

FINANCIAL CENTRALITY AND THE VALUE OF KEY PLAYERS

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ABSTRACT. Consider an economy in which agents face endowment income risk but have stochastic market access as they interact in a *stochastic financial network*. We define the *financial centrality* of an agent as the ex-ante marginal social value of injecting an infinitesimal amount of liquidity to that agent. We show financially central agents are not only those who trade often, but those who have *fewer* trading links with other agents given this frequency, trade when income risk is high, when income shocks are positively correlated, when attitudes toward risk are more sensitive in the aggregate, and when there are tail risks. Equivalently, financial centrality is the value of a personalized security which pays off over states in which a named trader participates in market exchanges. Evidence from village risk-sharing network data in which the Pareto weights of the model are determined from bargaining solutions is consistent with theory. We allow for endogenous market participation and conclude with normative, policy implications.

JEL CLASSIFICATION CODES: D14, E44, G01, L14, O16

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1. INTRODUCTION

Delirium: *You use that word so much. Responsibilities. Do you ever think about what that means?*

Dream: *Well, I use it to refer that area of existence over which I exert a certain amount of ... influence.*

Delirium: *It's more than that. The things we do make echoes.*

–Neil Gaiman, *The Sandman, Vol. 9: The Kindly Ones*

We focus on a measure of financial centrality and the identification of key players in financial markets and risk-sharing environments. We pay particular attention to settings in which disruptions to markets take the form of shocks to market participation. That is, in a given state of the world not every agent may be able to transact with every other agent. The market structure is modeled as stochastic; participation is modeled as either exogenous or endogenous to other fundamental shocks (such as income shocks).

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Stochastic market participation is a useful framework for several contexts. First, the framework is useful to study and augment classical risk-sharing models, but extend them to settings whereby agents only interact in exogenous or endogenously determined subgroups, and therefore the agents vary considerably in their value in smoothing risk.¹ These limits to interaction may be for exogenous reasons corresponding to opportunities or costs or for endogenous reasons such as asymmetric information or moral hazard concerns. Second, participation shocks are consistent with and nest several types of models that are widely used to think about financial markets. One class of models include search frictions with bilateral and stochastic matching as in [Duffie et al. \(2005\)](#), and, in particular, directed search models in which subsets of agents group together facing a tradeoff between offers and matching rates as in [Armenter and Lester \(2015\)](#). Another class of models with disruption to markets include market participation models used in finance and monetary economics, in particular, when the number and composition of traders who can deal with one another is limited and potentially stochastic as in [Kiyotaki and Wright \(1989\)](#), [Trejos and Wright \(1995\)](#), [Freeman \(1996\)](#) and, [Santos and Woodford \(1997\)](#).² Our focus here, in this paper, relative to each of these applications, is on the resulting risk of thin markets, as this is a particular concern to the agents individually and to overall social welfare.

In this paper we consider agents in a standard risk-sharing environment: a collection of agents with income shocks can make transfers to smooth risk. Unlike in the standard model where one can think of it as having all agents trading in a single market, we study the case where agents can be heterogenous as to with whom and when they can transact with others. That is, agents interact in a *stochastic financial network*.

What this means is that in a given state of the world, only certain subsets of agents can mutually exchange, which we call participating in the market.^{3,4} We study both the case of exogenous participation as well as endogenous participation; the stochastic financial network can have the distribution being determined *exogenously or endogenously* in equilibrium. This environment allows for considerable restriction on the extent of risk-smoothing. Individuals may participate in the market in many or few states and when they participate the market may be thick or thin. Both

¹Our work of course is thematically related to a large literature studying risk-sharing networks. Examples include, but are not are certainly not limited to, [Fafchamps and Lund \(2003\)](#); [Bramouille and Kranton \(2007\)](#); [Bloch, Genicot, and Ray \(2008\)](#); [Ambrus, Mobius, and Szeidl \(2010\)](#); [Jackson, Barraquer, and Tan \(2012\)](#); [Ambrus and Elliott \(2018\)](#). As described below, our framework and methodology is quite different.

²See also [Grossman and Weiss \(1983\)](#); [Fuerst \(1992\)](#); [Green et al. \(1999\)](#); [Green and Zhou \(2002\)](#); [Manuelli and Sargent \(2010\)](#).

³That is, a clique is drawn with a distribution according to the stochastic financial network model. This structure is closely related to the subgraph generated models (SUGMs), econometric models of network formation, studied in [Chandrasekhar and Jackson \(2016\)](#).

⁴Stochastic networks are often studied and are perhaps more sensible than non-random networks in numerous contexts wherein interactions need not be with the exact same agents in every state of the world. For example, while interactions among some set of agents (say i, j, k) may be more likely, it can be the case that in other states perhaps i only interacts with m , while j interacts with k and l . Stochasticity is in many ways a useful generalization and of course a sub-case is just the degenerate distribution on a single deterministic financial network. For simplicity, we consider the case where there is only one trading room at a time—that is, there is only one clique drawn or there is only one collection of say, i_1, \dots, i_k who can mutually exchange in a given period. This is easily generalizable to a setting where there are numerous “parallel markets” or subsets of agents who mutually interact with each other but not across the cliques (e.g., ijk, lm, opq are the participants in three different markets in one state of the world). We study this in Online Appendix C.1, but the single market assumption provides cleaner exposition without extraneous notation.

low participation as well as thin markets restrict risk-smoothing. To our knowledge our modeling approach is rather different than anything in the literature.

Against this backdrop, we are interested in what it means to be *financially central* in such a stochastic financial network. We take a price theoretic approach to identify key players in a financial network. We want to price agents to get a sense of their social value in this risk-sharing environment, so we define financial centrality as measuring the marginal social benefit of giving a small unit of extra income to an agent i every time i is in the market.⁵ Equivalently, this means that financial centrality amounts to the equilibrium price (or fundamental value in the sense of Lucas Jr (1978)) of an agent-specific asset (personalized debt) that pays when the agent is present in the market. There are also normative interpretations which we come to later.

We are able to characterize financial centrality in the stochastic financial networks and our analysis provides a new perspective. What we find is that, while a key component is the number of states in which i accesses the market, there is a new and perhaps more critical piece that affects financial centrality. Consider the case where i and j have the same marginal probability of participating in the market; if i tends to participate in smaller cliques (thinner markets) than j in the sense of first-order stochastic dominance, then i is more financially central than j . That is, holding fixed frequency of interaction, i is more valuable if the agent has have *fewer*, rather than more realized links. This is in stark contrast to oft-used models in which an agent's value increases in its reach (i.e., direct and indirect links) and captured by notions like degree and eigenvector-like centralities and their generalizations (e.g., Katz-Bonacich centrality). In such models an agent tends to be more central the better connected her links are; the opposite is true here. An intuition is as follows.

Trade is inherently limited by participation frictions, so idiosyncratic shocks do not typically net to zero.⁶ Thus there can be considerable market risk: that is, idiosyncratic shocks hitting traders are a source of aggregate risk, especially when markets are thin. Hence, the value of a trader has to do with being around to mitigate this risk, being able to trade with others at key times. Concavity is key for us as individuals and societies are risk averse and hence care about this risk ex-ante. As a result, we show that our measure of financial centrality features risk aversion, prudence, and the coefficient of variation of income. We then identify and quantify the value of key players in terms of whether they are likely to be able to smooth the resulting market participation risk and how valuable that smoothing would be when they are there. Our measure of financial centrality is general enough to include endogenous participation.

Outline of the Paper. Section 2 presents the baseline environment. The simplest one we feature in notation has one good, many states allowing independence or dependence in income and either exogenous or endogenous market participation, and is otherwise static. But as we point out,

⁵At an abstract level, this is analogous to defining centrality in a diffusion context as measuring the increase in social welfare (there measured by take-up of a product) by giving a unit of information to an agent.

⁶This is particularly true when the network is sparse, meaning the average agent is linked in a meaningful way to a small share of agents rather than many/most agents. As is well-known in the literature, real-world economic and social networks tend to be sparse and this is documented across numerous disciplines (e.g., economics, sociology, computer science, statistics) and contexts (e.g., financial networks, information networks, social media networks, friendship networks). See for example discussions in Watts and Strogatz (1998); Jackson (2008); Hüser (2015); Chandrasekhar (2016); De Paula (2017) and references therein.

the baseline easily extends to include multiple goods and securities, dynamics within periods as in bilateral links and dynamics across multiple periods, and partitioning of agents into multiple segregated clusters (or simultaneous segmented markets). In Online Appendix C.1 we demonstrate how all our results extend to the case of multiple segmented markets. An important feature of our model is that whether agents get autarky value or some other value for not accessing the market is actually unimportant: centrality is defined as conditioning on agents being in the market. Therefore, the utility they get if they do not have market access is not relevant in the financial centrality measure. That is the reason why we derive the same formulas in the multi-market model as in our single-market model that we focus on for expositional simplicity.

We define our notion of financial centrality in Section 3. The centralized planning problem that delivers Pareto optimal allocations is the problem of maximizing a Pareto weighted sum of ex-ante expected utilities of agents subject to shock contingent resource and to market participation constraints. Financial centrality of an agent i is then the increment in ex-ante social value, a marginal increase in the objective function of planner, derived from injecting an infinitesimally small amount liquidity to i ex-ante—that is,

$$FC_i := \text{Marginal Social Value of giving } \epsilon > 0 \text{ to } i \text{ whenever she can trade.}$$

The first order conditions with respect to this liquidity ϵ , when ϵ is driven to zero, is then the value of liquidity and the correct measure of financial centrality of each trader i . It is the expectation of the joint product of the value of liquidity as the shadow price in the resource constraint and the participation indicator of that player i .

Section 4 shows how financial centrality relates to fundamentals, then generalizes the results to incorporate heterogeneity in ex-ante Pareto weights, heterogeneity in means and in variances of trader-specific idiosyncratic shocks, correlation across these shocks, and differential risk aversion. We also allow all agents to see in advance an aggregate shock that contains information about both market participation and income risk. This serves as a reduced form way to capture that fact that the importance of a player can depend on aggregate macro conditions. To summarize results in this more general context, the most financially central agents are those who trade often; are more likely to trade when there are few traders; more likely to trade when income risk is high, and income shocks are positively correlated; more likely to trade when attitudes towards risk are more sensitive in the aggregate (high average risk aversion); more likely to trade with “distressed” institutions; and trade when there are tail risk, macro shocks which co-determine both income and risk characteristics but also limited participation—a shock to cross sectional dispersion, in the spirit of Lehman type events.

Section 5 explores several stylized market formation processes—that is, processes that micro-found and operationalize the stochastic financial network distribution itself. For example, we study a tractable model (which we dub the “Poisson market formation model”) which describes markets such as various OTC and Municipal bond markets that feature no centralized exchanges and a small number of bilateral transactions over a given asset. In fact between OTC and centralized

markets lie hybrid platforms in which a small number of invited dealers are asked to bid in a short duration auction. One such platform is Markit.⁷

Section 6 presents applied counterparts to financial centrality. First, we consider an equivalent Walrasian, decentralized interpretation of our measure of financial centrality when participation is determined by exogenous aggregate shocks. In an ex-ante securities market, agents will buy and sell claims to receive and give income transfers contingent on subsequent market participation shocks and contingent on subsequent income draws for those in a market. We show that the social planner of Section 2 can implement its optimal consumption allocation via a Walrasian equilibrium with transfers, by showing a version of the second welfare theorem in our environment. At the implementing Walrasian equilibrium, prices of securities will be such that net excess demand for securities is zero. These Arrow Debreu securities are priced by the Lagrange multiplier on the resources constraint or, roughly, by the marginal utility of consumption. Thus we can determine the value of a bundled-security associated with the name of the trader i as a security paying off one unit whenever i is in the market. All agents can buy and sell these security bundles, and they have prices which correspond to the expected product of i being in the market and the shadow price of resources at those times. Hence this is fully equivalent with the measure of financial centrality we described earlier and has an analogue, in principle, in security prices. The next part of Section 6 interprets market participation shocks as transaction chains, implementing pre-determined target allocations.

The third and final part of Section 6 turns to a positive analysis of the determination of Pareto weights in the planner's representation of the consumption allocation. Pareto weights of traders can differ. We show that if the solution concept is Nash bargaining, then there is a positive linear relationship between the Pareto weight of an agent in the planner's problem and her financial centrality measure. Similarly, with Kalai-Smorodinsky bargaining, we show that the Pareto weights depends on the aforementioned market thickness terms that are contained in our measure of financial centrality (such as market size when the trader is present, volatility, and so on). In sum, agents with higher centrality get rewarded with higher consumption.

In Section 7 we apply this latter perspective to the data. In an observational analysis, we investigate whether there is empirical content in our theoretical approach. Using the Townsend Thai village data with a panel over 15 years, we look at 338 households across 16 villages where we have detailed data on consumption, income, and transactions across villagers. We use whether a household has reported making or receiving a transfer to any other household in a given month as a measure of being active in the network in a given period. Our positive analysis under Nash or Kalai-Smorodinsky bargaining suggest that those who participate when the market is thin (few active trader, greater volatility) should receive higher Pareto weights in the planner's representation. We use the panel to estimate from a standard risk sharing equation a household fixed-effect of consumption which would be a monotonically increasing function of the Pareto weight. We then regress this fixed-effect on variables that capture whether the market has few household traders in months when the household is active and whether the market has households with more income volatility when the household is active. We show that, indeed, a one standard deviation increase

⁷See <https://ihsmarkit.com/products/auctions.html>.

in market thinness by either measure corresponds to roughly a 0.1 standard deviation increase in the consumption fixed-effect ($p = 0.034$ and $p = 0.012$), hence consistent with the theory.

Until now we have a wide class of models where the market participation is either exogenous or endogenous but an infinitesimal liquidity injection did not alter the distribution of participation. We term these models *inert to an infinitesimal liquidity injection*. Such an assumption covers a wide set.

For instance, consider a case where there is a probability distribution over which agents could possibly participate in the market if they choose to do so. Income draws happen after agents are in the market. Other than these public initial participation draws and ex post income draws, all agents are identical. Market entry has some fixed cost k and agents choose, endogenously, whether or not to participate after seeing the set of eligible participants. Intuitively more people going means a given agent faces less consumption risk if present, due to a reduction in variance. So a given agent will go if and only if there is at least a given amount of consumption risk reduction that outweighs the cost k . If no one is on the margin of indifference, which would be the generic situation in the environments of this class of models, then an infinitesimal liquidity injection cannot change this discrete equilibrium cutoff. Despite there being endogenous participation decisions, there is no response to the injection.

In contrast in Section 8, we focus on the class of models where the participation distribution is *responsive to an infinitesimal liquidity injection*. The centrality formula has two pieces. The first, which we dub the *risk sharing effect*, measures how an additional dollar given to agent i propagates through the economy, taking the market participation process as given. The second effect, which we call the *participation effect*, measures how giving an additional dollar to agent i whenever she trades changes the (endogenous) participation decision by all agents in the economy.

Consider the following example. Suppose every individual decides whether or not to access the market at some fixed cost drawn at random before income is realized. An agent sees her own cost draw and not the draws of others; that is, unlike the first example above, the agent does not have knowledge of an exogenous participation opportunity network. Here, in a Bayes Nash Equilibrium, there will be response in the participation distribution to an infinitesimal liquidity injection, as can be surmised from thinking about an out-of-equilibrium experiment. If someone's fixed cost of entry goes up, this increases the likelihood that the room size itself is smaller, causing a chain reaction among others, feeding back to the original agent, and so on, as in classical Leontief inversion of an input output matrix.

We also present a model with private information about income shocks, a moral hazard model for market access featuring in essence a first order condition embodying tradeoffs, along with a fourth model of team production (which like the others correlates income and participation but nevertheless exhibits inertness). More generally, beyond these examples, the extra term in the financial centrality measure when there is responsiveness weights the percent change in the market participation decision of each agent, by the Pareto-weighted utilities of all agents. Essentially, in cases that are responsive to the injection, agents are more (less) central if the marginal liquidity injection to the agent corresponds to an increase (decrease) in participation configurations where

more valuable agents enter the market or generates smaller cliques of agents holding fixed one's frequency of interaction.

