denote the revenue function associated with the downstream market, we can rewrite the objective function as

$$(\,y; Q; c_P) = R(y + mQ) \quad mC^U(Q) \quad C^D(y);$$

where

$$C^U(Q) = \min_{q:[\underline{c};\bar{c}]\to \mathbb{R}\ 0} \int_{\underline{c}}^{\bar{c}} [\,(\,c)q(c) + g(q(c))]\ dF(c) : \int_{\underline{c}}^{\bar{c}} q(c)\ dF(c) = Q;\ q(\,)\text{ decreasing}$$
$$C^D(y) = c_P y + g(y).$$

That is, we can essentially think of the platform acting as a downstream monopoly that has access two two separate production technologies: it can produce output using the competitive fringe (which requires that the platform cover the productive costs of these producers, in addition to paying them an information rent) and it can produce output using its own production technology. We now compute the corresponding marginal cost functions. The marginal cost associated with the platform's own production is simply given by

$$MC^D(y) := \frac{dC^D(y)}{dy} = c_P + g'(y).$$

Computing the marginal cost associated with the output produced by the competitive fringe requires slightly more work. Letting μ denote the Lagrange multiplier associated with the quantity constraint we consider the dual problem

$$C^U(Q) = \max_{\mu \in \mathbb{R}} \min_{q:[\underline{c},\bar{c}] \to \mathbb{R}_{\geq 0}} \int_{\underline{c}}^{\bar{c}} [(\psi(c) - \mu)\, q(c) + g(q(c))]\, dF(c) + \mu Q \; : q(\cdot) \text{ decreasing}$$

For all $c \in [\underline{c}, \bar{c}]$, the first-order condition that pins down the optimal allocation rule $q^M(c)$ is then given by

$$g'(q^M(c)) = (\mu - \psi(c))_+ ; \tag{11}$$

while the optimal value μ^M of the Lagrange multiplier is characterized by

$$\int_{\underline{c}}^{\bar{c}} q^M(c)\, dF(c) = Q.$$

The marginal cost associated with output produced by the competitive fringe is then

$$MC^U(Q) := \frac{dC^U(Q)}{dQ} = \mu^M.$$

Let's restrict attention to the case where $y^M > 0$. Then the optimal level y^M of the platform's own output and the optimal level of aggregate production Q^M at the competitive fringe must satisfy

$$\left.\frac{dC^U(Q)}{dQ}\right|_{Q=Q^M} = \left.\frac{dC^D(y)}{dy}\right|_{y=y^M} \Rightarrow \mu^M = c_P + g'(y^M).$$

This in turn implies that

$$Q^M = \int_{\underline{c}}^{\bar{c}} q^M(c)\, dF(c);$$

where $q^M(c)$ satisfies

$$g'(q^M(c)) = \left(c_P + g'(y^M) - c\right)_+$$

for all $c \in [\underline{c}, \bar{c}]$. Consequently, we have

$$\frac{dQ^M}{dc_P} = \int_{\underline{c}}^{\bar{c}} \left(1 + g''(y^M)\frac{dy^M}{dc_P}\right) \frac{1}{g''(q^M(c))}\, dF(c)$$

as well as

$$\frac{d\left(mQ^M + y^M\right)}{dc_P} = m \int_{\underline{c}}^{\bar{c}} \left(1 + g''(y^M)\frac{dy^M}{dc_P}\right) \frac{1}{g''(q^M(c))}\, dF(c) + \frac{dy^M}{dc_P}.$$

We need to show that this last derivative is negative. From Proposition 2 we know that $\frac{dy^M}{dc_P} \leq 0$ and $\frac{dQ^M}{dc_P} \geq 0$. Moreover, by assumption we have $g'' \geq 0$ by assumption. Consequently, we must have

$$1 + g''(y^M)\frac{dy^M}{dc_P} \geq 0. \tag{12}$$

Next, taking

$$MR^D(y) := \frac{dR(y + mQ)}{dy} = P(y + mQ) + (y + mQ)P'(y + mQ)$$

and using $MR^D(y^M) = MC^D(y^M)$ shows that we must also have

$$P(y^M + mQ^M) + (y^M + mQ^M)P'(y^M + mQ^M) = c_P + g'(y^M).$$

Differentiating both side of this last equation with respect to c_P we have

$$\left(2P'(y^M + mQ^M) + (y^M + mQ^M)P''(y^M + mQ^M)\right) \frac{d(mQ^M + y^M)}{dc_P} = 1 + g''(y^M)\frac{dy^M}{dc_P},$$

which implies that

$$\frac{d(mQ^M + y^M)}{dc_P} = \frac{1 + g''(y^M)\frac{dy^M}{dc_P}}{2P'(y^M + mQ^M) + (y^M + mQ^M)P''(y^M + mQ^M)}.$$

Since $R''(Q) = QP''(Q) + 2P'(Q)$ and R is a strictly concave function we have $QP''(Q) + 2P'(Q) < 0$ for all $Q > 0$. Combining this with (12) then yields $\frac{d(mQ^M + y^M)}{dc_P} < 0$ as required.

