Lecture 12
Feedback control systems: static analysis

- feedback control: general
- example
- open-loop equivalent system
- plant changes, disturbance rejection, sensor noise

Feedback control systems

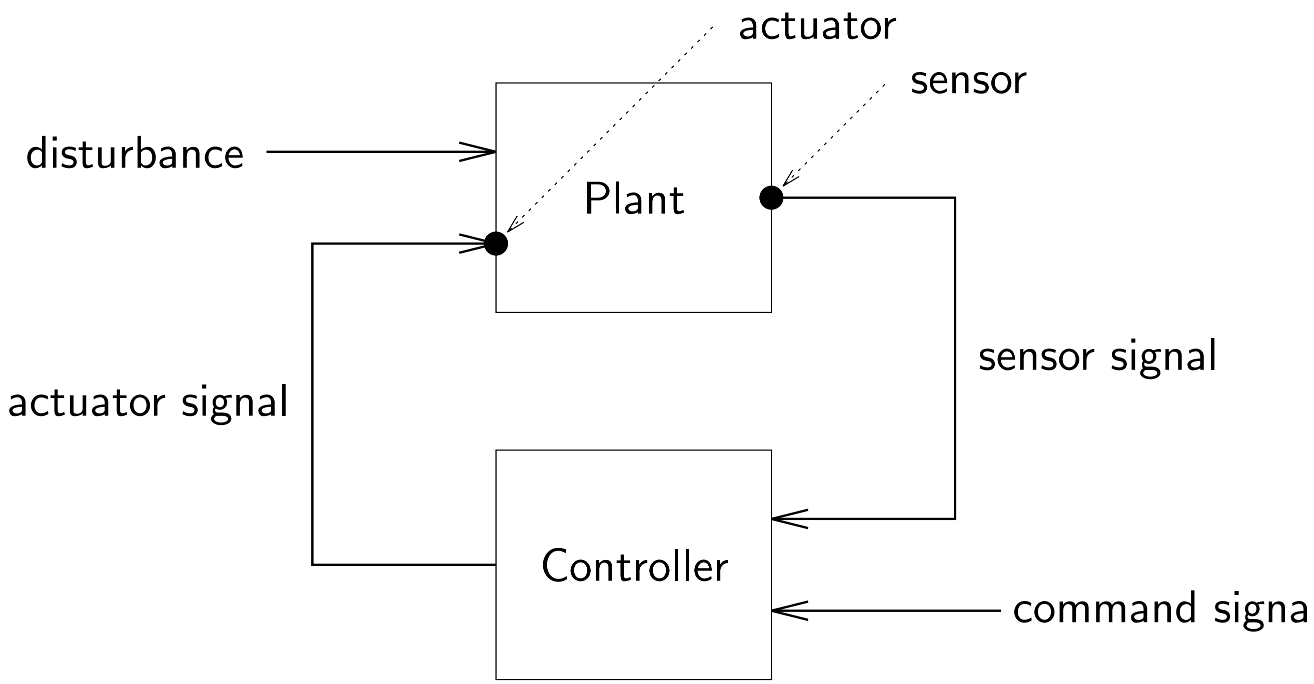
feedback is widely used in *automatic control*

terminology:

- the system to be controlled is called the *plant*
- a *sensor* measures the quantity to be controlled
- an *actuator* affects the plant
- the *controller* or *control processor* processes the sensor signal to drive the actuator
- the *control law* or *control algorithm* is the algorithm used by the control processor to derive the actuator signal

Block diagram

(often the sensors and actuators are not shown separately)



Examples

plants: CD player, disk drive mechanics; aircraft or missile; car suspension, engine; rolling mill; high-rise building, XY stage on stepper machine for IC lithography; computer network; industrial process; elevator

sensors: radar altimeter; GPS; shaft encoder; LVDT; strain gauge; accelerometer; tachometer; microphone; pressure and temperature transducers; chemical sensors; microswitch

actuators: hydraulic, pneumatic, electric motors; pumps; heaters; aircraft control surfaces; voice coil; solenoid; piezo-electric transducer

disturbances: wind gusts; earthquakes; external shaking and vibration; road surface variations; variation in feed material

control processors: human operator; mechanical; electro-mechanical; analog electrical; general purpose digital processor; special purpose digital processor