In Section 9 also generalizes the model to include larger discrete liquidity injections, turns to normative policy considerations, and confirms the earlier measures of valuable traders should be used to direct these liquidity injections. Thus, just as the earlier notions of financial centrality consistent with ex post shocks and contagion have influenced the way policy makers think about prudential regulation, here our ex-ante measure of financial centrality could be used to think about monetary policy. We are reminded of Jeremy Stein's 2013 discussion⁸ of how central bank liquidity should be priced ex-ante in an auction—a price which in turn could serve a guide to policy makers concerning market conditions, a feed back loop to policy. Our contribution would be a measure of which traders or institutions have a key value in channeling the incremental liquidity to the market. This normative perspective presents a moral hazard policy for our ex-ante measure of financial centrality.

Section 10 is a conclusion.

Related Literature. Our work speaks to the literature studying risk-sharing networks (see e.g., Bramouille and Kranton (2007); Bloch, Genicot, and Ray (2008); Ambrus, Mobius, and Szeidl (2010); Jackson, Barraquer, and Tan (2012); Ambrus, Gao, and Milán (2017); Ambrus and Elliott (2018)). The literature studies these informal insurance networks from a variety of different angles: e.g., how the shape of an exogenous non-random network affects the extent of insurance sustained, endogenous formation of a non-random network to be able to sustain favors or risk-sharing, and so on. Perhaps the closest works are Ambrus et al. (2017) and Ambrus and Elliott (2018). These too are different. The former studies risk-sharing where transfers among pairs of agents can only depend on income realizations of agents whom both are linked to in a pre-existing network. Here central agents tend to be well-connected in a manner akin, though not identical, to Bonacich centrality (and other eigenvector-like centralities). The focus of the latter is particularly on link investments in a bargaining game whereby once the network is constructed, all connected components perfectly share risk and the focus is on the efficiency or lack thereof in terms of over or under-investment in links.

There is also a literature on contagion in financial networks. Much of this literature focuses on a kind of non-linearity, whereby positive and negative shocks propagate asymmetrically through a financial network. For instance, in the presence of solvency constraints, a positive shock may leave the network intact whereas a large enough negative shock may have a large impact on welfare. Intriguingly, agents may vary in whether they are central for positive versus negative shocks and further, the optimal network structure may vary in the size of the shock (e.g., for small shocks the complete graph but for large shocks the empty graph). The theory is developed in, among others, Allen and Gale (2000), Freixas et al. (2000) and Eisenberg and Noe (2001), Gai and Kapadia (2010), Blume, Easley, Kleinberg, Kleinberg, and Tardos (2011), Battiston et al. (2012), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Elliott, Golub, and Jackson (2014), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), Farboodi (2015), Babus (2016), Elliott and Hazell (2016),

⁸<https://www.federalreserve.gov/newsevents/speech/stein20130419a.htm>

Cabrales, Gottardi, and Vega-Redondo (2017), and Farboodi et al. (2017). Empirical work includes Upper and Worms (2004), Bech and Atalay (2010), Boss et al. (2004), Cohen-Cole et al. (2013), Craig and Von Peter (2014), Cont et al. (2013), Langfield et al. (2014), Jaramillo (2012), and others. See Allen and Babus (2009), Summer (2013), and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2016) for surveys.

Both the contagion literature and our paper have implications for policy, though distinct, through regulation of exposures or liquidity injections, respectively. Our setting assumes complete contracts (with a complete markets implementation) and does not exhibit this kind of contagion. Agents face market participation risk, but without pecuniary externalities and balance sheet effects that generate the typical contagion and asymmetric shock amplification mechanisms studied in this literature. This is because consumption allocations in our model are allowed to be fully contingent on both income distribution draws and realized financial networks. Our model could be extended to include these features, by imposing contractual constraints on the set of allocations allowed (which we discuss briefly in Section 3.2). Our model exhibits “symmetric contagion”: our notion of centrality measures the welfare impact of a marginal increase in the endowment of agent i , which directly increases the consumption of said agent, but also has an effect on the consumption of all other agents that trade with i , through the underlying risk sharing contracts in the economy.

2. ENVIRONMENT

2.1. Setup. We consider an endowment economy with a set $I = \{1, \dots, n\}$ of agents and one good. As we indicate explicitly below, this is much more general than it may first appear to be. We study a one period economy, but, again, this can be easily generalized to multiple periods. Agents face idiosyncratic income risk, where $y = (y_1, \dots, y_n)$ denotes the vector of income realizations for all agents in the economy, which we assume are drawn from some distribution $F(y)$. Let $\mu_i = \mathbb{E}(y_i)$ and $\sigma_i^2 = \mathbb{E}(y_i - \mu_i)^2$ and Σ denote the variance-covariance matrix of y .

Agents have expected utility preferences, with utility function $u_i(c_i)$, which we assume to be strictly increasing, strictly concave, and sufficiently smooth (i.e., all derivatives exist). We will also assume that $u_i'''(c) > 0$, making agents prudent. Risk aversion can vary with wealth. We will assume that $\lim_{c \rightarrow 0} u_i'(c) = +\infty$ for all $i \in I$.

The leading example in this paper (as in much of the finance literature) is an environment with CARA preferences and normal income shocks (henceforth the CARA-Normal model):

$$u_i(c_i) = -r_i^{-1} \exp(-r_i c_i) \text{ and } y \sim \mathcal{N}(\mu, \Sigma).$$

This parametrization will prove useful to obtain closed form expressions in Section 4.

The only point of departure with the usual risk sharing environment is on trading opportunities or *market participation*. Not every agent is present in the market in every state; only a random set of agents gets access to the market, which can be thought of as a meeting place where they can trade. If agents don't have access to this market, they are in autarky and have to consume their endowment. In Online Appendix C we explore a generalization with several segmented markets and show how to map all of the results of this special model to the general case. We focus on the single market case purely for expositional simplicity.

Thus, the *stochastic financial network* we study is a probability distribution over all elements of the power set of n agents.⁹

Formally, let $\zeta \in \{0, 1\}^n$ be the market participation vector, which we model as a shock to the consumption set of agents. This is general and includes both exogenous market participation as well as a wide class of endogenous market participation models as well, described below. Here if $\zeta_i = 0$ then $c_i = y_i$. However, if $\zeta_i = 1$, then consumption and income do not have to coincide as agents can make transfers in such states. The relevant state, in the Arrow and Debreu sense of enumerating all shocks and indexing the commodity space by them, is then $s = (y, \zeta) \in S := \mathbb{R}_+^n \times \{0, 1\}^n$. A *feasible consumption allocation* is a function $c(s) = (c_i(s))_{i \in I}$ such that, for every $s = (y, \zeta)$, $c_i(s) = y_i$ whenever $\zeta_i = 0$ and it is resource feasible: i.e., $\sum_i \zeta_i c_i \leq \sum_i \zeta_i y_i$ for all s .

State s is drawn from a probability distribution $P(y, \zeta)$ which is common knowledge among agents, and we assume the support of the distribution, S , to be discrete for most proofs, for expositional simplicity. This is a primitive of our baseline environment. The distribution models the market participation process: examples nested include classical search models (as in [Duffie et al. \(2005\)](#)), directed search models (as in [Armenter and Lester \(2015\)](#)), and random matching models (as in [Kiyotaki and Wright \(1989\)](#); [Trejos and Wright \(1995\)](#), etc.).

The timing of the realization of income and market shocks matter and will give rise to different measures of centrality. The baseline assumption in most of the paper is to consider income and market participation shocks as independent random variables. One can simply think about this as a case where market participation, ζ , is assigned first and then independent of this income shocks y are drawn. This may describe exogenous settings in which transaction opportunities arise, to first order, from a set of pre-determined agents (e.g., relatives or individuals with whom trust has been established over many years) and where shocks to availability, awareness, or costs further affect participation. It also describes some endogenous settings more generally, described below.

Suppose the optimum can be determined as if there were a planner who tries to chooses among resource feasible allocations to maximize a linear welfare functional, with Pareto weights vector $\lambda \in \mathbb{R}_+^n$, effectively choosing $c(s)$ to solve:

$$(2.1) \quad V(\lambda) := \max_{(c_i(\cdot))_{i=1, \dots, n}} \mathbb{E}_s \left\{ \sum_{i=1}^n \lambda_i u_i [c_i(s)] \right\}$$

subject to

$$(2.2) \quad \sum_{i=1}^n \zeta_i c_i(s) \leq \sum_{i=1}^n \zeta_i y_i(s) \text{ for all } (y, \zeta)$$

and

$$c_i(s) = y_i \text{ for all } s = (y, \zeta) : \zeta_i = 0.$$

We therefore consider a setting where a set of n agents who may have heterogeneous preferences, heterogeneous income processes, endogenous participation decisions, and for whom the planner has heterogeneous Pareto weights, are assigned consumption allocations that maximize the planner's

⁹This is intimately related to a draw from the subgraph generated model (SUGM) studied in [Chandrasekhar and Jackson \(2016\)](#). In the case of multiple markets, this generalizes to distributions over partitions of n agents.

objective function, as a way of generating and characterizing constrained Pareto optimal allocations. In Section 6 we also discuss foundations for the Pareto weights λ .

2.2. Discussion. The basic setup can be generalized considerably. First of all, we can index by time, with long or even infinite horizon. We can entertain Markov process on shocks. Our timeline can be divided into sub-periods: traders meet in a market for two or more periods before the next market participation draw (and we allow both implementation via bilateral links of a multi-person outcome as well as borrowing and lending with risk contingencies within the longer period). Though dynamics could easily be incorporated throughout most of the paper, we spare the reader the requisite notation.

We are featuring one good but we can easily generalize the notation and allow commodity vectors over goods. Then there would be a sequence of resource constraints (market clearing), one for each good; utility functions still strictly concave though. Likewise we can reinterpret goods as securities and endowments as portfolios.

Trivially, our setting could be partial equilibrium with prices of all goods, or assets, fixed outside, as in a small open economy, one market at a time, or one village at a time. In this case value functions would be strictly concave over a selected numeraire good, taking outside prices as given. It is also easy to allow preference shocks rather than endowment shocks.

Moreover, we can generalize this to many cliques of agents meeting or, in other words, many segmented markets that are drawn in parallel, with ζ now being an $n \times k$ matrix and ζ_i^m is a dummy for whether i participates in market m . We study this in Online Appendix C.1.

3. FINANCIAL CENTRALITY

3.1. Definition. We define our measure of financial centrality of an agent i as the increment in value for the planner of providing liquidity to agent i whenever she can trade. The liquidity injection corresponds to giving $\epsilon > 0$ to agent i each time she is in the market, so

$$\forall (\zeta, y) : \zeta_i = 1 \implies y'_i = y_i + \epsilon.$$

i.e., the injection is an increase in the expectation of y_i conditional on the agent having market access ($\zeta_i = 1$). Let $V_{i,\epsilon}(\lambda)$ be the maximum value of program (2.1) given such an injection to agent i .

Definition 3.1. We define *financial centrality* of agent $i \in I$ as

$$FC_i := \left. \frac{\partial V_{i,\epsilon}(\lambda)}{\partial \epsilon} \right|_{\epsilon=0}.$$

So our measure of centrality measures the marginal social benefit of giving a small unit of extra income to agent i every time i is in the market and trades with it as per contracts and markets.

One justification for this is as follows. From the planner's perspective, the agents in the economy can be thought of as assets, in the sense of Lucas Jr (1978). When a planner considers injecting liquidity, the role an agent plays is to be available to trade: the agent only fulfills the role when she is available of course. This corresponds precisely to the idea of an asset that pays only in certain states—in this case being present. Consequently, the fundamental value of the asset corresponds

precisely to integrating over the marginal increments in social welfare, given by the equilibrium pricing kernel, over all states where the asset pays (again, here being present).

We can now formally define models that are responsive to infinitesimal liquidity injection versus inert to this liquidity injection. We say that an environment is *inert to infinitesimal liquidity injection* if $P(\zeta | y)$ is constant under changes in $\mathbb{E}(y_i | \zeta_i = 1)$, for all $i \in I, y \in Y$ and $\zeta : \zeta_i = 1$. This would be the case if the market formation process (either exogenously determined or endogenously determined) is completely independent from the income distribution, and would have the feature that a marginal liquidity injection has no effect on the market participation distribution. Recall the example from the introduction wherein individuals decide whether or not to enter a market knowing the set of others who have the opportunity to participate in this state of the world; participation has some known fixed cost. In (the pure strategy maximal entry) equilibrium, all or no such agents choose to participate and in the homogenous parameter case agents' decisions purely depend on the number of other agents who have the opportunity: there is a threshold participation opportunity size above which all agents will participate. An infinitesimal liquidity injection clearly cannot change this endogenous distribution of participation decisions and, further, if the equilibrium with maximal entry is inert to liquidity injections, then so is a mixed strategy equilibrium with independent mixes.

Environments where the above property fails are models that are *responsive to infinitesimal liquidity injection*. Note that all exogenous market participation models are inert. Endogenous market participation environments may be inert or may be responsive, and this depends on the details of the model. An example of such an environment (which we will study in more detail in Section 8.2) is one where agents have to decide whether to (costly) access the market or not, before observing income draws. In this environment, agents draw fixed market participation costs $k_i \geq 0$ from some distribution $G(k_1, \dots, k_n)$ which has full support in an interval in \mathbb{R}^n , and decide to access the market if the expected utility of having market access (integrating over income draws and market participation decisions of other agents) net of the trading cost k_i exceeds the expected autarky value. In any equilibrium, agents will have a cutoff cost such that they only access the market for low enough k_i . This model will typically display responsiveness to infinitesimal liquidity injection since the liquidity injection would, in particular, increase the expected utility of for agent i from getting market access ($\zeta_i = 1$), therefore changing the equilibrium market participation distribution.

Whether an environment is responsive can be delicate. While having correlation between income and participation is important for a model to exhibit responsiveness, it is not sufficient. In Section (8.5) we provide an example studying team production, where output is not independent of participation and yet the environment is inert.

3.2. Characterizing Financial Centrality. In what follows, we will explore the properties of which agents are more financially central as a function of fundamentals such as propensity to be an active trader, composition of those who are active when the agent is active, variances and covariances of incomes of active traders, risk preferences, and so on.

We next develop a useful formulation of financial centrality in terms of the multipliers of the maximization problem in (2.1).

Let $\hat{q}(y, \zeta)$ be the Lagrange multiplier for the first condition and define an auxiliary multiplier vector

$$(3.1) \quad q(y, \zeta) : \hat{q}(y, \zeta) := q(y, \zeta) P(y, \zeta),$$

and let $\gamma_i(s)$ be the corresponding Lagrange multiplier for the non negativity constraint $c_i \geq 0$. The Lagrangian for (2.1) is then

$$(3.2) \quad \mathcal{L} = \mathbb{E}_s \left\{ \sum_{i \in I} \lambda_i u_i [c_i(s)] + q(s) \zeta_i [y_i - c_i(s)] + \gamma_i(s) c_i(s) \right\}.$$

In the baseline model, we assume market participation is independent of income draws. In this case, financial centrality can be expressed using the envelope theorem on program (2.1).

Proposition 3.1. *Suppose the environment is inert to infinitesimal liquidity injection, and let $q(s)$ and $\gamma_i(s)$ be the multipliers of Lagrangian (3.2) for program (2.1). If for all $i \in I$ and $s \in S$ we have $c_i(s) > 0$ whenever $\zeta_i = 1$, then*

$$(3.3) \quad FC_i = \mathbb{E}_s \{ \zeta_i q(s) \}.$$

If not, then $FC_i = \mathbb{E}_s \{ \zeta_i [q(s) + \gamma_i(s)] \}$

Proof. We use the classical envelope theorem on a variation of program (2.1), changing the income of agent i to $\hat{y}_i = y_i + \zeta_i \epsilon_i$. Then, the envelope theorem implies

$$\frac{\partial V}{\partial \epsilon_i} \Big|_{\epsilon=0} = \mathbb{E}_s \left[\zeta_i \frac{\partial \mathcal{L}}{\partial y_i} \times \frac{\partial y_i}{\partial \epsilon_i} \right] = \mathbb{E}_s \{ \zeta_i [q(s) + \gamma_i(s)] \}.$$

If $c_i(s) > 0$ for almost all s , then complementary slackness implies $\gamma_i(s) = 0$ for almost all s , and hence $\mathbb{E}_s \{ \zeta_i \gamma_i(s) \} = 0$, proving the desired result. \square

The multiplier $q(s)$ is, of course, the marginal value of consumption, at the (constrained) efficient allocation $c_i(\cdot)$. As we will see below, when defining a Walrasian equilibrium in an Arrow Debreu economy defined on this environment, $q(s)$ will correspond to the equilibrium price of the Arrow Debreu security that pays only at state s . As such, equation (3.3) is effectively the price of a fictitious asset that pays 1 consumption unit whenever $\zeta_i = 1$, using $q(s)$ as its pricing kernel.