□

A.6 Proof of Lemma 3

Proof. For this proof it's more convenient if we denote direct mechanisms by $\langle q_1; T_1 \rangle$ where $T_1(c) = q_1(c) t_1(c)$ specifies the total payment made by a producer of type c to the platform. Given any incentive compatible and individually rational mechanism direct mechanism $\langle q_1; T_1 \rangle$, each type c then selects a non-platform quantity $q_2(c) \geq 0$ and makes a corresponding payment of $t_2 q_2(c)$. However, the platform can always replicate this outcome by simply offering the menu of contracts corresponding to the direct mechanism $\langle q_1 + q_2; T_1 + t_2 q_2 \rangle$. Under this mechanism each producer then optimally sets $q_2(c) = 0$. Moreover, this mechanism generates weakly more revenue for the platform. Conversely, under any incentive compatible and individually rational direct mechanism that satisfies the constraint given in the proposition, we have $q_2(c) = 0$ for all $c \in [\underline{c}; \bar{c}]$. □

B Linear production costs with capacity constraints

B.1 Setup

We consider a setting in which sellers produce a homogeneous good that is traded in a two-sided market. A platform supplies an intermediate input to sellers in an upstream market. Sellers then compete with the platform (as well as each other) in a competitive downstream market.

We begin by describing the buyers and sellers who participate in the downstream market. Specifically, we assume that there is a unit mass of buyers and a mass $m > 0$ of sellers. Buyers have unit demand and are distinguished by their valuations $v \in [\underline{v}, \bar{v}]$. The mass of buyer values is distributed according to the absolutely continuous distribution F_B whose density f_B has full support on $[\underline{v}, \bar{v}]$. Sellers have unit capacity and are distinguished by their marginal costs $(c_0, c_1) \in [\underline{c}_0, \bar{c}_0] \times [\underline{c}_1, \bar{c}_1]$, where c_0 denotes the marginal cost of producing the input that can be outsourced to the platform and c_1 denotes the residual marginal cost of production. These costs are distributed according to an absolutely continuous distribution F_S whose density f_S has full support on $[\underline{c}_0, \bar{c}_0] \times [\underline{c}_1, \bar{c}_1]$. To ensure that we do not have a trivial market where either full trade or no trade is optimal absent the platform, we assume that $\underline{v} \le \underline{c}_0 + \underline{c}_1 < \bar{v} \le \bar{c}_0 + \bar{c}_1$. We also introduce the standard mechanism design assumptions that all agents in the downstream market are risk-neutral, have quasi-linear utility and have an outside option of 0.

Absent the platform, sellers of type (c_0, c_1) have a marginal cost of production of $c_0 + c_1$ in the downstream market. The total cost of producing $x \in [0, 1]$ units in the downstream market is thus given by $x(c_0 + c_1)$. Letting p^* denote the market-clearing price in the downstream market, it is weakly profitable for a seller to produce in the downstream market if and only if

$$p^* - c_0 - c_1 \ge 0$$

and without loss of generality we can assume that sellers that produce in the downstream market produce 1 unit of output (their maximum capacity). The competitive market-clearing price then satisfies

$$1 - F_B(p^*) = m \cdot P[c_0 + c_1 \le p^*] = m \int 1_{\{(c_0, c_1): c_0 + c_1 \le p^*\}} \, dF_S(c_0, c_1).$$

Next, we describe the platform and the upstream market. We assume that sellers can either purchase an input from the platform in the upstream market or produce the input themselves at a marginal cost of c_0.[16] We also assume that the platform produces the input in the upstream market at a marginal cost of 0 and can produce in the downstream market at a marginal cost of c. Once again, we denote the downstream market-clearing price by p. However, this price now depends on the upstream selling mechanism selected by the platform, as well as the level of output $y \geq 0$ produced by the platform in the competitive downstream market.

The timing of the game is as follows. First, the platform announces a selling mechanism $\langle x, t \rangle$ for the upstream market and commits to a level of production $y \geq 0$ in the downstream market. Given y and $\langle x, t \rangle$, sellers decide how much of the productive input to purchase from the seller in the upstream market. The sellers and the platform then participate in the downstream market.

By the revelation principle it is without loss of generality to consider incentive compatible direct mechanisms, which we denote by $\langle x, t \rangle$. Direct mechanisms consist of an allocation rule x and a payment rule t. The allocation rule $x : [\underline{c}_0, \bar{c}_0] \times [\underline{c}_1, \bar{c}_1] \to [0, 1]$ is such that $x(c_0, c_1) \in [0, 1]$ specifies the total output produced by a seller of type (c_0, c_1) using the platform's input. The payment rule $t : [\underline{c}_1, \bar{c}_1] \to \mathbb{R}_+$ is such that $t(c_0, c_1)$ is the payment made by consumers of type (c_0, c_1). Given an allocation rule x, a seller of type (c_0, c_1) can produce a quantity of $x(c_0, c_1)$ at a marginal cost of c_1 and a quantity of $1 - x(c_0, c_1)$ at a marginal cost of $c_0 + c_1$ in the downstream market. Note that we can assume without loss of generality that sellers either produce 1 unit of output or no output in the downstream market.[17] The indirect utility of a seller of type (c_0, c_1) is therefore given by

$$((p - c_1)_+ x(c_0, c_1) + (p - c_1 - c_0)_+ (1 - x(c_0, c_1)) - t(c_0, c_1))_+ .$$