Example

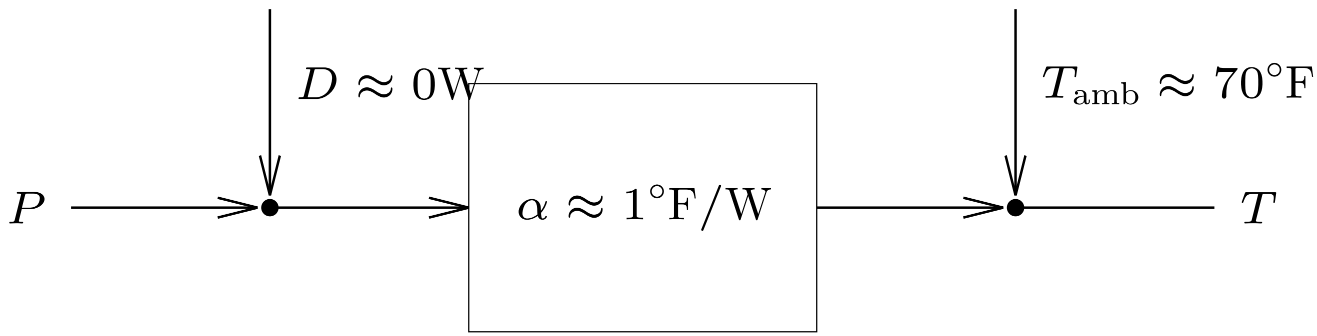
a plate is to be heated to a desired temperature T_{des} by an electrical heater

- the *plant* is the plate
- the *actuator* is the electrical heater
- the *controller* sets the heater power, given T_{des}
- the input u is the heater power P
- the output y is the plate temperature T

in steady-state, $T = T_{\text{amb}} + \alpha(P + D)$

- T_{amb} is the ambient temperature, $T_{\text{amb}} \approx 70°\text{F}$
- α is a thermal resistance coefficient, $\alpha \approx 1°\text{F}/\text{W}$
- D is a thermal disturbance to the plate, $D \approx 0\text{W}$
  (represents other heat flow, in or out of plate)

block diagram:



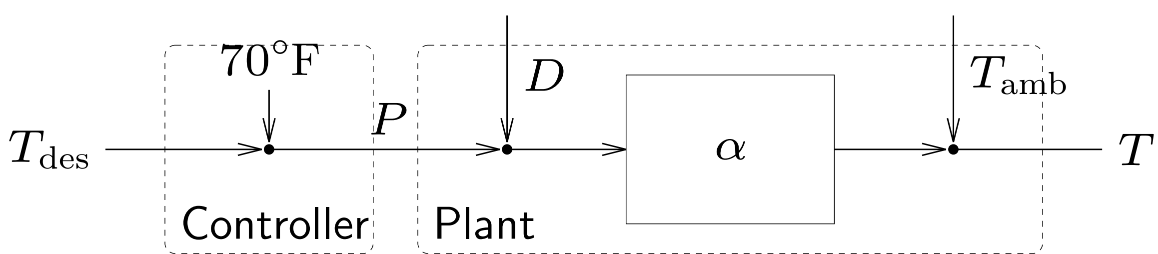
An open-loop controller

obvious control law: use power that yields $T = T_{\rm des}$ when $T_{\rm amb} = 70°{\rm F}$, $\alpha = 1°{\rm F/W}$, and $D = 0{\rm W}$, *i.e.*,

$$P = (T_{\rm des} - 70°{\rm F})/(1°{\rm F/W})$$

(we assume here $T_{\rm des} \geq 70°{\rm F}$, so $P \geq 0{\rm W}$)

called *open-loop* or *feedforward* because sensor signal is not used



How well does it work when $T_{\mathrm{amb}} \neq 70°\mathrm{F}$, $\alpha \neq 1°\mathrm{F/W}$, and $D \neq 0\mathrm{W}$?

temperature error is $e = T - T_{\mathrm{des}}$

$$= (\alpha - 1°\mathrm{F/W})(T_{\mathrm{des}} - 70°\mathrm{F}) + (T_{\mathrm{amb}} - 70°\mathrm{F}) + \alpha D$$

some scenarios, with $T_{\mathrm{des}} = 150°\mathrm{F}$:

T_{amb}	α	D	e
$70°\mathrm{F}$	$1°\mathrm{F/W}$	$0\mathrm{W}$	$0°\mathrm{F}$
$65°\mathrm{F}$	$1°\mathrm{F/W}$	$0\mathrm{W}$	$-5°\mathrm{F}$
$70°\mathrm{F}$	$0.9°\mathrm{F/W}$	$0\mathrm{W}$	$-8°\mathrm{F}$
$70°\mathrm{F}$	$1°\mathrm{F/W}$	$5\mathrm{W}$	$5°\mathrm{F}$

A closed-loop controller

add sensor to measure plate temperature T

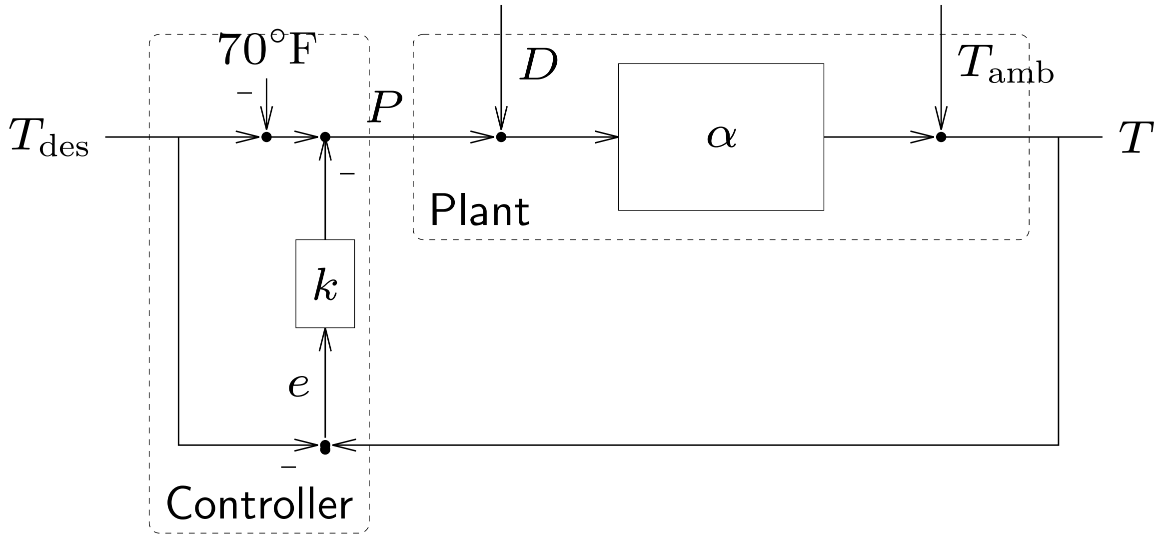
modify heater power to

$$P = \underbrace{(T_{\rm des} - 70°{\rm F})/(1°{\rm F/W})}_{\text{open-loop control}} - k \underbrace{(T - T_{\rm des})}_{\text{error}}$$

where $k > 0{\rm W}/°{\rm F}$

- called *proportional control* since we are feeding back a signal proportional to the error
- k is called the proportional feedback gain
- extra term 'does the right thing': $T < T_{\rm des} \;\Rightarrow\; e < 0 \;\Rightarrow$ increase P
  $T > T_{\rm des} \;\Rightarrow\; e > 0 \;\Rightarrow$ decrease P

block diagram:



How well does this controller work?