Next, we can consider the case where the participation distribution is responsive to infinitesimal liquidity injection. A key feature in responsive settings is that participation and income become correlated. Consider several examples that we develop in detail in Section 8. First, imagine each individual draws private costs of entry and endogenously chooses whether or not to enter the market. Second, imagine that each individual gets a signal about income draws in society before it is realized. Given the signal individuals must decide whether or not to participate at some fixed cost. Third, consider a moral hazard environment wherein agents apply effort that affects their probability of attending the market. Each of these have responsiveness, described below.¹⁰ Here $P(\zeta|y)$ plays a crucial role and how participation responds to changes in income draws for i also

¹⁰Correlation of income and participation is not sufficient to generate responsiveness however. The fourth example we develop in Section 8 considers a team production environment wherein the income distribution for those participating depends on those accessing the market. This is inert to the injection but correlates income and participation nonetheless.

drives the financial centrality of i . Define

$$\mathbf{S}_i(\zeta | y) := \frac{\partial \log P(\zeta | y)}{\partial \epsilon_i} \Big|_{\epsilon_i=0}$$

as the score of the likelihood function $P(\zeta | y)$ with respect to $\mathbb{E}(y_i | \zeta_i = 1)$.

Proposition 3.2. *Suppose the environment is responsive to infinitesimal liquidity injection, and that $c(\cdot) \gg 0$ solves program (2.1). Then financial centrality can be written as:*

$$FC_i := \mathbb{E}_s[\zeta_i q(s)] + \mathbb{E}_s \left\{ \left[\sum_{j \in I} \lambda_j u_j(c_j(s)) \right] \mathbf{S}_i(\zeta | y) \right\}.$$

Proof. All proofs are in Appendix A unless otherwise noted. \square

As noted above, it immediately follows that if the environment is inert— $\zeta \perp y$ —then $FC_i = \mathbb{E}_s[\zeta_i q(s)]$. This centrality measure has two components. The first is the *risk sharing effect*. This measures how an additional dollar given to agent i propagates through the economy, taking the market participation as given. The second effect is the *participation effect*, which measures how giving an additional dollar to agent i changes the endogenous participation decisions by all agents in the economy.

Finally, note in principle that $V(\cdot)$ need not only reflect the program (2.1). Consider a modification where we have constrained efficiency. So we add to program (2.1) some additional constraints

$$\Xi(c, y, \zeta) \leq 0 \text{ for all } (y, \zeta) : \zeta_i = 1.$$

This function represents some (arbitrary) frictions in consumption allocations and transfers. For instance it could represent constraints as to who is able to make (how much) transfers to whom. As an example, perhaps agent i can never make transfers if j and k are active in a trading room, but can make transfers up to size \bar{c} if j and l are in the trading room. Another example of constraints that $\Xi(c, y, \zeta)$ could capture is the structure studied in Ambrus et al. (2017). There they study risk-sharing where transfers among pairs of agents can only depend on income realizations of agents both have links to in an underlying social network.

In a case with constraints $\Xi(c, y, \zeta)$ financial centrality under inert environments would be

$$FC_i = \mathbb{E}_s \left\{ \zeta_i [q(s)] + p(s) \cdot \frac{\partial \Xi(s)}{\partial y_i} \right\},$$

where $p(s)$ is the Lagrange multiplier of the friction constraints and with responsive environments,

$$FC_i := \mathbb{E}_s \left[\zeta_i q(s) + p(s) \cdot \frac{\partial \Xi(s)}{\partial y_i} \right] + \mathbb{E}_s \left\{ \left[\sum_{j \in I} \lambda_j u_j(c_j(s)) \right] \mathbf{S}_i(\zeta | y) \right\}.$$

Note that the first term in all these expressions is the same, regardless of the increased generality. We focus on that first part now, recognizing that the implications from that part will carry over to general settings.

4. HOW DOES FINANCIAL CENTRALITY RELATE TO FUNDAMENTALS?

In this section we study how financial centrality relates to underlying economic fundamentals in environments that are inert to infinitesimal liquidity injections. We show that those who are more financially central are active in the trading room (1) more often, (2) when there are fewer others present (and consequently there is greater income volatility), (3) when those who have higher Pareto weights are present, (4) when more risk-averse agents are present, and (5) when incomes are more correlated.

4.1. Homogenous Fundamentals. We consider here first, for simplicity homogeneous preferences, independent and identically distributed income draws, and an utilitarian planner. Such an environment has all agents being virtually identical in almost all aspects, except their relative positions through market participation shocks ζ . If agents have different financial centralities in this setting, it can only come from their heterogeneity in participation, for example through their positions in the financial network—the new dimension we introduce in this paper. Formally, we assume $u_i(\cdot) = u(\cdot)$ for all $i \in I$ and $\{y_i\}_{i \in I}$ are independent and identically distributed where $\mathbb{E}_y(y) = \mu$ and $\text{var}(y) = \sigma^2$. Under this assumption, the financial centrality measure can be further decomposed as

$$FC_i = \mathbb{E}_\zeta \{ \zeta_i \mathbb{E}_y [q(y, \zeta) \mid \zeta] \} = \mathbb{E}_\zeta \{ \zeta_i h(\zeta) \}.$$

where $h(\zeta) := \mathbb{E}_y [q(y, \zeta)]$. Our aim will be to get a closed form approximation to $h(\zeta)$. Two quantities will become particularly important in our analysis: the number of agents that are able to trade and the average income of agents that are able to trade. Therefore let

$$n_\zeta := \sum_{i=1}^n \zeta_i$$

denote the number of agents that are able to trade and

$$\bar{y}(s) := \frac{1}{n_\zeta} \sum_{i=1}^n \zeta_i y_i$$

as the average income of agents able to trade. In this very simple case, we get the following approximation.

Proposition 4.1. *Suppose $u_i = u$ and $\lambda_i = 1/n$ for all i , and income draws are independent and identically distributed across agents. Then $q(s) = u'[\bar{y}(s)]$ and $c_i(s) = \zeta_i \bar{y}(s) + (1 - \zeta_i) y_i$. Moreover, if u is analytic then we can approximate $h(\zeta) \approx u'(\mu) \left(1 + \gamma \frac{\sigma^2}{n_\zeta}\right)$, where $\gamma := (1/2) u'''(\mu) / u'(\mu)$. Therefore,*

$$(4.1) \quad FC_i \approx \mathbb{E}_\zeta \left[\zeta_i u'(\mu) \left(1 + \gamma \frac{\sigma^2}{n_\zeta}\right) \right].$$

The approximation of financial centrality given in Proposition 4.1 gives us a summary of the relevant moments of the market participation process, for the purpose of calculating financial centrality. The assumption of homogeneity in preferences and equal Pareto weights imply that consumption is equalized ex-post across agents in the market; i.e., $c_i(s) = \bar{y}_\zeta$ whenever $\zeta_i = 1$. Then, because

average income available in the market has the the same conditional expectation (because income draws are i.i.d.), we make an approximation $h(\zeta) := \mathbb{E}_y \left[u'(\bar{y}_\zeta) \right]$ around $\mathbb{E}_y(\bar{y}_\zeta | \zeta) = \mu$.

Note that we can rewrite equation (4.1) as

$$FC_i \approx u'(\mu) P(\zeta_i = 1) \times \left[1 + \gamma \sigma^2 \mathbb{E} \left(\frac{1}{n_\zeta} \mid \zeta_i = 1 \right) \right].$$

What matters here is the variance of mean income, which is the relation between income volatility (σ^2) and market size $n(\zeta)$, giving us the only relevant moment of the distribution of ζ and income volatility σ^2 .

This shows that centrality can be decomposed into two pieces. Financial centrality is higher when (1) the agent has a higher probability of trading ($P(\zeta_i = 1) \uparrow$) and (2) the market size conditional on the agent entering is smaller. Finally, the degree to which each of these matters can depend on the mean income, degree of risk aversion, degree of prudence (convexity of marginal utility of consumption, which governs precautionary savings), and variability of income (measured by the coefficient of variation). For example with log utility $\gamma = \frac{1}{\mu^2}$, with CES utility $u = \frac{c^{1-\rho}-1}{1-\rho}$ we get $\gamma = \frac{\rho(1+\rho)}{2\mu^2}$, and with CARA preferences we have $\gamma = \frac{r^2}{2}$.¹¹

In the particular example of the CARA-Normal model, we get an exact expression for financial centrality. Under homogeneity of preferences and an utilitarian planner, we have $c_i(s) = \bar{y}(s)$ whenever $\zeta_i = 1$, and $q(s) = u'[\bar{y}(s)] = \exp[-r\bar{y}(s)]$. Since $y_i \sim_{i.i.d.} \mathcal{N}(\mu, \sigma^2)$ we have that $\bar{y}(s) | \zeta \sim \mathcal{N}(\mu, \sigma^2/n(\zeta))$. Therefore

$$h(\zeta) = \mathbb{E}_y [q(y, \zeta)] = \mathbb{E}_y \{ \exp[-r\bar{y}(s)] \} = \exp \left(-r\mu + \frac{r^2 \sigma^2}{2 n_\zeta} \right)$$

so

$$FC_i = \mathbb{E}_\zeta [\zeta_i h(\zeta)] = \underbrace{\exp(-r\mu)}_{u'(\mu)} \mathbb{E}_\zeta \left\{ \zeta_i \exp \left[\gamma \frac{\sigma^2}{n_\zeta} \right] \right\}$$

where, as we saw before, $\gamma = u'''(\mu) / 2u'(\mu) = r^2/2$.

4.2. Heterogeneous Fundamentals. Next we turn to the case of heterogenous fundamentals. The above approximation can be extended to the case where income draws have identical means, but a richer correlation structure, where $\text{cov}(y_i, y_j) = \sigma_{ij} \neq 0$ and potentially agent-specific volatility. In this case, and under suitable conditions, financial centrality can be well approximated as

$$FC_i \approx \mathbb{E}_\zeta \left[\zeta_i u'(\mu) \left(1 + \gamma \times \frac{\bar{\sigma}_\zeta^2}{n_\zeta} \right) \right]$$

where $\bar{\sigma}_\zeta^2 := n_\zeta^{-1} \sum \zeta_i \zeta_j \sigma_i \sigma_j$. This formula implies that agents are more central if they are more likely to trade when (a) income volatilities are higher and (b) trading agents have positively correlated income shocks ($\sigma_{ij} > 0$). To better understand the intuition behind (b), consider for example a risk sharing model where agents can only meet pairwise. If the planner was able to choose the meeting distribution, he would pair agents with $\sigma_{ij} < 0$ in order to smooth the income shocks. If $\sigma_{ij} > 0$ then both agents do very well or poorly, exacerbating risk. Thus agents that are present whenever

¹¹As a technical aside, Taylor expansions around the mean are not always valid. See [Loistl \(1976\)](#) and [Levy and Markovitz \(1979\)](#) for a discussion. But if $u = -r^{-1} \exp(-rc)$, the approximation at $y = \mu$ is always valid.

this happens are more valuable. What matters is the average variance of agents in the market, that is, not only the size of the market matters, but also the average volatility of the agents to whom agent j is connected.

We specialize the setting to consider a setting where the social planner has preferences parametrized by a vector of pareto weights $\lambda \in \mathbb{R}_+^n$ and we have the assumptions of the CARA-Normal model; i.e., $u_i(c_i) = -r_i^{-1} \exp(-r_i c_i)$ and $y \sim \mathcal{N}(\mu, \Sigma)$, where $\mu = (\mu_1, \mu_2, \dots, \mu_n)$ is the vector of expected income for each agent, and Σ is its variance-covariance matrix. In this environment, $h(\zeta)$ has an intuitive closed form expression, which allows us to write financial centrality as

$$FC_i = \mathbb{E}_\zeta \left\{ \underbrace{\zeta_i \exp(-\bar{r}_\zeta \bar{\mu}_\zeta) \bar{\lambda}_\zeta \exp\left(\frac{\bar{r}_\zeta^2}{2} \times \frac{\bar{\sigma}_\zeta^2}{n_\zeta}\right)}_{=h(\zeta)} \right\}$$

where $\bar{\mu}_\zeta := n_\zeta \sum_i \zeta_i \mu_i$ is the mean income of agents present at market ζ , $\bar{r}_\zeta = \left(n_\zeta \sum_i \zeta_i r_i^{-1}\right)^{-1}$ is the harmonic mean of income at that market and $\bar{\lambda}_\zeta = \exp\left[n_\zeta^{-1} \sum_i \zeta_i (\bar{r}_\zeta / r_i) \ln(\lambda_i)\right]$ is a geometric weighted average of the pareto weights of agents present at ζ , weighted by how risk averse they are compared with the market average.

This environment introduces, in a tractable manner, a number of intuitive features of the aspects that make an agent more or less central than others. Specifically an agent i has higher centrality (i.e., makes $h(\zeta)$ higher more often) when she is present when (a) the average agent in the market has a lower endowment in expectation, (b) the average agent in the market is more important to the planner, or (c) the degree of risk aversion of other agents in the market is higher, all of which once again capture a sort of generalized notion of market thinness.

4.3. Heterogenous Fundamentals with Correlated Income and Participation. Next let us consider an extension that naturally correlates participation with income. This demonstrates that the core intuitions are robust to such correlation. We suppose CARA utility and a jointly normal income distribution with heterogeneous mean, variance, and covarying income draws.

Assume that agents observe shocks to income volatility and expected income to certain agents. To begin with, there is an aggregate (fundamental) shock $z \in Z$ with some distribution $G(z)$. This fundamental shock affects preferences expected income $\mu(z)$, income variance $\Sigma(z)$, preferences $u_i(c, z) = -\frac{1}{r_i(z)} \exp(-r_i(z)c)$, and even the planner's preferences $\lambda(z)$.

Formally, we assume $y | z \sim \mathcal{N}(\mu(z), \Sigma(z))$ and $\zeta \sim F(\zeta | z)$, and are such that $(y | z) \perp (\zeta | z)$. This is without loss of generality and nests the case where participation is endogenous prior to observing the realized income, but after observing the realized shock z . With these assumptions, income is Gaussian and $(y | z) \perp (\zeta | z)$. The conditional independence will buy us a simple characterization.

Let $\bar{\mu}_\zeta := \frac{1}{n_\zeta} \sum_i \zeta_i \mu_i$ be the average expected income of those trading, $\bar{\sigma}_\zeta^2 := \frac{1}{n_\zeta} \sum_{i,j} \zeta_i \zeta_j \sigma_{i,j}$ be the average volatility, $\bar{r}_\zeta := \left(\frac{1}{n_\zeta} \sum_{i:\zeta_i=1} \frac{1}{r_i}\right)^{-1}$ be the mean of the risk parameter of those trading, and $\bar{\lambda}_\zeta := \left(\prod_{i:\zeta_i=1} \lambda_i^{\bar{r}_\zeta / r_i}\right)^{\frac{1}{n_\zeta}}$ be the risk-weighted geometric mean of Pareto weights.