Recall that p depends on the allocation rule x, and that firms take p as given. The outside option of a seller of type (c_0, c_1) in the upstream market also depends on p as follows

$$(p - c_1 - c_0)_+ ;$$

[16] Equivalently, we could think of c_0 as the price of sourcing the input from some other external supplier.

[17] This follows from our linear specification of seller utility, which ensures that sellers either strictly prefer to produce 0 units of output, strictly prefer to produce 1 unit of output or are indifference over all levels of output $x \in [0, 1]$.

and the marginal value of the firm's input to a seller of type (c_0, c_1) is therefore

$$(p - c_1)_+ - (p - c_1 - c_0)_+ = \begin{cases} c_0 & \text{if } p - c_1 > c_0; \\ p - c_1 & \text{if } p - c_1 \in [0, c_0]; \\ 0 & \text{if } p - c_1 < 0 \end{cases} = \min\{(p - c_1)_+, c_0\}.$$

The demand function $D(p)$ in the competitive downstream market is given by

$$D(p) = 1 - F_B(p).$$

Given any incentive compatible and individually rational direct mechanism $\langle x, t\rangle$ and level of platform production y, the supply function $S(y; p; x)$ in the competitive downstream market is given by

$$S(y; p; x) = \underbrace{m \int [1 - x(c_0, c_1)] \mathbf{1}_{c_0 + c_1 \leq p} \, dF_S(c_0, c_1)}_{\text{output produced using platform's input}} + \underbrace{m \int x(c_0, c_1) \mathbf{1}_{c_1 \leq p} \, dF_S(c_0, c_1)}_{\text{output produced without platform's input}} + y.$$

The market-clearing price p^* therefore satisfies $D(p^*) = S(y; p^*; x)$ or, equivalently,

$$\frac{1 - F_B(p^*)}{m} - y = \int [1 - x(c_0, c_1)] \mathbf{1}_{c_0 + c_1 \leq p^*} \, dF_S(c_0, c_1) + \int x(c_0, c_1) \mathbf{1}_{c_1 \leq p^*} \, dF_S(c_0, c_1). \tag{13}$$

As we will see shortly, this last equation represents a moment constraint on the allocation function.

B.2 Mechanism design analysis

In this section we characterize the optimal selling mechanism of the platform in the upstream market. Our solution derives the allocation function as the extreme point of an infinite-dimensional convex set of all possible allocation functions, which shares common ground with technical results by Manelli and Vincent (2007), Doval and Skreta (2018), Kleiner et al. (2020) and Kang (2021a).

The objective of the platform is to maximize its total profit across both the upstream and downstream markets. We can think of the platform's optimization problem as consisting of two steps. First, take the platform's level of output y and the price p^* in the downstream market as given. The platform then selects the direct mechanism $\langle x, t\rangle$ that maximizes its total profit subject to the incentive compatibility and individuals rationality constraints of the sellers, as well as the constraint that a market clearing price of p^* is induced in the competitive downstream market. This yields a profit function of $\Pi(y, p^*)$. Second, the platform determines the optimal level of production y and the optimal market-clearing price p^* to be induced in the downstream market.

Before proceeding with the analysis, we first require a characterization of the price p^* that is induced in the downstream market. To that end, we have the following useful lemma.

Lemma B.1. An incentive compatible and individually rational upstream selling mechanism $\langle x, t \rangle$ and output choice y induces a price p^* in the competitive downstream market if and only if the market clearing condition $D(p^*) = S(y, p^*, x)$ is satisfied.

Given a level of output y, the maximum price $\bar{p}^*(y)$ that can be induced in the downstream market satisfies

$$1 - F_B(\bar{p}^*(y)) = m \int 1_{c_0 + c_1 \le \bar{p}^*(y)} \, dF_S(c_0, c_1) + y,$$

which corresponds to the case where the platform does not sell any output in the upstream market. The minimum price $\underline{p}^*(y)$ that can be induced in the downstream market satisfies

$$1 - F_B(\underline{p}^*(y)) = m \int 1_{c_1 \le \underline{p}^*(y)} \, dF_S(c_0, c_1) + y,$$

which corresponds to the case where all sellers outsource all production of the input to the platform in the upstream market. In light of Lemma B.1, the platform's optimization problem is then given by

$$\max_{y, p^*, \langle x, t \rangle} \int t(c_0, c_1) \, dF_S(c_0, c_1) + (p^* - c)y$$
$$s.t.: \quad D(p^*) = S(y, p^*, x).$$