Solve

$$T = T_{\mathrm{amb}} + \alpha(P + D), \quad P = T_{\mathrm{des}} - 70°\mathrm{F} - k(T - T_{\mathrm{des}})$$

for error $e = T - T_{\mathrm{des}}$ to get

$$e = \frac{(\alpha - 1°\mathrm{F/W})(T_{\mathrm{des}} - 70°\mathrm{F}) + (T_{\mathrm{amb}} - 70°\mathrm{F}) + \alpha D}{1 + \alpha k}$$

Thus

$$\text{closed-loop error} = \frac{\text{open-loop error}}{1 + \alpha k}$$

some scenarios, with $T_{\mathrm{des}} = 150°\mathrm{F}$ and $k = 10$:

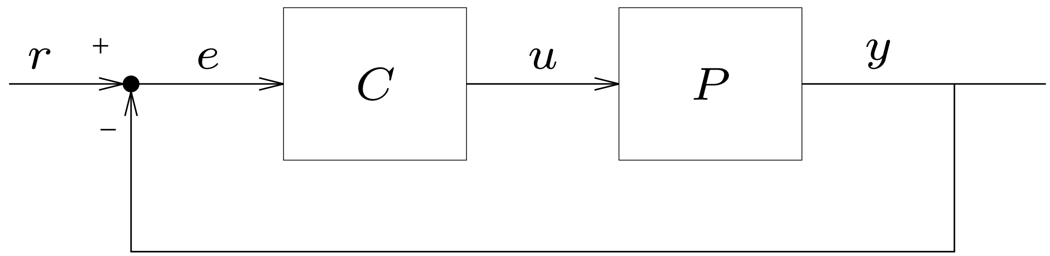
T_{amb}	α	D	e
70°F	1°F/W	0W	0°F
65°F	1°F/W	0W	−0.45°F
70°F	0.9°F/W	0W	−1°F
70°F	1°F/W	5W	0.45°F

- closed-loop controller handles changes in ambient temperature/ thermal resistance and disturbances much better than open-loop controller

- improvement is $1/(1 + \alpha k)$ — the sensitivity of the feedback loop — so large $k \Rightarrow$ good performance

Feedback control

Common setup for feedback control:



- P is the plant; C is the controller
- u is the plant input (actuator signal); y is the plant output (sensor signal)
- r is the reference or command input (what we'd like y to be)
- $e = r - y$ is the (tracking) error

can also be other signals $e.g.$, disturbances and noises

goal: make $y \approx r$, $i.e.$, e small (despite variations in P, disturbances, . . .)

Consider static, linear case (u, y, P, C, ... are numbers)

closed-loop gain from r to y, $T = \dfrac{PC}{1+PC} = 1 - S$ is called the *closed-loop input/output (I/O) gain*

closed-loop gain from r to e, $S = \dfrac{1}{1+PC}$ is the sensitivity

for small δP, $\dfrac{\delta T}{T} = S \dfrac{\delta P}{P}$

large loop gain $L = PC$ (positive or negative) $\Rightarrow$ S small $\Rightarrow$

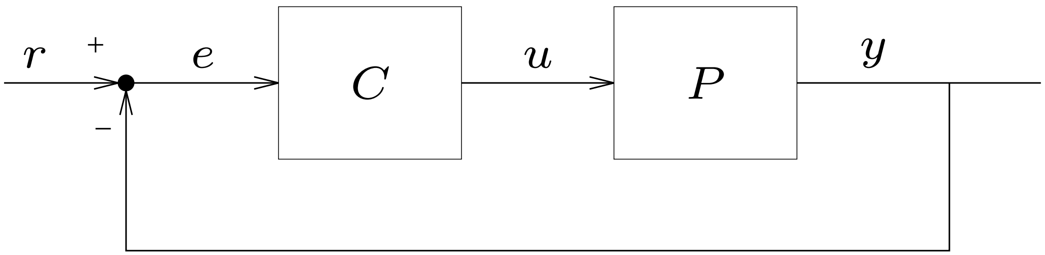
- $T \approx 1$
- e is small (for a given r)
- I/O gain is insensitive to changes in plant gain

example: $L \approx 20\text{dB} \Rightarrow$

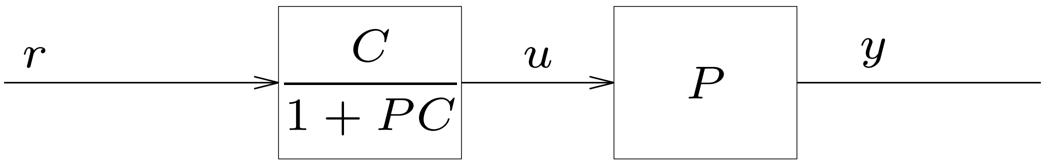
- $T \approx 1$ (within about 10%)
- $y \approx r$ within about 10% (tracking error is roughly 10%)