Proposition 4.2. *Under the above assumptions, we have that*

$$\mathbb{E}[q(y, \zeta) \mid z, \zeta] = \exp[-\bar{r}_\zeta(z) \bar{\mu}_\zeta(z)] \times \bar{\lambda}_\zeta(z) \times \exp\left[\frac{\bar{r}_\zeta^2(z)}{2} \times \frac{\bar{\sigma}_\zeta^2(z)}{n_\zeta}\right]$$

so financial centrality is given by

$$FC_i = \mathbb{E}_{z, \zeta} \left\{ \zeta_i \times \exp[-\bar{r}_\zeta(z) \bar{\mu}_\zeta(z)] \times \bar{\lambda}_\zeta(z) \times \exp\left[\frac{\bar{r}_\zeta^2(z)}{2} \times \frac{\bar{\sigma}_\zeta^2(z)}{n_\zeta}\right] \right\}$$

where the market averages $(\bar{r}_\zeta, \bar{\mu}_\zeta, \bar{\lambda}_\zeta, \bar{\sigma}_\zeta^2)$ are functions of market fundamentals $z \in Z$.

The robust implications of this are as follows. First, again we see that agents who tend to participate when the trading room is small (low n_ζ) are more financially central. Second, agents who tend to participate when those whom the planner values more are more central. Third, those who participate when there is greater volatility are more central. Fourth, agents are more central if the average agent in the market has a lower endowment in expectation when the agent in question is in the market. Fifth, agents are more central if the degree of risk aversion when the agent is in the market is higher.

Overall, the notion of financial centrality captures a generalized notion of market thinness. The planner values the agents precisely whom are able to provide transfers to others who need it when they are in particular dire need. In doing so, the planner takes into account who will be in the room in equilibrium. Though this is intuitive, it provides an economically relevant relationship between our fundamentals and a notion of centrality.

5. MARKET FORMATION PROCESS

Until now our discussion of the stochastic financial network has been rather abstract. It has been a fairly unrestricted distribution over the space of all subsets of agents: the realized market can be comprised of any subset of agents and then there is a distribution over each possibility. It is nonetheless instructive to examine specific examples that may micro-found the stochastic financial network distribution.

To make matters simple, consider the homogenous parameter case. Since

$$(5.1) \quad FC_i \propto \mathbb{E}_\zeta \left\{ \zeta_i \left(1 + \frac{\gamma}{n(\zeta)} \right) \right\}$$

we need to calculate $\mathbb{E}\left\{\frac{1}{n(\zeta)} \mid \zeta_i = 1\right\}$ and $\mathbb{P}(\zeta_i = 1)$.

Let us begin with a very simplistic perspective. The idea is that in a given state an agent is chosen to organize the market and the agent merely invites a subset of agents then who also arrive and they all can mutually exchange. A natural parable is that agents have social relationships. For instance, in a village there are kin and friends; people will leverage these to share risk. For the sake of discussion, consider that the collection of agents are members of some such non-random kin/friend network. Importantly note that this is not the stochastic financial network that we are interested in: the kin/friend network captures the list of deep, permanent relationships (kin/friend). In such a setting, the question is when i is called to organize the market, what affects whether some j from society participates in the market? And, moreover, what might determine whether i is called

to organize the market in the first place? We study four examples below. Note that while these are presented as exogenous processes, there could just as well be endogenous foundations for the same distribution of participation as long as it is inert to infinitesimal liquidity injections.

5.1. Degree Model: Market Comprised of Network Neighbors. In the first case, imagine that when an agent i is called to trade in some state, he invites all his kin and friends. This is, the entire neighborhood of agent i in the kin/relative network participates.¹² In such a case, the resulting stochastic financial network is simply a distribution over all neighborhoods of agents $j = 1, \dots, n$ in the kin/relative network with weights given by the probability that j is called to be the market organizer.

So, the first example imagines a network where in every period, a single node is chosen as a host and all of its neighbors are activated to trade. Let $g = (I, E)$ denote the kin/relative network with E the set of edges and $g_{ij} = 1 \{ij \in E\}$. For simplicity this is an undirected, unweighted graph and assume each node has a self-loop ($g_{ii} = 1$). Let $d_i := \sum_j g_{ij}$ denote the degree of node i and let $N_i := \{j \in I : g_{ij} = 1\}$ denote the neighborhood of i .

Market participation is drawn as follows. With probability $z_i = \frac{1}{n}$, each agent is selected to be the host. Then $\zeta_i = 1$ and also $\zeta_j = 1 \{j \in N_i\}$. We can compute financial centrality as

$$FC_i = \frac{1}{n} \left\{ d_i + \gamma \sum_j \frac{g_{ij}}{d_j} \right\}.$$

Agents who have larger neighborhoods in the kin/relative network are more financially central (from the d_i term), but in particular holding that fixed agents that have neighbors who have smaller neighborhoods are more financially central (from the $\frac{1}{d_j}$ term). The notion of financial centrality derived from our model may be quite different from traditional notions of centrality, such as degree, betweenness, eigenvector-like (e.g., Katz-Bonacich) centralities, among others. In Figures 5.1a and 5.1b, we compare two agents i and j in different parts of a large kin/relative network (so n is the same for both of them). Observe that agent j in Figure 5.1b is more central than the one in Figure 5.1a, i , according to most commonly used centrality measures, since she can reach more agents in the same number of steps (higher eigenvector centrality, for example). However, the agent j is less financially central in the induced stochastic financial network than i , since (a) it has the same probability of having market access, but (b) the markets she has access to are bigger (in the first order stochastic dominance sense) to those that agent i reaches, and is hence less important. This is because of the logic of consumption variance reduction: a dollar given to the agent i will reduce consumption variance a lot more than agent j .

5.2. General Poisson Models. The second example generalizes the above.¹³ We call this a generalized Poisson model for reasons that become clear below, but a simple way to understand it is to think of it like a model of invitations. One interpretation is that when i is the market host, then i invites each j in the community with some probability \mathbf{p}_{ij} . Note that i need not only invite

¹²To give an example for the case of endogenous participation, imagine this graph to be such that for every node every neighborhood represents the set of individuals who have the opportunity to participate and in equilibrium this neighborhood does attend the market. In such settings, it is without loss to proceed as if participation is exogenous.

¹³We present a detailed exposition of this section along with the technical derivations in Online Appendix B.

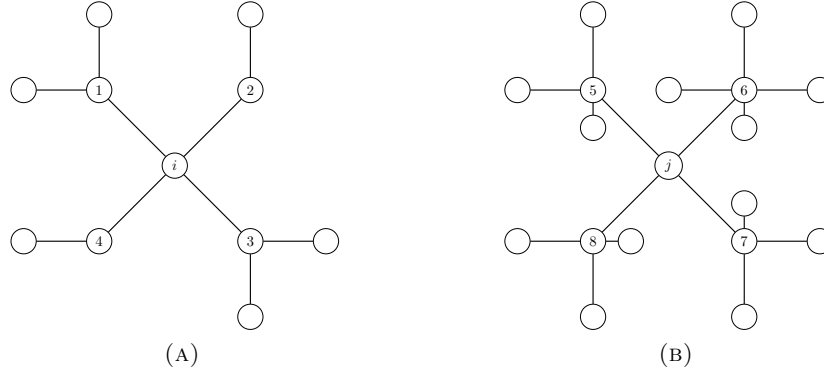


FIGURE 5.1. Two agents in a large network (not necessarily fully pictured), i and j , in Panels (A) and (B) respectively. Many typical measures of centrality (e.g., eigenvector-like centralities) would treat j as being more central than i , ceteris paribus, since they have the same degree and j 's neighbors' degrees are higher. Our notion of financial centrality ranks $FC_j < FC_i$.

her kin/friends, but perhaps her kin's friends or her friends' friends' friends and so on. A natural model of this is in Jackson and Wolinsky (1996), where $\mathbf{p}_{i,j} = \alpha^{\delta_{ij}}$ for some probability α where δ_{ij} is the (shortest path) distance from i to j in the kin/relative network. That is, the invitation probability declines in the number of steps the invitation has to travel for j to attend i 's market. We can explicitly characterize financial centrality of the stochastic financial network in this setting because this model allows us to directly compute the expected size of the trading room when any given agent i is called to form the market using Poisson distribution calculations.

Let $z_i \in [0, 1]$ denote the probability that an agent gets selected as the host. Then let \mathbf{p} denote a matrix with entries $\mathbf{p}_{i,j}$ denoting the probability that j is in the market when i is the host, which is independent across j . We set $\mathbf{p}_{i,i} = 1$.

This nests some obvious special cases. For example, in the degree model above, $\mathbf{p} = g$. Another example is an invitations model, described above. But of course the setup is considerably more general.

It is useful to define an individual specific parameter, which is the expected number of individuals in the trading room when i is selected as host, ν_i , noting $\nu_i = \sum_j \mathbf{p}_{i,j}$. To characterize financial centrality, we need to know the expected sizes of the trade rooms when i is host and conditional on i being in the room, integrating across the other possible hosts.

Letting $p_i = \mathbb{P}(\zeta_i = 1) = \sum_j z_j \mathbf{p}_{j,i}$, we can write financial centrality as

$$FC_i \approx \hat{FC}_i = p_i \left[1 + \gamma z_i \mathbb{E} \left(\frac{1}{n_\zeta} \mid i \text{ is host} \right) + \gamma \sum_{j \neq i} z_j \mathbb{E} \left(\frac{1}{n_\zeta} \mid j \text{ is host and } \zeta_i = 1 \right) \right].$$

We need to compute the conditional expectations of the inverse market size conditional on both (a) agent i being present and (b) each potential host. We can approximate these using a shifted Poisson distribution for market size (details in Online Appendix B), and find

$$\mathbb{E} \left(n_\zeta^{-1} \mid i \text{ is host} \right) = \frac{1 - \exp(1 - \nu_i)}{\nu_i - 1} =: m_1(\nu_i) \quad \text{and} \quad \mathbb{E} \left(n_\zeta^{-1} \mid j \text{ is host and } \zeta_i = 1 \right) = \frac{1 - m_1(\nu_{ji})}{\nu_{ji} - 1} =: m_2(\nu_{ji})$$

where $\nu_{ji} = \nu_j - \mathbf{p}_{j,i}$, both strictly decreasing functions of $\nu_i \geq 1$.

As a consequence, we can simply calculate financial centrality as

$$\hat{FC}_i \approx p_i \times \left\{ 1 + \gamma z_i m_1(\nu_i) + \gamma \sum_{j \neq i} z_j m_2(\nu_j - \mathbf{p}_{j,i}) \right\}.$$

This shows the following. First, nodes with a larger expected reach as measured by ν_i are more central (as long as n is large enough relative to γ). Second, nodes that have larger expected inverse room size when they are hosts are more central. Third, i is more central when $\mathbf{p}_{j,i}$ increases, particularly when ν_j is small. So when j tends to invite few individuals, but i is likely to be in such a j 's room, then i is more valuable.

5.3. Sequential Market Formation. In the Poisson models, for $j \neq k \neq i$, note that $\zeta_j \perp \zeta_k$ conditional on i hosting. But trading groups may be determined sequentially, along a chain of meetings. In this case the study of random walks on graphs provides the right vocabulary to capture this.

We can model this in a simple way, though the analytic characterization is hard to come by. Let $(z_i)_{i \in I}$ denote the probabilities that each node is the host, let $(p_{ij})_{i,j \in I}$ denote the probability that i meets j , and let β be the probability that at each stage the chain continues. With complementary probability $1 - \beta$, the chain terminates exogenously. However, the chain also terminates if an agent is revisited (and hence no new agents are added to the market).

This process, at termination, determines the size of the trading room. While it is easy to describe, and easy to simulate, it is hard to analytically compute moments for the distribution of $\frac{1}{n_\zeta}$ (Aldous and Fill, 2002; Durrett, 2007), even if chains are not terminated upon revisiting an agent. This is because what matters is the number of distinct agents in the market, not just the number of steps the chain makes (which, in that case, would simply follow a geometric random variable). In the special case with large n , $z_i = 1/n$, $\beta = 1$ (no random exogenous termination) and $p_{ij} = 1/d_i$ (i.e., uniform random walk, with equal probability among first degree neighbors) and g comes from an Erdős-Renyi process, Tishby et al. (2017) get closed form expressions for the distribution of chain length (or market size in our setup), showing that it follows a product of an exponential and a Rayleigh distribution.¹⁴

5.4. Market Participation Shocks as Transaction Chains. Now we give an alternative interpretation of the market participation shocks. Any market participation shock can be interpreted as a realization of a chain of bilateral transactions among a subset of agents in the economy, which are allowed to run short-run deficits. Formally, a simple transaction chain is a set of agents that can only trade with adjacent agents. Namely, there is a set of agents $J = \{i_1, i_2, \dots, i_k\} \in I$ (which are selected randomly), such that i_j can trade only with agents i_{j-1} and i_{j+1} , for $j \in \{0, 1, \dots, k\}$ (except for the first agent i_1 , who can only trade with i_2 , and the last member i_k , who can only trade with i_{k-1}). Agent j can make or receive transfers $\hat{T}_{j,h} \in \mathbb{R}$ for $h \in \{j-1, j+1\}$, which might be such that $\hat{T}_{j,h} + y_j < 0$ (i.e., giving agent h more than the endowment she has at the moment of the transaction). If $\hat{T}_{j,h} > 0$ it means that j sends resources to agent h , while $\hat{T}_{j,h} < 0$ means that j receives

¹⁴This, of course, can be adapted by allowing $\beta \in (0, 1)$.

resources from k . The budget constraint that j faces is then $T_{j,j-1} + T_{j-1,j} + T_{j,j+1} + T_{j+1,j} \leq y_j$. Defining $T_{j,h}$ as net transfers instead of gross transfers, we then have that $T_{j,j+1} = -T_{j+1,j}$. Therefore, we can work only with the net transfers $T_j = T_{j,j+1}$ for agents $j = 1, 2, \dots, k-1$, and the simplified budget constraint for each agent is

$$T_j \leq y_j + T_{j-1}$$

for every $j = 1, \dots, k-1$. There is a clearing house that, at the end of the day, settle all transactions. That is, agents can have short run deficits, but at the end of the period, payments are settled simultaneously, once all transactions are agreed upon. Without loss of generality, let's assume $i_j = j$, so that $C = \{1, 2, \dots, k\}$. A consumption profile of the agents in the chain C , is a description of consumption amounts $c = (c_1, c_2 \dots c_k)$. A consumption allocation is feasible if and only if $\sum_{i=1}^k c_i = \sum_{i=1}^k y_i$. We say that a consumption bundle is *transfer-feasible* if and only if it is feasible and there exist transfers $\{T_{i,j}\}_{i=1}^n$ such that

- (1) $c_j = y_j + T_{j-1} - T_j \geq 0$
- (2) $\sum_{j=1}^{k-1} (T_{j-1} - T_j) = 0$.

In order to be able to define this objects for all j , we set $T_{1-1} = T_{k,k+1} = 0$. Therefore, for $i = 1$ we have $c_1 = y_1 - T_2$ and for $i = k \in \{1, \dots, n\}$ we have $c_k = y_k + T_{k-1}$. For such a consumption allocation, we say the sequence of net transfers $\{T_j\}$ *implements the allocation* c . The (rather obvious) result is that the set of feasible consumption profiles is equal to the set of transfer feasible allocations. This then implies that by modeling the interactions among agents as trades as if everyone was trading with each other is just an useful representation.

So, the basic assumptions in this environment is that (1) agents can only trade bilaterally with adjacent agents (with a predetermined order) in the chain and (2) promises to pay (i.e., net transfers) have to be settled jointly, after all trades have been agreed upon. This is the most important assumption which abstracts away from leverage or run-away constraints (which would limit the short-run deficits agents can have in any given moment). In Proposition 5.1 we show that, if we allow agents to run short-run deficits until the end of the day, where all transactions are settled, than any feasible consumption allocation among k agents can be implemented by a trading chain (in no particular order of agents).