Here, we maximize over $y \in [0, 1]$, $p^* \in [\underline{p}^*(y), \bar{p}^*(y)]$ and all incentive compatible and individually rational mechanisms $\langle x, t \rangle$ that satisfy the market clearing constraint $D(p^*) = S(y, p^*, x)$ in the competitive downstream market. This can be rewritten as the nested optimization problem

$$\max_{p^*, y} \left\{ \max_{\langle x, t \rangle} \left\{ \int t(c_0, c_1) \, dF_S(c_0, c_1) : D(p^*) = S(y, p^*, x) \right\} + (p^* - c)y \right\}.$$

We now turn our attention to the inner problem. Solving this problem yields a function

$\Pi : [0,1] \times [0,1] \to \mathbb{R}_+$ given by

$$\Pi(y;p) = \max_{\langle x,t\rangle} \int t(c_0,c_1)\, dF_S(c_0,c_1)$$
$$s.t.: \quad D(p) = S(y;p;x)$$

We solve this problem by showing that it can be completely rewritten in terms of the effective types introduced in Section B.2.

Fix the output of the platform y and the price p to be induced in the competitive downstream market. Given a direct mechanism $\langle x,t\rangle$, the indirect utility of sellers of type $(c_0,c_1) \in [\underline{c}_0,\overline{c}_0] \times [\underline{c}_1,\overline{c}_1]$ who report to be of type $(\hat{c}_0,\hat{c}_1) \in [\underline{c}_0,\overline{c}_0] \times [\underline{c}_1,\overline{c}_1]$ is given by

$$\begin{aligned}&(p-c_1)_+ x(\hat{c}_0,\hat{c}_1) + (p-c_1-c_0)_+(1-x(\hat{c}_0,\hat{c}_1)) - t(\hat{c}_0,\hat{c}_1)\\ &= \min\{(p-c_1)_+, c_0\} x(\hat{c}_0,\hat{c}_1) + (p-c_1-c_0)_+ - t(\hat{c}_0,\hat{c}_1).\end{aligned}$$

Incentive compatibility then requires that, for all $(c_0,c_1),(\hat{c}_0,\hat{c}_1) \in [\underline{c}_0,\overline{c}_0] \times [\underline{c}_1,\overline{c}_1]$,

$$\min\{(p-c_1)_+, c_0\} x(c_0,c_1) - t(c_0,c_1) \geq \min\{(p-c_1)_+, c_0\} x(\hat{c}_0,\hat{c}_1) - t(\hat{c}_0,\hat{c}_1),$$

while individual rationality requires that, for all $(c_0,c_1) \in [\underline{c}_0,\overline{c}_0] \times [\underline{c}_1,\overline{c}_1]$,

$$\min\{(p-c_1)_+, c_0\} x(c_0,c_1) - t(c_0,c_1) \geq -(p-c_1-c_0)_+.$$

These constraints are of the same functional form as those found in standard mechanism design settings. However, the type (c_0,c_1) of each seller is replaced with their marginal value for the firm's input in the upstream market. Moreover, we also have a type-dependent outside option.

We now modify our mechanism design problem as follows. First, we consider a continuous function θ that maps the type (c_0,c_1) of each seller to an effective type $\theta(c_0,c_1) = \min\{(p-c_1)_+, c_0\} \in [\underline{\theta},\overline{\theta}]$, where $\underline{\theta} = \min\{(p-\overline{c}_1)_+, \underline{c}_0\}$ denotes the lowest possible effective type and $\overline{\theta} = \min\{(p-\underline{c}_1)_+, \overline{c}_0\}$ denotes the highest possible effective type. We then introduce an effective allocation rule $\tilde{x} : [\underline{\theta},\overline{\theta}] \to [0,1]$ and an effective transfer rule $\tilde{t} : [\underline{\theta},\overline{\theta}] \to \mathbb{R}$ such that, for all (c_0,c_1),

$$x(c_0,c_1) = \tilde{x}(\theta(c_0,c_1)) \quad \text{and} \quad t(c_0,c_1) = \tilde{t}(\theta(c_0,c_1)).$$

An immediate implication of incentive compatibility is that, for all $(c_0; c_1)$ and $(\hat{c}_0; \hat{c}_1)$,

$$(\theta(c_0; c_1) - \theta(\hat{c}_0; \hat{c}_1))(x(c_0; c_1) - x(\hat{c}_0; \hat{c}_1)) \leq 0:$$

Hence, incentive compatibility dictates that if $\min\{(p - c_1)_+ ; c_0\} \leq \min\{(p - \hat{c}_1)_+ ; \hat{c}_0\}$ then $x(c_0; c_1) \geq x(\hat{c}_0; \hat{c}_1)$. Or, equivalently, that $\tilde{x}$ is increasing on its domain.