Open-loop equivalent system

closed-loop system:



open-loop equivalent (OLE) system:



(has same I/O gain as closed-loop system, $i.e.$, T)

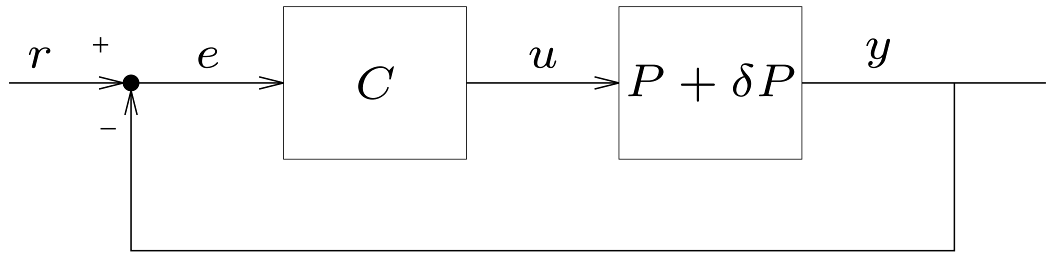
OLE system is used to compare open- & closed-loop arrangements

we'll look at

- changes in P
- input & output disturbances
- sensor noise

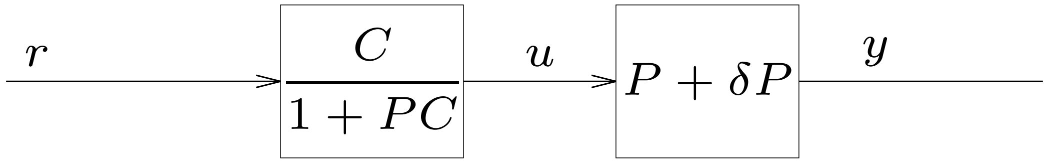
Changes in P

closed-loop system:



$$\delta T_{\mathrm{cl}} = \frac{PC + \delta PC}{1 + PC + \delta PC} - \frac{PC}{1 + PC}$$

open-loop equivalent system:



$$\delta T_{\text{ole}} = \frac{PC + \delta PC}{1 + PC} - \frac{PC}{1 + PC}$$

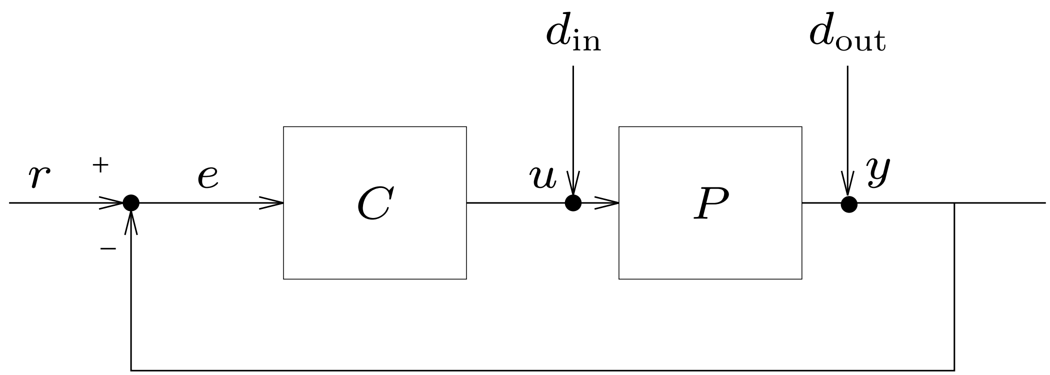
hence (after some algebra)