Proposition 5.1. *Let $c = (c_i)_{i=1}^{i=k}$ be a feasible consumption allocation (so $\sum_i c_i = \sum_i y_i$). Then, the net transfers T_j defined as*

$$(5.2) \quad T_j = T_{j \rightarrow j+1} := \sum_{i=1}^{i=j} (y_i - c_i)$$

implement c . Moreover, the following gross transfers implement c

$$\hat{T}_{j \rightarrow j+1} = \max\{0, T_j\} \quad \text{and} \quad \hat{T}_{j+1 \rightarrow j} = \max\{0, -T_j\}$$

so either $\hat{T}_{j \rightarrow j+1} = T_j > 0$ and $\hat{T}_{j+1 \rightarrow j} = 0$, or $\hat{T}_{j \rightarrow j+1} = 0$ and $\hat{T}_{j+1 \rightarrow j} = -T_j \geq 0$.

6. APPLIED COUNTERPARTS TO FINANCIAL CENTRALITY

We now take a *positive* approach. First, we study a complete market General Equilibrium model, where agents can trade Arrow Debreu state-contingent assets, where the state space is S with $s = (\zeta, y) \in S$. We show that market participation shocks can be modeled as realizations of bilateral transaction chains among agents that are allowed to have short-run deficits, but which have to clear at the end of the trading period; i.e., all transactions must clear by a clearing house institution at the end of the day. In Section 6.1 we show two important results. First, we show that under the assumption that market access is inert to infinitesimal liquidity injection, the optimal allocation defined in program (2.1) can always be implemented by a complete markets Walrasian Equilibrium with lump-sum transfers redistributing wealth, a Second Welfare Theorem, and that a Walrasian Equilibrium is one of these optima, without such transfers, a first welfare theorem for our environment. We then show that, in such an equilibrium, financial centrality for agent i can be measured as the price of a personalized bond; i.e., an asset that pays whenever agent i is able to trade. Therefore, centrality can be measured using classical asset pricing techniques, once the equilibrium pricing kernel is estimated.

We also study a different decentralized environment, where agents engage in ex-ante cooperative Nash bargaining. In this model, agents' natural threat points are their autarky values, and bargain over state-contingent consumption allocations. We show that there is a positive linear relationship between the “representing Pareto weight” of an agent and her financial centrality measure. However, as we saw in financial centrality is itself a function of the Pareto weights vector, which makes the determination of the representing Pareto weights vector a fixed point equation problem. We then study the Kalai-Smorodinsky bargaining solution, which unlike the Nash bargaining solution, does not explicitly depend on the financial centrality of the agents. However, the representing Pareto weights (which, again, map into higher consumption for the agent in the CARA preferences environment) does depend on moments that typically make financial centrality of an agent higher, such as smaller market sizes conditional on the agent being trading.

6.1. Decentralization as Arrow Debreu Economies. The main assumptions we need for results in this section are inert market participation and that income and market participation shocks are independent; without this assumption, there could be non-pecuniary externalities in the market participation decision which will not be reflected in the equilibrium prices (i.e., we may lose the constrained-efficiency result).

We consider an Arrow Debreu economy, where agents can buy and sell claims on income and consumption, contingent on the configuration of the market and the nature of income shocks. However, agents cannot buy or sell income claims that will pay off in states where they are unable to trade (since there is no physical way to make such transfers), which we formalize as “consumption space shocks” (as in Mas-Colell et al. (1995)). Formally, let $A_{\hat{s}}$ denote the Arrow Debreu (AD) asset¹⁵ that pays 1 unit of the consumption good if the state is $s = \hat{s}$, nothing if $s \neq \hat{s}$, and $a_i(\hat{s}) \in \mathbb{R}$ the demand of asset $A_{\hat{s}=s}$ by agent i . Consumption for agent i at state $s = (y, \zeta)$ is then

¹⁵Formally, $A_{\hat{s}}(s) = \begin{cases} 1 & \text{if } s = \hat{s} \\ 0 & \text{otherwise} \end{cases}$ is the return matrix of the AD security paying only at state \hat{s} .

$c_i(s) = y_i + a_i(s)$. The market participation constraint can be introduced by imposing a *physical constraint*: whenever $\zeta_i = 0$ we must have $a_i(s) \in \{0\}$ (i.e., agents cannot trade in assets that they will not be able to be present in the market to clear the trades ex-post).

To simplify proofs and exposition, we consider cases where there is only a countable number of possible income shocks, so that $S = \prod_i (Y_i \times \{0, 1\})$ is also countable, and where $P(s \in S) > 0$ for all $s \in S$. Given an Arrow Debreu prices $\hat{r}(s)$ for each A_s and a vector of lump sum transfers $\tau = (\tau_i)_{i \in I}$, that is a given distribution of wealth, such that $\sum_{i \in I} \tau_i = 0$, agents choose consumption and asset purchases to maximize expected utility, given her budget constraint:

$$(6.1) \quad \max_{\{c_i(s), a_i(s)\}} \mathbb{E}_s \{u_i [c_i(s)]\}$$

$$(6.2) \quad s.t : \begin{cases} c_i(s) = y_i(s) + a_i(s) & \text{for all } s \in S \\ a_i(s) = 0 & \text{for all } s \in S : \zeta_i = 0 \\ \sum_{s \in S} a_i(s) \hat{r}(s) \leq \tau_i. \end{cases}$$

As we did when defining the Lagrange multipliers for the planning problem, we normalize the price function as $r(s) = \hat{r}(s) / P(s)$, changing the budget constraint in the consumer problem as

$$(6.3) \quad \mathbb{E}_s [a_i(s) r(s)] := \sum_{s \in S} a_i(s) r(s) P(s) \leq \tau_i.$$

A *Walrasian Equilibrium with a lump-sum redistribution of wealth* τ is a triple $(c, a, r) = (\{c_i(s), a_i(s)\}_{i \in I, s \in S}, \{r(s)\}_{s \in S})$ such that

- $\{c_i(s), a_i(s)\}_{s \in S}$ solves (6.1) with budget constraint (6.3) for all $i \in I$, given (normalized) prices $r(s) = \hat{r}(s) / P(s)$ and lump-sum redistribution of wealth $\tau = (\tau_i)_{i \in [n]}$,
- asset markets clear: $\sum_{i \in I} a_i(s) = 0$ for all $s \in S$,¹⁶ and
- consumption good markets clear: $\sum_{i \in I} \zeta_i c_i(s) \leq \sum_{i \in I} \zeta_i y_i$ for all $s \in S$.

A Walrasian Equilibrium is an equilibrium (c, r) with no transfers ($\tau = 0$), that is, the net value of purchases and sales of securities in the ex-ante budget must sum to zero. In Proposition 6.1 below, we show a version of the First and Second Welfare Theorems for this economy, which is an application of the classical welfare theorems to this environment (See Mas-Colell et al. (1995)). This can be qualified as a welfare theorem with “constrained efficiency,” since the constraint that lack of market access (i.e., $\zeta_i = 0$) implies autarkic consumption is interpreted as a physical constraint (i.e., a social planner could not change an inactive agent’s consumption either).

Proposition 6.1 (Welfare Theorems). *Suppose the environment exhibits market participation inert to infinitesimal liquidity injection, with $\zeta \perp y$. Take a planner’s problem (2.1) with Pareto weights $\lambda \in \Delta^I$, and an optimizing allocation $c = (c_i(s))_{i \in I, s \in S}$, with normalized Lagrange multipliers $q(s)$ (as defined in (3.1)). Then, (c, r) is a Walrasian Equilibrium with lump-sum transfers of wealth τ , where $r(s) = q(s)$ for all $s \in S$ and $\tau_i = \mathbb{E}_s \{[c_i - y_i(s)] q(s)\}$. On the other hand, if (c, r) is such an equilibrium with lump-sum transfers τ , then \exists Pareto weights $\lambda \in \Delta^I$ such that c is the allocation solving planner’s problem (2.1) (where we again have $q(s) = r(s)$).*

¹⁶If $\exists \bar{s} \in S : P(\bar{s}) = 0$, then we can interpret this condition as imposing the constraint that $a_i(\bar{s}) = 0$ for all $i \in I$ (i.e., agents cannot trade in probability zero events).

An important Corollary of Proposition 6.1 (and most classical proofs of Second Welfare Theorems in various settings) is that it gives us an explicit formulation for the equilibrium Arrow Debreu security prices at the implementing equilibrium, which coincide with the shadow values $q(s)$ at the resource constraint at each state s .¹⁷ But then, since we can interpret this economy as one with complete markets (once we interpret market participation shocks as consumption sets shocks) $r(s)P(s)$ is a *pricing kernel*, which greatly simplifies the pricing of additional assets, if available to the market. More explicitly, if we add to this economy, on top of the Arrow Debreu securities offered, an asset with return payoff function $\rho(s) \in \mathbb{R}$, its (no arbitrage) equilibrium price in this economy would be

$$\text{Price} = \mathbb{E}_s [\rho(s) \times r(s)] := \sum_{s \in S} \rho(s) r(s) P(s).$$

Using the results from 6.1, we can then show that financial centrality can be thought as the equilibrium price of an asset (which we dubbed personalized debt) with return payoff matrix $\rho^i(s) = 1$ if $s : \zeta_i = 1$.

Proposition 6.2. *Suppose $y \perp \zeta$ and let (c, r) be the Walrasian Equilibrium with transfers $\tau = (\tau_i)_{i \in I}$ that implements the planner's problem 2.1 optimal allocation c with Pareto weights $\lambda \in \Delta$. Then*

$$FC_i(\lambda) = \sum_{s \in S} \rho^i(s) \hat{r}(s) = \sum_{s \in S} \rho^i(s) r(s) P(s).$$

That is, financial centrality is the price of a personalized debt asset implementing Walrasian Equilibrium with transfers.

Of course, there is a mapping between a Walrasian Equilibrium without lump-sum transfers and its corresponding utilitarian planner representation, with its Pareto weight vector λ . Two special cases are of interest. In the benchmark case of the CARA-Normal model, assuming constrained efficient allocations are implemented without lump sum transfers, we obtain a fixed point equation mapping the primitives of the model (income distribution moments and preferences) to the Pareto weights of the planner's problem which we derive in Online Appendix D.

We also show that in the case where the planner has uniform Pareto weights (i.e., $\lambda_i = 1/n$ for all i), preferences are identical and shocks are i.i.d. Gaussian variables, then the planner's problem can be implemented by a Walrasian Equilibrium with no transfers with $q(s) = \exp(-r\bar{y}_\zeta)$ and $c_i(s) = \zeta_i \bar{y}_\zeta + (1 - \zeta_i) y_i$, where $\bar{y}_\zeta := \frac{1}{n_\zeta} \sum_{j \in [n]} \zeta_j y_j$ is the mean income of agents in the market, and $n_\zeta := \sum_{j \in I} \zeta_j$ is the market size at state s . Moreover, the price of personalized debt is simply $FC_i = \mathbb{E}_\zeta \left\{ \zeta_i \exp\left(\gamma \frac{\sigma^2}{2}\right) \right\}$.

6.2. Bargaining Foundations.

6.2.1. *Nash Bargaining.* Suppose agents decide the social contract by bargaining among themselves. Agents receive an expected utility $U_i = \mathbb{E}[u_i(c_i(s))]$ in a contract. If they reject the proposed social contract, then agents get their “disagreement point,” or autarky value, $U_i^{aut} = \mathbb{E}_{y_i}[u_i(y_i)]$. The

¹⁷If the environment had endogenous participation where agents choose whether or not to trade, as in Section 8.3, then there typically will be pecuniary externalities from this choice. This will not be reflected in the equilibrium prices. A richer model where agents could pay others for their market participation (e.g., a Lindahl equilibrium) would restore efficiency.

social contract is the choice of a feasible consumption allocation $c(s) = \{c_i(s)\}_{s=(y,\zeta)}$. If the bargaining process satisfies Pareto optimality, linearity in utilities, and independence of irrelevant alternatives, then the optimal contract solves

$$\max_{c(s)} \prod_{i \in I} \{\mathbb{E}_s \{\zeta_i u_i [c(s)] + (1 - \zeta_i) u_i (y_i)\} - \mathbb{E}_{y_i} [u_i (y_i)]\}^{\alpha_i}$$

subject to $\sum_i \zeta_i c_i(s) \leq \sum_i \zeta_i y_i$, for some vector α such that $\sum_i \alpha_i = 1$ and $\alpha_i \geq 0$. This is equivalent to solve the following program

$$\begin{aligned} & \max_{c(s)} \sum_{i \in I} \alpha_i \ln \{\mathbb{E}_s \zeta_i [u_i (c_i(s)) - u_i (y_i)]\} \\ & \text{s.t.} : \sum_{i \in I} \zeta_i c_i(s) \leq \sum_{i \in I} \zeta_i y_i \text{ for all } (y, \zeta). \end{aligned}$$

The first order conditions of this problem are

$$\frac{\alpha_i}{\mathbb{E}_s \zeta_i [u_i (c_i(s)) - u_i (y_i)]} u' (c_i(s)) = q(s)$$

where, again, $q(s)$ are the Lagrange multipliers of the resource constraints. Therefore, the equivalent Pareto weight in the planner's problem is exactly

$$\lambda_i = \frac{\alpha_i}{\mathbb{E}_s \zeta_i [u_i (c_i(s)) - u_i (y_i)]}$$

In the next proposition, we investigate the fix point equation for the CARA-Normal model.

Proposition 6.3. *Suppose $u_i(c) = -r_i^{-1} \exp(-r_i c)$ and $y \sim \mathcal{N}(\mu, \Sigma)$. Then, the Pareto weights associated with the Nash bargaining solution with bargaining weights $\alpha \in \Delta^n$ satisfy the following fix point equations:*

$$(6.4) \quad \lambda_i = \frac{\alpha_i r_i + FC_i(\lambda)}{p_i \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right)} \text{ for all } i.$$

This proposition shows why the Nash bargaining solution is a nice application in our setting. In the fixed point equation, holding everything else fixed, agents with higher financial centrality have also higher Pareto weights. This suggests that, if agents bargain over risk sharing contracts, holding autarky as a threat point of the negotiation, then agents with higher centrality should have higher portions of aggregate income.

In particular, for the symmetric Nash bargaining solution ($\alpha_i = 1$) and homogeneous preferences and i.i.d. income, we get that the representing Pareto weights are not uniform ($\lambda_i \neq 1/n$) but rather satisfy

$$(6.5) \quad \ln(\lambda_i) = \kappa + \ln[r + FC_i(\lambda)] - \ln(p_i)$$

so the heterogeneity in the market participation process has a bite, unlike (as we saw above) the Walrasian Equilibrium with the same preferences.

6.2.2. Kalai-Smorodinsky Bargaining. The second most used bargaining solution in the literature is the Kalai-Smorodinsky solution. It also gives closed form solutions to Pareto weights and the

weights are expressed as a function of fundamentals of the environment, rather than a fixed point equation.

The most important parameter in the *bliss point*. The bliss point for agent i , \bar{U}_i , is defined as the utility she would achieve if she consumed all the available income in the market in every state where she can trade and only her own income otherwise:

$$\bar{U}_i := \mathbb{E}_{\zeta, y} \left[\zeta_i u_i \left(\sum_j \zeta_j y_j \right) + (1 - \zeta_i) u_i(y_i) \right]$$

and $\bar{U} = (\bar{U}_1, \bar{U}_2, \dots, \bar{U}_n)$. Likewise, the disagreement point \underline{U}_i is the value of autarky in this environment for each agent

$$\underline{U}_i := \mathbb{E}_{y_i} [u_i(y_i)]$$

and $\underline{U} := (\underline{U}_1, \underline{U}_2, \dots, \underline{U}_n)$. The Kalai-Smorodinsky solution consists on finding the linear combination of \bar{U} and the \underline{U} that lies on the Pareto frontier of the utility possibility set; i.e, find $\alpha \in [0, 1]$ such that $\alpha \bar{U} + (1 - \alpha) \underline{U} \in \mathbb{P}(\mathcal{U})$, and the solution is $U^* = \alpha \bar{U} + (1 - \alpha) \underline{U}$. Since $\bar{U} > \underline{U}$, the Kalai-Smorodinsky solution here would be

$$\max_{\alpha \in [0, 1], \{c_i(y, \zeta)\}_{i \in I}} \alpha$$

subject to

$$\begin{cases} \mathbb{E}_{y, \zeta} [\zeta_i u_i(c_i(y, \zeta)) + (1 - \zeta_i) u_i(y_i)] \geq \alpha \bar{U}_i + (1 - \alpha) \underline{U}_i & \text{for all } i \\ \sum \zeta_i c_i(y, \zeta) \leq \sum \zeta_i y_i & \text{for all } (y, \zeta). \end{cases}$$

One of the most attractive properties of the Kalai-Smorodinsky solution is that the Pareto weights derived from it have a closed form formula and is not a fixed point equation (as in the Nash Bargaining solution case).