We now argue that it is without loss of generality to limit attention to allocation rules such that, for all $(c_0; c_1) \in [\underline{c}_0; \bar{c}_0] \times [\underline{c}_1; \bar{c}_1]$, $x(c_0; c_1) = \tilde{x}(\theta(c_0; c_1))$, where $\tilde{x}$ is increasing on its domain. Of course, the platform is not restricted to offer the same allocation to all sellers with the same effective types. However, incentive compatibility dictates that, for all $(c_0; c_1); (\hat{c}_0; \hat{c}_1) \in [\underline{c}_0; \bar{c}_0] \times [\underline{c}_1; \bar{c}_1]$ with $\theta(c_0; c_1) = \theta(\hat{c}_0; \hat{c}_1)$, we have

$$\theta(c_0; c_1)x(c_0; c_1) - t(c_0; c_1) = \theta(\hat{c}_0; \hat{c}_1)x(\hat{c}_0; \hat{c}_1) - t(\hat{c}_0; \hat{c}_1):$$

That is, sellers of type $(c_0; c_1)$ must be indifferent between reporting truthfully and reporting any other type $(\hat{c}_0; \hat{c}_1)$ such that $\theta(c_0; c_1) = \theta(\hat{c}_0; \hat{c}_1)$. Let G_S denote the distribution of effective types and g_S denote the corresponding density function. We also let $F_S(c_0; c_1 | \theta)$ denote the conditional distribution of types given the effective type θ, with corresponding density function

$$f_S(c_0; c_1 | \theta) = \frac{f_S(c_0; c_1)}{g(\theta)}:$$

Given an incentive compatible allocation rule x and an effective type $\theta \in [\underline{\theta}; \bar{\theta}]$, if we modify this allocation rule by offering all types $(c_0; c_1)$ such that $\theta(c_0; c_1) = \theta$ the average allocation and payment,

$$\int_{\{(c_0;c_1):\ \theta(c_0;c_1)=\theta\}} x(c_0; c_1)\, dF_S(c_0; c_1 | \theta) \quad \text{and} \quad \int_{\{(c_0;c_1):\ \theta(c_0;c_1)=\theta\}} t(c_0; c_1)\, dF_S(c_0; c_1 | \theta);$$

respectively, this has no impact on the expected payoff of the platform or the incentive compatibility constraints of sellers. Therefore, if we solve for the optimal effective allocation rule $\tilde{x}$, subject to the constraint that $\tilde{x}$ is increasing in θ, it is without loss of generality for the platform to set $x(c_0; c_1) = \tilde{x}(\theta(c_0; c_1))$ for all $(c_0; c_1) \in [\underline{c}_0; \bar{c}_0] \times [\underline{c}_1; \bar{c}_1]$.

Next, we show that Myerson's lemma holds in this setting and that the effective payment rule $\tilde{t}$ is pinned down, up to a constant, by the effective allocation rule $\tilde{x}$. Since consumers of effective type $\theta \in [\underline{\theta}, \bar{\theta}]$ solve

$$\max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}]} \{\theta \tilde{x}(\hat{\theta}) - \tilde{t}(\hat{\theta})\};$$

applying the envelope theorem yields

$$\theta \tilde{x}(\theta) - \tilde{t}(\theta) = \underline{\theta}\tilde{x}(\underline{\theta}) - \tilde{t}(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} \tilde{x}(\hat{\theta})\, d\hat{\theta}.$$

Under the optimal mechanism, the individual rationality constraint must bind for consumers with effective type $\theta = \underline{\theta}$. Imposing this and rearranging yields

$$\tilde{t}(\theta) = \theta \tilde{x}(\theta) - \int_{\underline{\theta}}^{\theta} \tilde{x}(\hat{\theta})\, d\hat{\theta}.$$

Summarizing, all of this we have the following proposition.

Proposition B.1. Conditional on p^* and y, it is without loss of generality for the platform to restrict attention to direct mechanisms $\langle x, t \rangle$ such that

(i) $x(c_0; c_1) = \tilde{x}(\min\{(p^* - c_1)_+; c_0\})$ for some increasing function $\tilde{x}$; and

(ii) $t(c_0; c_1) = \tilde{t}(\min\{(p^* - c_1)_+; c_0\})$ for some function $\tilde{t}$ that satisfies

$$\tilde{t}(\theta) = \theta \tilde{x}(\theta) - \int_{\underline{\theta}}^{\theta} \tilde{x}(\hat{\theta})\, d\hat{\theta}.$$

Combining Proposition B.1 with standard Myersonian mechanism design arguments, the profit of the platform in the upstream market can be written

$$\Pi(\tilde{x}; p^*) = \int_{\underline{\theta}}^{\bar{\theta}} \phi(\theta) \tilde{x}(\theta)\, dG(\theta);$$

where ϕ is the effective virtual valuation function given by

$$\phi(\theta) = \theta - \frac{1 - G(\theta)}{g(\theta)}.$$

It only remains to rewrite the constraint in terms of effective types. To that end, we start by noting that the market-clearing condition given in (13) can be rewritten

$$\frac{1 - F_B(p^*) - y}{m} - P[c_0 + c_1 \leq p^*] = \int x(c_0; c_1) 1_{c_0 \leq p^* \leq c_0 + c_1}\, dF_S(c_0; c_1).$$