$$\delta T_{\text{cl}} = \frac{1}{1 + PC + \delta PC} \delta T_{\text{ole}}$$

so for small δP, $\delta T_{\text{cl}} \approx S \delta T_{\text{ole}}$

Input & output disturbances

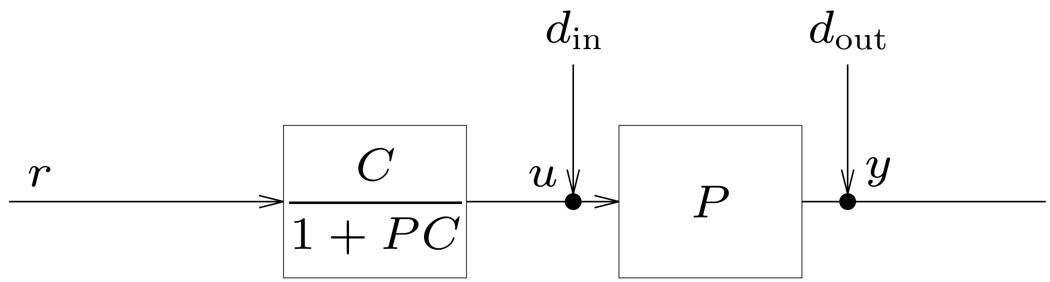
suppose disturbances $d_{\rm in}$, $d_{\rm out}$ act on the plant



effect on y (with $r = 0$)

$$y_{\rm cl} = \frac{P}{1+PC} d_{\rm in} + \frac{1}{1+PC} d_{\rm out}$$

open-loop equivalent system:



effect on y (with $r = 0$):

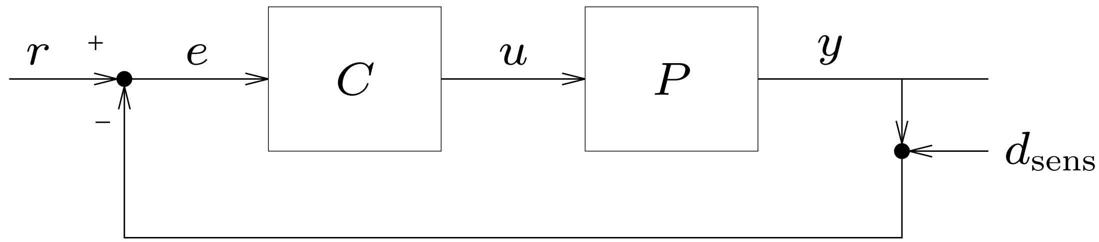
$$y_{\text{ole}} = Pd_{\text{in}} + d_{\text{out}}$$

hence $y_{\text{cl}} = Sy_{\text{ole}}$, *i.e.*, effect of disturbances multiplied by S

L large $\Rightarrow$ S small $\Rightarrow$ effect of disturbances small

Sensor noise

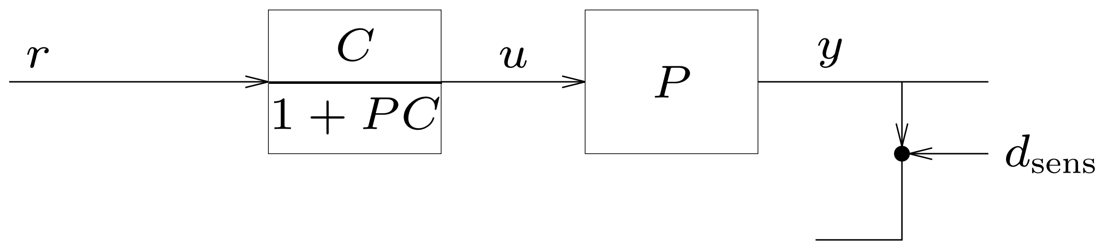
suppose sensor has noise d_{sens}



effect on y (with $r = 0$):

$$y_{\mathrm{cl}} = \frac{-PC}{1+PC} d_{\mathrm{sens}}$$

open-loop equivalent system:



effect on y (with $r = 0$):

$$y_{\text{ole}} = 0,$$

much better than closed-loop system!

finally, a *disadvantage* of feedback: output can be affected by sensor noise

Summary

- benefits of feedback determined by the sensitivity $S = 1/(1 + PC)$
- large loop gain $L = PC$ (positive *or* negative) yields small S, hence benefits of feedback

benefits of feedback include (when $|S| \ll 1$):

- good tracking ($y \approx r$)
- low sensitivity of I/O gain w.r.t. plant gain
- reduction of effect of input & output disturbances on output

some *disadvantages* of feedback control:

- cost (or reliability) of sensor
- sensor noise affects output

(we'll see others later)