Proposition 6.4. *If the risk sharing contract is the Kalai-Smorodinsky solution over the utility possibility set, then the Pareto weights associated with the solution are*

$$\lambda_i = \frac{1}{\mathbb{E}_s \{ \zeta_i [u_i(Y(s)) - u_i(y_i)] \}}$$

where $Y(s) = \sum_j \zeta_j y_j$ is the aggregate income in state $s = (y, \zeta)$. If $u_i(c) = -r_i^{-1} \exp(-r_i c)$ and $y \sim \mathcal{N}(\mu, \Sigma)$, then

$$\lambda_i = \frac{\beta}{p_i \times \mathbb{E}_{\zeta} \left\{ \exp\left(-r\mu + \frac{\sigma^2}{2}\right) - \exp\left[n_{\zeta} \left(-r\mu + \frac{\sigma^2}{2}\right)\right] \mid \zeta_i = 1 \right\}}$$

where $\beta = r / (-r\mu + \sigma^2/2)$.

The following corollary is an immediate consequence.

Corollary 6.1. *In the CARA-Normal model, with homogeneous preferences and i.i.d. income shocks, if $\mu > \frac{r}{2}\sigma^2$, then Lagrange multipliers are decreasing (in the FOSD sense) in market size.*

Unlike Nash bargaining, Pareto weights in this environment have a closed form solution, so comparative statics are easier to interpret, and the comparative statics are the same as the one

suggested by the Nash Bargaining fixed point equations. The most important feature, of course, is that the elements determining λ are the same as those that determine our measure of financial centrality. This correspondence allows us to operationalize financial centrality in empirical analysis.

7. EMPIRICAL EXAMPLE: RISK SHARING IN THAI VILLAGES

We modeled the equilibrium division of surplus in a setting where prior to engaging in a risk-sharing environment, agents bargain either through Nash or Kalai-Smorodinsky processes for division of surplus. This captures the idea that agents recognize that they have heterogenous interaction relationships and therefore stand to capture differential surplus, which seems a priori sensible for this application where we consider informal insurance networks in rural villages.

In taking our perspective to data, we seek to investigate, with an observational analysis, whether there is empirical content in our theoretical approach.¹⁸ Our theory has a unique prediction. In such a game, agents that provide more value—those with higher measures of financial centrality—are exactly those that are in the market when the market is thin (in a generalized sense including few active traders and greater per-trader-volatility in income). And those who are more central in this sense claim a greater share of the surplus. In this case that corresponds to higher average consumption.

We look at the Townsend Thai village data over 15 years, and we focus on 338 households across 16 villages where we have detailed data on consumption, income, and transactions across villagers (Townsend, 2016). In particular, in this setting we have variation in the number of transactions per time period. We use whether a household has reported making or receiving a transfer to any other household in the given month as a measure of being active (or a member of the clique drawn from the stochastic financial network) in that period.

We proceed in two steps. First, we need a measure that reflects $FC_i = \frac{\partial V}{\partial t_i}$. As we showed in Sections 6.2.1 and 6.2.2, if Pareto weights are determined by Nash bargaining or Kalai-Smorodinsky bargaining, then a more financially central individual i has a higher Pareto weight λ_i . This is made explicit and without the use of fixed points under Kalai-Smorodinsky bargaining in an analytically tractable manner. So, though we do not observe financial centrality, we can use observations on consumption in panel data to obtain an estimate of a function for each agent i which is monotonically increasing in the Pareto weight λ_i . Specifically we obtain the household fixed-effect of consumption, using only active periods:

$$c_{ivt} = \alpha_i + \beta y_{ivt} + \delta_{tv} + \epsilon_{ivt}$$

where t is indexing a set of active periods, α_i is a household fixed-effect, and δ_{vt} is a village-by-time fixed-effect. Under CARA utility the α_i is a monotone function of Pareto weights λ_i .

We also know from our theory the crucial component in our financial centrality measure is market thinness. So we next compute measures of market thinness for each household using observable variables. Here we can observe the number of active agents in a given village in a given period as well as the volatility due to the composition of active agents. As such, we define

$$\rho_i^\zeta := \text{cov}_t \left(\zeta_{it}, \frac{1}{n_{vt}} \right) \quad \text{and} \quad \rho_i^\sigma := \text{cov}_t (\zeta_{it}, \bar{\sigma}_t),$$

¹⁸We make no causal claims here.

TABLE 1. Do Pareto weights correlate with measures of market thinness when the agent is active?

VARIABLES	(1)	(2)	(3)	(4)	(5)	(6)
	α_i	α_i	α_i	α_i	α_i	α_i
ρ_i^ζ	0.095 (0.041)		0.112 (0.045)	0.078 (0.041)		0.093 (0.044)
ρ_i^σ		0.103 (0.050)	0.118 (0.051)		0.121 (0.051)	0.131 (0.052)
Observations	338	338	338	338	338	338

Robust standard errors in parentheses. The dependent variable is a (mean zero, standardized) Pareto weight estimate of a given household, obtained from using the vectors of household fixed effects from a regression of consumption on household income. Regressors are each standardized as well. Wealth controls are added for columns 4, 5 and 6. Wealth controls includes a third-degree polynomial in household wealth.

where n_{vt} is the number of active participants in period t in a village v , computed from the transfers data as mentioned, and where $\bar{\sigma}_t^2 := \frac{1}{n_{\zeta_t}} \sum_{i,j} \zeta_{it} \zeta_{jt} \hat{\sigma}_{i,j,t}$ is an estimate of the volatility at period t , where $\hat{\sigma}_{i,j,t}$ is the measured covariance between households i and j 's income.

Finally, we study the relationship of these two pieces, implied Pareto weights and crucial components of financial centrality, by running a multi-variate regression

$$\alpha_i = \beta_0 + \beta_1 \rho_i^\zeta + \beta_2 \rho_i^\sigma + X_i' \beta_3 + u_i$$

where X_i is a polynomial of wealth. Our theory suggests that $\beta_1 > 0$ and $\beta_2 > 0$ as being present in generalized thin markets should correspond to higher endogenously determined Pareto weight and therefore higher average consumption. We note that this is an observational claim, but it is not mechanical: that those who are present exactly when the market is thin tend to receive a greater mean consumption, even conditional on wealth, is consistent with our model.

Table 1 presents the results. Columns 1-2 and 4-5 include each measure of market thinness when the agent enters one-by-one and columns 3 and 6 include them together. Columns 1-3 include no subsequent controls, whereas columns 4-6 include a third-degree polynomial in wealth. We see a one-standard deviation increase in the tendency to enter when the market is thin in numbers is associated with a corresponding 0.095 standard deviation increase in mean consumption (column 1, $p = 0.021$). Similarly, a one-standard deviation increase in the tendency to enter when the market is thin in the sense of high volatility is associated with a corresponding 0.103 standard deviation increase in the mean consumption (column 2, $p = 0.04$). These estimates are stable to being jointly included (column 3), as well as when we include wealth controls (and including controls does not change the coefficients in a meaningful nor statistically significant way).

Taken together, the results are consistent with a story where agents have determined Pareto weights through a bargaining process, and those who have higher weights and therefore higher financial centrality are precisely those who tend to be active traders when the market is thin either

in terms of numbers of individuals or volatility. We emphasize that this observation is new to the literature and, to our knowledge, unique to our model.

8. WHEN PARTICIPATION IS RESPONSIVE TO INFINITESIMAL LIQUIDITY INJECTION

This section now allows agents' choices to determine whether or not they participate in contexts wherein the infinitesimal liquidity injection affects their participation distribution. This not only correlates y and ζ through this endogenous decision making process (which alone does not necessarily make the model responsive to injections), but changes the financial centrality expression as we noted in Proposition 3.2. Centrality now captures how marginally increasing income in states that the agent trades in, increases both the likelihood that the agent trades and the concurrent market participation decisions of other agents. This stands in contrast to endogenous participation models wherein the participation distribution is inert to infinitesimal liquidity injections.

8.1. Overview. Let us begin by clarifying the role of endogenous participation. There are two cases. In the first, every agent can decide whether or not to participate, given a distribution of participation opportunities. For instance, in a given state of the world, agents i_1, \dots, i_m may have the opportunity to enter the market, but not all necessarily decide to participate. If this decision in equilibrium is unchanged by an ϵ liquidity injection to any agent, then we may as well imagine the participation distribution as being exogenous. Such an example was given in the introduction. Financial centrality is identical even in the case where agents choose to participate; after all, the choice has no meaningful bearing on altering this distribution, so the planner evaluates the injection in the same manner as if this participation distribution was indeed exogenous.

In contrast, in what follows below, we focus on cases where the injection affects the participation decision itself. This allows us to study the participation effect, which thus far has not been a factor in our analysis of financial centrality. Recall Proposition 3.2, wherein financial centrality included an extra term

$$\mathbb{E}_s \left\{ \left[\sum_{j \in I} \lambda_j u_j (c_j(s)) \right] \mathbf{S}_i(\zeta | y) \right\}$$

which we called the participation effect. The goal of this section is to study this term. Therefore we present three models that are responsive to infinitesimal liquidity injection. We also present for clarification and contrast, a fourth model in which participation and income are correlated and yet the response is inert.

8.2. Random Market Participation Costs. In our first example, the consumption allocation $c = (c_i(s))_{i \in I, s \in S}$ is common knowledge, but agents have random market participation costs, which are privately observed. Formally, agents observe a cost $k_i \in K_i$, and costs are jointly distributed according to distribution $G(k)$ with full support in an interval in \mathbb{R}^n , and are independent of the income shocks y . Given the consumption allocation, an equilibrium market participation is a set of mappings $\zeta_i^* : K_i \rightarrow \{0, 1\}$ such that if $\zeta_i^*(k_i) = 1$ then

$$(8.1) \quad \mathbb{E}_{y,k} \{ u_i [c_i(y, \zeta_{-i}^*(k_{-i}))] - u_i(y_i) \mid k_i \} \geq k_i.$$

That is, it is a Bayesian Nash Equilibrium in an incomplete information game where agents's strategies are their market participation decisions.

In this example, we assume $u_i = u$ for all i , $\lambda_i = 1/n$ for all n , and $y_i \sim_{i.i.d} \mathcal{N}(\mu, \sigma^2)$, so that $c_i = \bar{y}_\zeta$ whenever $\zeta_i = 1$. We also assume that the conditional distribution of $k_{-i} \mid k_i$ is FOSD increasing in k_i .¹⁹

Under these assumptions, we show that

- (1) $\zeta_i(k_i) = 0$ for all k_i is the lowest participation equilibrium.
- (2) There exist thresholds $\bar{k} = (\bar{k}_i)_{i \in [n]}$ and an equilibrium $\bar{\zeta}(k)$ such that $\bar{\zeta}_i(k_i) = 1$ if and only if $k_i \leq \bar{k}_i$. Moreover, $\bar{\zeta}_i(k_i) \geq \zeta_i^*(k_i)$ for all $k \in K^n$, all agents $i \in [n]$, and for any other equilibrium participation $\zeta^*(k)$. In what follows, we characterize the market participation equilibrium with highest market participation (i.e. highest n_ζ) for all realizations of private costs $\bar{\zeta}(k)$.

Define k_m^* as the threshold if agents had complete information about the market size: $k_m^* = \mathbb{E}_y \{u(\bar{y}_m) - u(y_i)\} \geq 0$.²⁰ Also, for $m \leq n$ define $\pi(m, \bar{k}) := \mathbb{P}(\sum_i \bar{\zeta}_i(k_i) = m)$, which can be written as a function of the thresholds \bar{k} as $\pi(m, \bar{k}) = \sum_{J: |J|=m} \mathbb{P}(k_j \leq \bar{k}_j \forall j \in J \text{ and } k_h > \bar{k}_h \forall h \notin J)$. Equation (8.1) can be used to obtain a fix point equation for the thresholds \bar{k} :

$$\bar{k}_j = \Psi_j(\bar{k}, \epsilon_i) := \sum_{m \leq n} \mathbb{E}_y \left\{ u\left(\bar{y}_m + \bar{\zeta}_i(k_i) \frac{\epsilon_i}{m}\right) - u(y_i) \right\} \times \pi(m, \bar{k} \mid \bar{k}_j)$$

where $\pi(m, \bar{k} \mid k_j) = \mathbb{P}(\sum_i \bar{\zeta}_i(k_i) = m \mid k_j)$. Finally, let $\mathbf{J}_k = \left[\frac{\partial \Psi_i}{\partial k_j} \right]_{i,j \in [n]}$ be the Jacobian (with respect to k) of the above vector Ψ , and define the matrix of *modified cross-centralities* $\mathbf{F}_{n \times n}$ as

$$\mathbf{F}_{ij} = \mathbb{E}_s \left[\zeta_i \zeta_j \frac{q(s)}{n_\zeta} \mid \bar{k}_j \right] \text{ where } \zeta_j = \bar{\zeta}_j(k_j).$$

Proposition 8.1 shows the decomposition of centrality into the risk sharing effect and the participation effect.

Proposition 8.1. *Under the above assumptions,*

$$FC_i = \mathbb{E}_s [\zeta_i q(s)] + \Lambda' (I - \mathbf{J}_k)^{-1} \cdot \mathbf{F}^{(i)} = \mathbb{E}_s [\zeta_i q(s)] + \Lambda' \sum_{t \in \mathbb{N}} [\mathbf{J}_k]^t \cdot \mathbf{F}^{(i)}$$

where $\mathbf{F}^{(i)} = (\mathbf{F}_{i1}, \mathbf{F}_{i2}, \dots, \mathbf{F}_{in})$ and $\Lambda = (\Lambda_j)_{j \in [n]}$ where $\Lambda_j := \sum_{m=1}^n m k_m^* \frac{\partial \pi(m, \bar{k})}{\partial \bar{k}_j} \geq 0$.

The risk-sharing component is as usual. The participation effect can be interpreted as follows. Consider a term $[\mathbf{J}_k^t]_{ij}$. If $t = 1$, this directly encodes the change in the participation of i when j 's threshold cost of entry changes infinitesimally. For higher t , as is usual for such positive matrices, this encodes a (weighted) chain of terms. If $t = 2$, it is easy to see it now sums over every chain, $\sum_l \frac{\partial \Psi_i}{\partial k_l} \frac{\partial \Psi_l}{\partial k_j}$, which captures both the change in the participation decision of i due to the increase in cost for l as well as change in participation for l due to an increase in cost for j . This can

¹⁹This is satisfied, for example, if costs are independent, $k_i \sim G_i(k_i)$ for all i . It is also satisfied if $k_i = K + \xi_i$, where K is a random variable, and $\xi_i \sim_{i.i.d} F(\xi)$ with zero mean.

²⁰Because $u(\cdot)$ is strictly increasing and concave, and \bar{y}_m is a mean preserving spread of \bar{y}_z with $z \leq m$ (since $\bar{y}_m \sim \mathcal{N}(\mu, \sigma^2/m)$)

be thought of as a chain rule, or the indirect effect of distance 2 by increasing the equilibrium threshold cost for j . Now more generally for higher orders of t , this encodes larger chains. This is typical of numerous notions of network centralities in the literature and analogously our (weighted) endogenous network here is \mathbf{J}_k .