Rewriting the right-hand-side in terms of effective types then yields

$$Q(p^*;y) = \int \tilde{x}(\varphi)\,\eta(\varphi)\,dG(\varphi);$$

where

$$Q(p^*;y) := \frac{1 - F_B(p^*) - y}{m} - P[c_0 + c_1\theta \leq p^*] \quad \text{and} \quad \eta(\varphi) := P[c_0 \leq p^* \leq c_0 + c_1 \mid \varphi]:$$

The platform's inner problem can thus be summarized by

$$\max_{\substack{\tilde{x}:[\underline{\varphi},\bar{\varphi}]\to[0,1];\\ \tilde{x}\text{ increasing}}} \int \psi(\varphi)\,\tilde{x}(\varphi)\,dG(\varphi)$$
$$\text{s.t.:} \quad \int \tilde{x}(\varphi)\,\eta(\varphi)\,dG(\varphi) = Q(p^*;y)$$

and the platform's full optimization problem is given by

$$\max_{\substack{y\in[0,1];\\ p^*\in[\underline{p}(y),\bar{p}(y)]}} \left\{ \max_{\substack{\tilde{x}:[\underline{\varphi},\bar{\varphi}]\to[0,1];\\ \tilde{x}\text{ increasing}}} \left\{ \int_{\underline{\varphi}} \psi(\varphi)\,\tilde{x}(\varphi)\,dG(\varphi) : \int \tilde{x}(\varphi)\,\eta(\varphi)\,dG(\varphi) = Q(p^*;y) \right\} + (p^* - c)y \right\}:$$

In short, the platform faces a mechanism design problem where the outside options for sellers (which depend on the equilibrium price in the downstream market) are endogenous to the platform's chosen output and selling mechanism (which pins down the downstream supply curve and the equilibrium price in the downstream market). We solve this fixed-point problem by rewriting the platform's problem as a nested optimization problem. The platform's inner problem involves a constraint that is linear in the effective allocation rule $\tilde{x}$ and is therefore analogous to a monopoly pricing problem involving a capacity constraint. Consequently, the optimal selling mechanism can be represented as a convex combination of at most two posted-price selling mechanisms. Formally, we have the following theorem.

Theorem B.1. *Suppose that the function η has full support on the interval $[\underline{\varphi},\bar{\varphi}]$. Then there exists an optimal choice of output y^* and an optimal selling mechanism $\langle x^*, t^*\rangle$ such that $|\mathrm{Im}(x^*)| \leq 3$, where $\mathrm{Im}(x^*) \subseteq \{0, q, 1\}$ for some $q \in (0,1)$. This entails an upstream selling mechanism involving at most two prices p_1 and p_2 with $p_1 > p_2$.*

Moreover, we also have the following proposition.

Proposition B.2. *Suppose that the platform faces a mechanism design problem that is* regular in the sense that the following function is increasing:

$$(\ ) = \frac{1}{(\ )} \qquad \frac{1 \quad G(\ )}{g(\ )} = \frac{1}{(\ )} (\quad):$$

Then an optimal selling mechanism in the upstream market consists of the the platform posting a single price p_1 and serving any seller who wishes to buy at this price.

Again, the proof of this result follows directly from the analogous monopoly pricing problem, where is analogous to a monopoly's marginal revenue curve. If we also introduce the integral of the marginal revenue function , then the function is analogous to the revenue curve of a monopoly. The platform then strictly bene ts from price discrimination in the upstream market if and only if is not concave at the \quantity" $Q(p \ ; y \ )$ and a market clearing price in the upstream market if and only if is concave at the \quantity" $Q(p \ ; y \ )$.

Our nal result of this section highlights the important role of the constraint that $y \quad 0$. First, notice that if we rewrite our nested optimization problem so that the downstream level of output y is chosen in the inner nest. Consequently, for a given value of p, the platform can always satisfy the market clearing constraint simply by choosing an appropriate value of y, provided the constraint that $y \quad 0$ doesn't bind. Absent a binding constraint, the optimal upstream selling mechanism necessarily involves a menu of at most one price. In sum, we have the following proposition.

Proposition B.3. The platform uses a pricing schedule involving a menu of two prices only if the $y \quad 0$ constraint is binding.

B.3 Consumer surplus analysis

Motivated by the consumer surplus standard in antitrust practice, in this section we consider the consumer surplus implications of our analysis from the previous section. For now, we focus on the regular case, where the platform posts a market clearing price in the upstream market.