The more subtle feature here is that not only do chains of participation effects matter, but also these are weighted by the very effect of the liquidity injection itself. A typical eigenvector-like centrality for adjacency matrix g would be of the form $x \propto \sum_t g^t \cdot 1$, where x_i is the centrality, and so all paths from i to j s of t lengths are counted and added up. In our case, we don't add up the terms with equal weight, but rather weight by $\frac{\partial \Psi_j}{\partial \epsilon_i}$ —the change in j 's participation decision due to the injection itself. (In the proof we show that the above term is equivalent, $\mathbf{F}_{ij} = \frac{\partial \Psi_j}{\partial \epsilon_i}$.) So returning to an overall term, we can write the participation effect as

$$\sum_j \Lambda_j \frac{\partial \Psi_j}{\partial \epsilon_i} \left\{ \sum_{t \in \mathbb{N}} [\mathbf{J}_k]^t \right\}_{ji}.$$

The interpretation is clear. It takes the weighted direct and indirect effects of the marginal change in participation due to a cost increase but then weights the effect of i on every other agent in the network by how much their participation is also directly affected by the liquidity injection itself, holding the entry cost fixed.

To understand the intuition, consider the following simplified case. Imagine that i was the only agent with endogenous entry (i.e., all other agents that may have market access when i enters have costs that are negative so entry is free or above the threshold for entry so they never enter). In this case, numerous terms drop from the above, and so the participation effect of financial centrality immediately becomes

$$\Lambda_i \cdot \mathbb{E}_s \left\{ \zeta_i q(s) \frac{1}{n \zeta} \right\}.$$

Note that this is a monotone function of the risk-sharing effect of financial centrality meaning the same agents who are financially central without the endogenous effect of liquidity injection will be financially central with such an effect. Of course the more general case involves the network of effects characterized above.

8.3. Private Information about Income Shocks. In this example, the consumption allocation is also common knowledge, but agents can only observe (objective) private information about both income shocks $y \in \mathbb{R}^n$, and about other agents information. We encode beliefs and higher order beliefs about income shocks and information using a *type space* structure, a modeling device introduced by [Harsanyi \(1967\)](#). Formally, we model agents' beliefs with a *signal structure* (or a *common prior type space*) $\mathcal{Z} = \{(Z_i, \beta_i : Z_i \rightarrow \Delta(Y \times Z_{-i}))_{i \in I}, \beta_0\}$ where $z_i \in Z_i$ is the agent's signal (or *type*). Here this represents the information she observes before observing the draw of $s = (y, \zeta)$. $\beta_0 \in \Delta(Y \times \prod_i Z_i)$ is a common prior distribution over income shocks and signals and $\beta_i(\cdot | z_i)$ is the conditional belief distribution over income shocks and signals of other agents, derived from β_0 using Bayes rule.²¹ Because Y is assumed to be finite and the choice set for

²¹That is, for all (y, z_i, z_{-i}) we have $\beta_i(y, z_{-i} | z_i) = \frac{\beta_0[y, (z_i, z_{-i})]}{\sum_{z_i} \beta_0[y, (z_i, z_{-i})]}$.

every agent is binary, we can focus also only on finite signal spaces. We also add the constraint that $\text{marg}_Y \beta_0 = F$ (i.e., the marginal distribution over income shocks coincide with the true distribution of shocks). Based on its type, agent i decides whether or not to access the market.

The timing is as follows:

- (1) Income shocks $y \in \mathbb{R}_+^n$ is drawn according to $F(y)$.
- (2) Agents observe only $z_i \in Z_i$, which are jointly drawn with probability

$$(8.2) \quad \mathbb{P}(z | y) = \beta_0(y, z) / \sum_{\hat{y} \in Y} \beta_0(\hat{y}, z).$$

- (3) Agents decide whether to access the market ($\zeta_i = 1$) or not (which may be costly, with commonly known participation costs k_i) given their private information $z_i \in Z_i$.
- (4) State $s = (y, \zeta)$ is publicly observed, and agents consume according to allocation $c(s)$.

To characterize the agents' market participation decisions, they need to form beliefs over the vector of income draws and market participations. We will model this as a game, where agent's strategies are the mappings from information to market participation. The natural solution concept here is the Bayesian Nash Equilibrium (BNE): a profile of functions $\zeta_i^* : Z_i \rightarrow \{0, 1\}$ ²² is a BNE if and only if, for all $i \in I$ and all $z_i \in Z_i$

$$\text{if } \zeta_i^*(z_i) = 1 \implies \mathbb{E}_s \{u_i[c_i(s)] | \zeta_i = 1, z_i\} - k_i \geq \mathbb{E}_s \{u_i(y_i) | z_i\}$$

where the expectations for each agent is taken with respect to the probability measure

$$\mathbb{P}(s = (y, \zeta) | z_i) := \sum_{y \in Y} \sum_{j \neq i} \left[\sum_{z_j \in Z_j: \zeta_j^*(z_j) = \zeta_j} \beta_i(y, z_{-i} | z_i) \right].$$

Given a signal structure \mathcal{Z} and a BNE profile $\zeta^* = (\zeta_i^*(\cdot))_{i \in I}$, we can then derive an ex-ante equilibrium distribution over states $s = (y, \zeta)$ as

$$\mathbb{P}(s = (y, \zeta)) = \mathbb{P}(y) \sum_{z \in Z: \zeta_i^*(z_i) = \zeta_i \forall i \in I} \mathbb{P}(z | y),$$

using (8.2). This would be the measure used by the social planner when measuring financial centrality, since she has to integrate over agents' signals from an ex-ante perspective, according to the assumed common prior distribution β_0 .

In the model proposed in Section 8.3, we assume that the credit line policy $t = (t_j)_{j \in J}$ from Section 3.1 is common knowledge among agents, and hence the policy has no effect on the information agents have access to. It does, however, affect the relative utility of market access. That is, the market access strategy (given transfer $t_i \geq 0$) is

$$(8.3) \quad \zeta_i^*(\theta_i | t) = 1 \iff \mathbb{E}_s \{u_i[c_i(y_i + t_i, y_{-i}, \zeta)] - u_i(y_i) | \theta_i\} \geq k_i.$$

If $c_i(\cdot)$ is a weakly increasing in own endowment (e.g., $c_i(s) = \bar{y}_\zeta$ in an environment with an utilitarian planner, and agents with homogeneous preferences) the transfer t_i acts as a subsidy for market participation, increasing the set of signals θ_i for which condition (8.3) is satisfied. However, since the transfer policy is assumed to be common knowledge, this also affects the marker

²²Without loss of generality, we focus on pure strategy equilibria.

participation decisions of other agents. If c_i is weakly increasing for all agents (e.g., also $c_i(s) = \bar{y}_\zeta$) then other agents also have higher incentives to access the market, since it is more likely that i will be trading, and i is more valuable, since i increases aggregate income whenever she trades. We summarize this result in the following corollary.

Corollary 8.1. *Consider the above model described in Section 8.3 and $\lambda \in \Delta^n$. If the allocation $c(\cdot)$ solving (2.1) is non-decreasing in y , then $FC_i > \mathbb{E}_s [\zeta_i q(s)]$.*

8.4. Moral Hazard and Effort in Accessing the Market. We briefly set up another example of endogenous market participation, without fully analyzing it, which concerns moral hazard. This is a generalization of the model analyzed in the preceding section. We take the exact same signal structure as before. The only difference is that instead of being a binary decision (whether to access the market or not) here we have a continuum of choices.

Assume that y is realized and every agent i observes only z_i , an imperfect signal about y (i.e., $z_i \sim \pi_i(z_i | y)$ for some conditional cdf π_i). Given this private information, agents simultaneously choose the probability of accessing the market, denoted by $p_i(z_i) \in [0, 1] = \mathbb{P}(\zeta_i = 1)$. Agents have to pay a disutility cost $\psi(p)$, where ψ is strictly increasing and convex.

Given the profile of functions $(p_i : Z_i \rightarrow [0, 1])_{i=1}^n$, the joint probability of market participation, given income draws, is given by

$$\mathbb{P}(\zeta | z) = \prod_{i=1}^n [p_i(z_i)]^{\zeta_i} [1 - p_i(z_i)]^{1 - \zeta_i}.$$

Then consumption is realized according to a feasible consumption allocation $\hat{c}(s) = \zeta_i c_i(s) + (1 - \zeta_i) y_i$, where $c_i(\cdot)$ is an (equilibrium) feasible allocation. For this example, we leave unspecified the choice of the consumption allocation, and it is only assumed that the consumption allocation as a function of the state $s = (y, \zeta)$ is common knowledge among agents.

Agents preferences (given $p_i(\cdot)$) are

$$U_i(y_i, p_i | (p_j(\cdot))_{j \neq i}) = p_i \mathbb{E}_{t-i, s} \left\{ \sum_{\zeta_{-i}} \prod_{j \neq i} [p_j(z_j)]^{\zeta_j} [1 - p_j(z_j)]^{1 - \zeta_j} u_i[c_i(y_i, y_{-i}, \zeta_i = 1, \zeta_{-i}) | z_i] \right\} + (1 - p_i) u_i(y_i) - \psi(p_i).$$

As in the private information example above, the solution concept once again is the BNE, with $p^*(t) = (p_i^*(z_i))_{i \in N}$ such that for all i and all $y_i \in Y$:

$$p_i^*(z_i) \in \operatorname{argmax}_{p_i \in [0, 1]} U_i \left(y_i, p_i | \{p_j^*(\cdot)\}_{j \neq i} \right).$$

8.5. Team Production Environments. Finally, we illustrate that simply having income and participation being correlated does not mean a model exhibits responsiveness. We consider a setting where market participation shocks are determined exogenously first and then the income distributions for agents with market access depend on the identities of those trading. Formally, the timing on the resolution of uncertainty would be as follows: (1) Market participation ζ is drawn according a distribution $G(\zeta)$; (2) Income distribution is drawn from $y \sim F(y | \zeta)$. A leading example of such an environment is one of *team production*. Agents without market access

draw income from their autarky income distribution $y_i \sim F_i(y_i)$. However, once agents are drawn together to form a market, income is drawn jointly, and then agents can divide aggregate income draws amongst them in any feasible consumption allocation.

Models like this also show correlation between market participation and income. However, it is straightforward to see that in such models, financial centrality is simply $FC_i = \mathbb{E}_s \{\zeta_i q(s)\}$ as before. This is simply because the injection policy of giving an injection to agent i has no effect on the market participation distribution, since it is assumed here to be exogenous to income draws. Clearly, this setting is one that is inert to infinitesimal liquidity injection. But unlike the baseline model, income and market participation are now not independent, and the expectation has to be calculated over market participation and income shocks jointly.

9. LARGE TRANSFERS AND NORMATIVE IMPLICATIONS

We consider a thought experiment: increase the endowment of a subset of agents $J \subseteq I$, across all values of income, whenever they can trade by a total amount $T > 0$ to finance this increase. That is, the policy consists of offering a “credit line” but really a transfer, contingent only on participation and without any repayment obligations,. More formally, $t = (t_j)_{j \in J} \geq 0$ changes the income process for agent $j \in J$ to $\hat{y}_j(s) = y_j + \zeta_j t_j$ for all $s = (\zeta, y)$ with $\sum t_j = T$. This is a commitment to a named trader j without knowing what situation the trader will be in.

If $V(t)$ is the maximization problem’s value function, with income process $y_j = \hat{y}_j$, the planner would choose $t = (t_j)_{j \in J} \geq 0$ to solve

$$(9.1) \quad \max_{t \in \mathbb{R}_+^{\#J}} V(t) \text{ s.t. } \sum_{j \in J} t_j \leq T.$$

Note that $V(t)$ here is a general value function, which could come from the corresponding to the solution V of program (2.1), but this not required. We can define financial centrality for transfer of size T of agent i generally in this way.

Definition 9.1. We define *financial centrality for total transfers T* of agent $i \in I$, where t^* is a maximizer of program (2.1), as

$$FC_i^T := V_i(t^*) = \frac{\partial V}{\partial t_i} \Big|_{t=t^*}.$$

The financial centrality for total transfers T is defined relative to a hypothetical transfer of T and computes the relative gain in the value due to rewarding agent i with a transfer t_i for any maximizing transfer vector t^* such that $\sum_j t_j^* = T$.²³

Any allocation that maximizes the objective function must, when giving a transfer to a set of agents K , not benefit at the margin by providing transfers to a set of agents $J \setminus K$. We show that in fact there will be a cutoff where the (endogenously determined) set of agents who are provided non-zero transfer will be financially more central than all other agents who receive no transfers in equilibrium. Further, we demonstrate that if the total to be transferred is small enough, then the unique solution is to provide the entirety of T to a single agent rather than a subset of agents, and

²³Program (9.1) will typically have a unique solution in our applications. However, if there is more than one maximizing transfer scheme, the choice of where to evaluate V for the definition of centrality is irrelevant, as long as $V(T)$ is differentiable (see Corollary 5 in Milgrom and Segal (2002)).

in this case $FC_i^T \approx \frac{\partial V}{\partial t_i} |_{t=0}$ and the agent has the highest financial centrality, which corresponds to the leading case we have been studying earlier in the paper. When we evaluate centrality at $T = 0$ (and hence $t^* = 0$ is the only possible solution) we write (with some abuse of notation) simply $FC_i^{T=0} = FC_i$.

Proposition 9.1. *Suppose $V(\cdot)$ is concave and differentiable, and let $v^* = \max_{j \in J} FC_j^T := V_j(t^*)$, where t^* solves (9.1).*

- (1) *If $t_i^* > 0$ then $FC_i^T = v^*$, and $FC_j^T < FC_i^T$ for any $j : t_j^* = 0$.*
- (2) *Moreover, suppose $\exists i \in J$ such that $FC_i > FC_j$ for all $j \in J \sim \{i\}$. Then, there exist $\hat{T}_J > 0$ such that if $T \leq \hat{T}_J$ the unique solution $t^* = (t_j^*)_{j \in J}$ to program (9.1) is $t_i^* = T$, $t_j^* = 0$ for all $j \in J \sim \{i\}$ and $FC_i > FC_j$.*

The intuition behind the second part of Proposition 9.1 relies on the fact that if V is differentiable at $t = 0$, then it is approximately a linear function, and hence it is locally maximized by allocating all the resources to the agent with highest marginal value, given by our notion of financial centrality. The first part relaxes the linearity, but the intuition extends to the set-level. Note that traders on the margin as having the next highest value after a discrete injection would likely be different from those of high value after infinitesimally small injections.

Next, we consider the situation where the transfers $T > 0$ are non-trivial in size but in the case of the CARA-Normal model with homogeneous preferences. We show that if $t_i^* > 0$ then we can calculate the financial centrality for transfer T for agent i as

$$FC_i^T = \sum_{\zeta \in \{0,1\}^n} \underbrace{P(\zeta) \bar{\lambda}_\zeta \exp(-r\bar{\mu}) \exp\left(r^2 \frac{\bar{\sigma}_\zeta^2}{2n_\zeta}\right)}_{=h(\zeta)} \exp(-r\bar{t}_\zeta).$$

Here $\bar{\lambda}_\zeta = \exp\left[n_\zeta^{-1} \sum_j \zeta_j \ln(\lambda_j)\right]$ is the simple geometric average of Pareto weights at market ζ , and $\bar{t}_\zeta = n_\zeta^{-1} \sum \zeta_j t_j$ the average liquidity injection made available at market ζ . As before, we can see that the average income, volatility, and market size when i is present all contribute to financial centrality in the usual way.

For any set of agents $A \subseteq I$, let $\zeta^A \in \{0,1\}^n$ denote the market where only agents belonging to A have market access.

Proposition 9.2. *Take the CARA=Normal model with $\zeta \perp y$ and homogeneous preferences ($r_i = r$ for all i) and let $t^* \in \mathbb{R}_+^n$ be a solution to 9.1 with $J = [n]$.*

- (1) *If $t_i^* > 0$ then we can calculate the financial centrality for transfer T for agent i as*

$$FC_i^T = \sum_{\zeta \in \{0,1\}^n} \underbrace{P(\zeta) \bar{\lambda}_\zeta \exp(-r\bar{\mu}) \exp\left(r^2 \frac{\bar{\sigma}_\zeta^2}{2n_\zeta}\right)}_{=h(\zeta)} \exp(-r\bar{t}_\zeta).$$

- (2) *If i, j are such that $t_i^* > 0$ and*

$$(9.2) \quad P(\zeta^{\{i,A\}}) h(\zeta^{\{i,A\}}) \geq P(\zeta^{\{j,A\}}) h(\zeta^{\{j,A\}}) \text{ for all } A \subseteq I \setminus \{i, j\}$$

then $t_i^ \geq t_j^*$. If there exist some $A \subseteq I \setminus \{i, j\}$ for which (9.2) is strict, then $t_i^* > t_j^*$.*

This says that if i is more central than j in a strong sense, then i will receive higher transfers than j . Of course, the condition for Proposition 9.2, part 2 implies that $FC_i \geq FC_j$ (i.e., around $T = 0$), since $FC_i = \sum_{\zeta} P(\zeta) h(\zeta)$, but this is a stronger requirement. However, this robustly captures the intuition from above, that even for non-marginal transfers, those who tend to be in rooms that are smaller, more volatile, more important, or require more insurance are indeed deemed to be more central, even if the exact formulation is not analytically tractable.