B.3.1 No platform

We begin our analysis by rst solving the model for the simple benchmark case involving no platform. To that end, given the total marginal cost $c_0 + c_1 \ 2 \ [\underline{c}_0 + \underline{c}_1; \bar{c}_0; \bar{c}_1]$ for a seller of type $(c_0; c_1)$, we let $F_S(c)$ denote the corresponding absolutely continuous distribution of total seller costs $c = c_0 + c_1$. We also let $f_S(c)$ denote the corresponding density function, which has full support on $[\underline{c}_0 + \underline{c}_1; \bar{c}_0; \bar{c}_1]$. The equilibrium price p^* in the competitive downstream market is then pinned down by

$$1 - F_B(p^*) = mF_S(p^*) \tag{14}$$

and the quantity produced in the competitive downstream market is given by $q^* = 1 - F_B(p^*) = F_S(p^*)$. Our assumptions from Section 2 ensure that $q^* \in [0; m]$ and $p^* \in [\underline{c}_0 + \underline{c}_1; \bar{c}_0 + \bar{c}_1]$. Letting $P_D(q) = F_B^{-1}(1 - q)$ denote the inverse demand function, the corresponding level of consumer surplus is thus

$$CS = \int_0^{q^*} P_D(q)dq - q^* p^* : \tag{15}$$

We then have the following simple proposition.

Proposition B.4. *Absent the platform, the market-clearing price p^* is decreasing in m and consumer surplus CS is increasing in m.*

That consumer surplus is increasing in the mass of sellers is unsurprising and very intuitive. An illustration of the comparative statics from Proposition B.4 can be found in Figure 2.

(a) Market-clearing price　　　　(b) Consumer surplus

Figure 2: An illustration of p^* and CS when consumer values v are uniformly distributed on $[0; 1]$ and the marginal costs c_0 and c_1 of sellers are uniformly distributed on $[0; 1]^2$.

B.3.2 Platform that does not produce downstream

The next simple benchmark case we consider is one in which the platform operates an upstream market but does not produce downstream. Since we are focusing on the regular case, without loss of generality we can assume that the platform posts a market clearing price in the upstream market. To analyze consumer welfare, we need to characterize the optimal upstream market price .

Given an arbitrary upstream market price , we start by determining the set of sellers that purchase from the platform and characterize the downstream market p . In particular, given an upstream market price of and a downstream market price of p , the payo of sellers that use the platform to trade is p c_1 and the payo of sellers that don't use the platform to trade is p c_0 c_1. Now, consider sellers with c_0 . These sellers will trade via the platform if $c_1 < p$ and will not trade otherwise. Next, consider sellers with $c_0 <$. These sellers will trade without the platform if $c_1 + c_0 < p$ and will not trade otherwise. So for a given value of , the market clearing price p satis es

$$1 \quad F_B(p) = m \int^{\bar{c}_0} \int_{\underline{c}_1}^{p} dF_S(c_0; c_1) + m \int_{\underline{c}_0} \int_{\underline{c}_1}^{p \quad c_0} dF_S(c_0; c_1)$$

and the pro t of the seller is given by

$$(\ ) = \ m \int^{\bar{c}_0} \int_{\underline{c}_1}^{p} dF_S(c_0; c_1):$$

The optimal price posted by the platform in the upstream market then satis es

$$= \arg\max_{2[\_;^{-}]} (\ )$$

and consumer surplus in this case is still given by (15). We then have the following comparative statics.

Proposition B.5. The equilibrium pro t of the platform is increasing in m. Moreover, relative to the case without a platform p is lower and consumer surplus is higher for every value of m.

The comparative statics from Proposition B.5 are illustrated in Figures 3 and 4. This proposition shows that consumers unambiguously bene t from the existence of the upstream platform, as this can only improve competition among sellers in the downstream market, resulting in lower equilibrium prices in the downstream market. In contrast, the impact of the platform on the pro t of sellers is heterogeneous: some sellers are harmed while others bene t.

(a) Prices (b) Platform profit

Figure 3: An illustration of (the platform price) and p (the price in the competitive downstream market) and the profit of the platform when consumer values v are uniformly distributed on the interval $[0;1]$ and the marginal costs c_0 and c_1 of sellers are uniformly distributed on the interval $[0;1]^2$.

(a) Market-clearing price (b) Consumer surplus

Figure 4: An illustration of the market-clearing price and consumer surplus with and without the platform when consumer values v are uniformly distributed on the interval $[0;1]$ and the marginal costs c_0 and c_1 of sellers are uniformly distributed on the interval $[0;1]^2$.

B.3.3 Platform that produces downstream

We now analyze consumer surplus in the full model analyzed in Section 3, where the platform can also produce in the downstream market at a marginal cost of . Once again, consumer surplus is given by (15) and to investigate the comparative statics of consumer surplus it suffices to study comparative statics of the equilibrium downstream market price p . To study the impact of downstream production on consumer surplus, we determine how consumer surplus varies with the downstream efficiency of the platform. Specifically, we use monotone comparative statics techniques to derive a sufficient condition under which consumer surplus monotonically decreases in the marginal cost c of the platform.

Given that we are restricting attention to the regular case, where posting a market clearing price in the upstream market is optimal, the platform's profit is

$$\max_{;y\ 0} f\, m\ P[\min f(p \quad c_1)_+; c_0 g \quad] + y(p \quad c) g$$
$$s{:}t{:} \quad D(p) = mS_1(p) + mS_2(p; \ ) + y;$$

where we define $S_1(p) := P[c_0 + c_1 \quad p]$ and $S_2(p; \ ) := P[c_0 > p \quad c_1 \quad]$. One difficulty with performing comparative statics here is that p is a non-linear function of and y. We circumvent this by exploiting the linearity in y in both the platform's objective function as well as the constraint,

$$\max_{;p\ 0} f\, m\ P[\min f(p \quad c_1)_+; c_0 g \quad] + [D(p) \quad mS_1(p) \quad mS_2(p; \ )](p \quad c) g$$
$$s{:}t{:} \quad D(p) \quad mS_1(p) \quad mS_2(p; \ ) \quad 0{:}$$

Next, we define the platform's objective function

$$(\ p; \ ; c) := m\ P[\min f(p \quad c_1)_+; c_0 g \quad] + [D(p) \quad mS_1(p) \quad mS_2(p; \ )](p \quad c){:}$$

Observe that

$$\frac{@^2}{@ \ @c} = m \frac{@S_2}{@} \quad 0;$$

where the inequality follows from the observations that

$$S_2(p; \ ) = E[P[p \quad c_0 < c_1 \quad p \quad j\, c_0]] \quad \text{and} \quad \frac{@}{@} P[p \quad c_0 < c_1 \quad p \quad] \quad 0{:}$$

Intuitively, the more efficient the platform is at downstream production, the more intense the downstream competition is between the platform and sellers. Next, we want to show that the objective function of the platform exhibits increasing differences in $(p; \ )$ so that we can apply the Monotone Selection Theorem. That is, we want to show that the partial derivative $\frac{@^2}{@p\,@}$ is of positive sign. However, directly signing this derivative requires that we are able to compare the magnitudes of p and , which we cannot do in general.

To address this issue We now make the change variables $7!\ p$. Here > 0 since it cannot be optimal for the platform to set an upstream market price that is higher than the downstream market price. Defining

$$\hat{S}_2(p; \ ) := P[\quad c_1 > p \quad c_0];$$

by the law of iterated expectations we have

$$\hat{S}_2(p;\ ) = E\left[P[p \quad c_0 < c_1 \quad \mid c_0]\right]:$$

Notably, for any c_0,

$$\frac{\partial}{\partial p}P[p \quad c_0 < c_1 \quad \mid c_0] \quad 0 \quad \text{and} \quad \frac{\partial}{\partial}P[p \quad c_0 < c_1 \quad \mid c_0] \quad 0:$$

That is, $\partial\hat{S}_2 = \partial p \quad 0$ and $\partial\hat{S}_2 = \partial \quad 0$. Writing the designer's objective function as

$$\hat{(}\ p;\ ) \quad m(p \quad)P[\min\{(p \quad c_1)_+ ; c_0\} \quad p \quad] + \Big[D(p) \quad mS_1(p) \quad m\hat{S}_2(p;\ ) \Big] (p \quad c);$$

we then see that

$$\frac{\partial^2 \hat{}}{\partial \ \partial c} = \frac{\partial\hat{S}_2}{\partial} \quad 0$$

as required. Applying the Monotone Selection Theorem, we then have the following result and exploiting the fact that the platform optimally posts a market-clearing price in the upstream market provided if constraint $y \quad 0$ binds, we then have the following result.

Theorem B.2. Suppose that the platform's objective function exhibits increasing differences in $(p; c)$ and $(; c)$. Then the equilibrium downstream market price p is increasing in c and consumer surplus is decreasing in c.

The conditions of Theorem B.2 are satisfied by the examples used to construct Figure 5, where we see that the equilibrium price p (and hence consumer surplus) is decreasing in the platform's marginal cost c. This figure also displays the equilibrium upstream market price , while Figure 6 illustrates the equilibrium output y and profit of the platform. Here, we see that a highly efficient platform does not sell any output in the upstream market in order to reap higher profits in the downstream market. We also see that in this case the platform competes more aggressively downstream as the mass of sellers m increases. In contrast, an inefficient platform derives relatively more of its profit from the upstream market. When the mass of sellers increases in this case, the platform competes less aggressively downstream in order to partially offset the corresponding decrease in the downstream market price p and the upstream market price . While this numerical example illustrates that this model gives rise to a rich set of behaviour and there are a number of counterveiling effects that influence consumer surplus.

(a) Upstream price

(b) Downstream price

(c) Upstream price

(d) Downstream price

Figure 5: An illustration of the upstream and downstream prices when consumer values v are uniformly distributed on the interval $[0;1]$ and the marginal costs c_0 and c_1 of sellers are uniformly distributed on the interval $[0;1]^2$.

(a) Platform output

(b) Platform profit

(c) Platform output

(d) Platform profit

Figure 6: An illustration of platform output and profit when consumer values v are uniformly distributed on the interval $[0;1]$ and the marginal costs c_0 and c_1 of sellers are uniformly distributed on the interval $[0;1]^2$.