In Online Appendix C.2 we also study an environment with a “passive” planner. That is, we consider a planner who does not choose the allocation and can only intervene through marginal liquidity injections, as in the definition of financial centrality. In such an environment, the consumption allocation is another fundamental in the model. If the allocation is Pareto optimal, then centrality will also be a function of the average “relative weights” that compare the Pareto weights of the planner (λ) with those that only represent the allocation (the weights φ of a fictitious planner who would find it optimal). An important example is a situation where the consumption allocation comes from a Walrasian Equilibrium without transfers, and the planner can only affect consumption by marginally changing the income endowment of agents in the economy.

Further, retuning to endogenous participation, it worth noting why our notion of centrality is defined with respect to injections happening ex-ante, before incomes are realized. Namely, our notion solves potential incentive problems. First, if income is privately observed, and injections are based on reported values, then agents may have incentives to report low income values in order to receive higher liquidity injections. Second, consider the case of moral hazard in income production. If income has to be produced (by investing or applying effort) and agents know that they will receive insurance from the planner, this dampens incentives for production, as in the standard moral hazard problem. Though making liquidity provision non-contingent may not be the optimal mechanism, it is robust in its ability to resolve the potential incentive problems in more general environments without having to spell out all the details of the model.

10. CONCLUSION

In a number of economic environments, agents in a market share risk, but there is heterogeneity in market access, in the ability to participate in exchange. This is true of financial markets with search frictions, matching with limited and stochastic market participation, and in some monetary models. This is observed in risk-sharing village networks, among other settings. A common, standard model which we extend to a stochastic financial network (exogenous or endogenous market participation shocks) is used to address the question of how one measures an agent’s importance in such settings. We define the financial centrality of an agent as the marginal social value of injecting an infinitesimal amount of liquidity to that agent. We show that the most valued agents are not only those who trade often, but trade when there are few traders, when income risk is high, when income shocks are positively correlated, when attitudes toward risk are more sensitive in the aggregate, when there are distressed institutions, and when there are tail risks. From a financial networks perspective, we provide a new contribution to the literature: an agent is more central, holding fixed frequency of trade, the fewer links or transaction partners she has.

Further, in our positive analysis, we study a complete market General Equilibrium model, where agents can trade Arrow Debreu state-contingent assets. After showing welfare theorem results, we characterize financial centrality as measuring the price of a personalized bond: i.e., an asset that pays whenever agent i is able to trade, and anyone can trade in that asset. Therefore, centrality can be measured using classical asset pricing techniques, once the equilibrium pricing kernel is estimated.

Additionally, we look at a different decentralized environment, where agents engage in ex-ante cooperative bargaining, which determines the Pareto weights. We show the resulting weights depend on exactly the same features (thinness of market when the agent is present, volatility when the agent is present, taste for risk when the agent is present, and so on), that is, in the same way as financial centrality. This allows us to study financial centrality in the data without observing it directly. We turn to a setting, where we have the requisite data, rural Thai villages. We provide observational evidence from village risk-sharing network data, consistent with our model, that the agents that receive the greatest share of the pie are indeed those who are not simply well-connected, but are active precisely when the market is otherwise thin in number of participants or consisting of participants with high ex-ante volatility of income.

The framework extends to both endogenous participation models, as in private information, moral hazard, or team production models where financial centrality may or may not have an extra component. In some contexts, endogenous participation has an identical form as exogenous participation; the infinitesimal liquidity injection does not generate a change in the participation distribution per se, so formally the participation decision can be thought of as exogenous. In other cases, endogenous participation leads to a change in the composition of participants in equilibrium due to the liquidity injection. Finally, normative analysis is straightforward with the intuitions from the small liquidity injection case carrying through exactly—in the case of inertness—to the case with large transfers by the policymaker to potentially a set of agents. Moral hazard concerns rationalize why we have taken an ex-ante perspective. The provision of liquidity and its characterization can be generalized to environments in which the policymaker has limited controls.

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APPENDIX A. PROOFS

Proof of Proposition 3.2. For simplicity of exposition, assume a finite state space (i.e., y is a discrete random variable), so the Lagrangian is

$$\mathcal{L} = \sum_{y \in Y} \sum_{\zeta \in \{0,1\}^n} \left[\sum_{j \in I} \lambda_j u_j(c_j) + \hat{q}(y, \zeta) \sum_{j \in I} \zeta_j (y_j - c_j) \right] \mathbb{P}(\zeta | y) \mathbb{P}(y).$$

Using the envelope theorem, we get that

$$\begin{aligned} FC_i &= \frac{\partial \mathcal{L}}{\partial y_i} \\ &= \sum_{y \in Y} \sum_{\zeta \in \{0,1\}^n} \zeta_i \hat{q}(y, \zeta) \mathbb{P}(\zeta | y) \mathbb{P}(y) \\ &\quad + \sum_{y \in Y} \sum_{\zeta \in \{0,1\}^n} \left[\sum_{j \in I} \lambda_j u_j(c_j) + \hat{q}(y, \zeta) \sum_{j \in I} \zeta_j (y_j - c_j) \right] \frac{\partial \mathbb{P}(\zeta | y_i)}{\partial y_i} \mathbb{P}(y) \end{aligned}$$

and using the facts that $q(y, \zeta) = \hat{q}(y, \zeta) / \mathbb{P}(y, \zeta)$ and complementary slackness implies

$$\hat{q}(y, \zeta) \sum_{i \in V} \zeta_i (y_i - c_i) = 0$$

for all (y, ζ) . We can simplify this expression as

$$FC_i = \mathbb{E}_{y, \zeta} \{ \zeta_i q(y, \zeta) \} + \mathbb{E}_{y, \zeta} \left\{ \sum_{j \in I} \lambda_j u_j(c_j) \underbrace{\frac{\partial \mathbb{P}(\zeta | y_i)}{\partial y_i} \frac{1}{\mathbb{P}(\zeta | y_i)}}_{:= \mathbf{S}_i(\zeta | y_i)} \right\}$$

proving the desired result. \square

Proof of Proposition 4.1. The first order conditions of program (2.1) with Lagrangian defined in (3.2) with respect to $c_i(s)$ whenever $\zeta_i = 1$ is $\lambda_i u'_i[c_i(s)] = q(s)$ (without taking into account the non-negativity constraint over consumption). Therefore, if $\lambda_i = \lambda_j = 1/n$ for all $i, j \in I$ and $u_i = u$ for all i , we then get that if $\zeta_i = \zeta_j = 1$ then $c_i(s) = c_j(s)$ (i.e., all agents participating in the market have equal consumption). Therefore, using the resource constraint, we obtain $c_i(s) = \bar{y}(s)$ whenever $\zeta_i = 1$, and obviously $c_i(s) = y_i$ otherwise. The first order condition also implies then that $q(s) = u'[\bar{y}(s)]$.

To obtain the approximation, we provide a heuristic proof. We first make a second order Taylor approximation $g(y) := u'(y)$ around $y = \mathbb{E}(y) = \mu$:

$$u'(\bar{y}) \approx u'(\mu) + u''(\mu)(\bar{y} - \mu) + \frac{u'''(\mu)}{2}(\bar{y} - \mu)^2$$

and then take expectations, we have

$$\mathbb{E}[u'(\bar{y}(s)) | \zeta] \approx u'(\mu) + u''(\mu) \mathbb{E}(\bar{y} - \mu | \zeta) + \frac{1}{2} u'''(\mu) \mathbb{E}[(\bar{y} - \mu)^2 | \zeta] = u'(\mu) + \frac{1}{2} u'''(\mu) \sigma^2/n(\zeta),$$

using the facts that $\mathbb{E}(\bar{y}) = \mu$ and that $\mathbb{E}(\bar{y} - \mu)^2 = \sigma^2/n(\zeta)$ if income draws are i.i.d. Reorganizing this expression, we get the desired result. \square

Proof of Expressions in 4.2. To see the first result, we use the same approximation around $y = \mu$ of Proposition 4.1, but now we have that $\mathbb{E}\{(\bar{y} - \mu)^2 | \zeta\} = \frac{1}{n_\zeta^2} \sum_{i:\zeta_i=1} \sum_{j:\zeta_j=1} \sigma_{ij} = \frac{1}{n_\zeta} \bar{\sigma}_\zeta^2$ as defined above. Following the same steps of the proof of Proposition 4.1 we obtain the result.

To see the second result, proceed as follows. The first order condition implies that $r_i c_i = \ln(\lambda_i) - \ln[q(s)]$. Substituting this equation into the resource constraint in state s , we obtain

$$\sum_{j:\zeta_j=1} r_j^{-1} \ln(\lambda_j) - \sum_{j:\zeta_j=1} r_j^{-1} \ln[q(s)] = n_\zeta \bar{y}$$

and using the definitions $\bar{r}_\zeta = \left(\frac{1}{n_\zeta} \sum \zeta_j \frac{1}{r_j}\right)^{-1}$ and $\hat{\lambda}_\zeta := \exp\left(\frac{1}{n_\zeta} \sum \zeta_j \frac{\bar{r}_\zeta}{r_j} \ln(\lambda_j)\right)$, we can rewrite this as

$$\frac{1}{n_\zeta} \sum_{j:\zeta_j=1} \frac{\bar{r}_\zeta}{r_j} \ln(\lambda_j) - \ln[q(s)] = \bar{r}_\zeta \bar{y}$$

if and only if

$$(A.1) \quad q(s) = \hat{\lambda}_\zeta \exp\left(-\bar{r}_\zeta \bar{y}_\zeta\right).$$

Since $\zeta \perp y$ we have that $\exp\left(-\bar{r}_\zeta \bar{y}_\zeta\right) | \zeta$ is independent of $\hat{\lambda}_\zeta | \zeta$. Moreover, because $y \sim \mathcal{N}(\mu, \Sigma)$ we know that $\bar{y} | \zeta \sim \mathcal{N}\left(\bar{\mu}_\zeta, \frac{1}{n_\zeta} \bar{\sigma}_\zeta^2\right)$, where $\bar{\mu}_\zeta := \frac{1}{n_\zeta} \sum \zeta_j \mu_j$ and $\bar{\sigma}_\zeta^2 := \frac{1}{n_\zeta} \sum \zeta_j \zeta_k \sigma_{jk}$. Using the moment generating function of the normal distribution, we obtain

$$(A.2) \quad \mathbb{E}_y \left[\exp\left(-\bar{r}_\zeta \bar{y}_\zeta\right) | \zeta \right] = \exp\left(-\bar{r}_\zeta \bar{\mu}_\zeta\right) \exp\left(\frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{2 n_\zeta}\right).$$

Using equation A.2 in the definition of financial centrality, we obtain:

$$FC_i = \mathbb{E}_\zeta \{ \zeta_i \mathbb{E}_y [q(y, \zeta) \mid \zeta] \} = \mathbb{E}_\zeta \left\{ \zeta_i \mathbb{E}_y \left[\hat{\lambda}_\zeta \exp \left(-\bar{r}_\zeta \bar{y}_\zeta \right) \mid \zeta \right] \right\}$$

which yields

$$(A.3) \quad \mathbb{E}_\zeta \left\{ \zeta_i \hat{\lambda}_\zeta \exp \left(-\bar{r}_\zeta \bar{\mu}_\zeta \right) \exp \left(\frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{2 n_\zeta} \right) \right\}$$

proving the desired result. \square

Proof of Proposition 6.1. We will focus on allocations where $c_i(s) > 0$ for all $s = (y, \zeta) : \zeta_i = 1$ for simplicity. Since (6.1) is a convex optimization problem and $u(\cdot)$ is strictly concave and differentiable, Kuhn-Tucker conditions are necessary and sufficient to characterize the optimum. This is also true for the planner's problem (2.1). Let $\mu_i > 0$ be the Lagrange multiplier of the AD budget constraint in (6.2) (this constraint will always be binding). The first order conditions of the consumer problem with respect to $a_i(s)$ at states $s = (y, \zeta) : \zeta_i = 1$

$$(A.4) \quad u'_i [c_i(s)] P(s) = \mu_i r(s) \text{ for all } s : \zeta_i = 1 \text{ where } c_i(s) = y_i(s) + a_i(s)$$

where $c_i(s) = y_i(s) + a_i(s)$. Also see that the choice of $a_i(s)$ is superfluous in the consumer's problem if $c_i(s) = 0$ for all $s : \zeta_i = 0$, and that the budget constraint can be written as $\sum \zeta_i c_i(s) r(s) \leq \sum \zeta_i y_i r(s)$. Hence $c = (c_i(s))_{i \in I, s \in S}$ is a Walrasian Equilibrium with transfers allocation if $\exists \mu_i > 0 \forall i \in I$ such that conditions (A.4) and the resource constraint (2.2) are satisfied, and such that $c_i(s) = y_i$ for all $s : \zeta_i = 0$. The corresponding Walrasian Equilibrium has $a_i(s) = c_i(s) - y_i$, $r(s) = (1/\mu_i) u'_i [c_i(s)] P(s) > 0$ and $\tau_i = \sum_s a_i(s) r(s) = (1/\mu_i) \mathbb{E}_s \{ [c_i(s) - y_i] u'_i [c_i(s)] \}$.

Doing the same exercise for the planner's problem (2.1), we get that a consumption allocation $c_i(s)$ solves the planner's problem with Pareto weights $\lambda \in \Delta$ if and only if it satisfies the resource constraint (2.2) for all $s \in S$, $c_i(s) = y_i$ for all $s : \zeta_i = 0$ and all $i \in I$, and satisfies for all $i \in I$:

$$(A.5) \quad \lambda_i u'_i [c_i(s)] = q(s) \text{ for all } s : \zeta_i = 1$$

where $q(s)$ is the (normalized) Lagrange multiplier of the resource constraint at state s .

Therefore, a Walrasian Equilibrium with transfers consumption allocation c will also be the solution to the planner's problem (2.1) with Pareto weights $\lambda_i = 1/\mu_i$. Likewise, for given $\lambda \in \Delta$, the solution to the planner's problem (2.1) will be a Walrasian Equilibrium with transfers if we take $\mu_i = 1/\lambda_i$. Moreover, the implementing price function $r(s)$ and transfers τ_i satisfy:

$$(A.6) \quad r(s) = (1/\mu_i) u'_i [c_i(s)] P(s) = q(s) P(s)$$

$$\tau_i = \mathbb{E}_s \{ [c_i(s) - y_i] q(s) \}$$

since $1/\mu_i = \lambda_i$. \square

Proof of Proposition 5.1. The fact that $\sum_{j=1}^k (T_{j-1} - T_j)$ comes from equation (5.2): we have

$$T_{j-1} - T_j = \sum_{i=1}^{i=j-1} (y_i - c_i) - \sum_{i=1}^{i=j} (y_i - c_i) = c_j - y_j$$

and hence

$$\sum_{j=1}^k (T_{j-1} - T_j) = \sum_{j=1}^k (c_j - y_j) = 0$$

since c is feasible. The consumption attained for each agent is

$$\hat{c}_j = y_j + T_{j-1} - T_j = y_j + (c_j - y_j) = c_j$$

i.e., it achieves the target consumption allocation. \square

Proof of Proposition 6.3. From the first order conditions of the planner's problem we have

$$\lambda_i \exp(-r_i c_i) = \bar{\lambda}_\zeta \exp(-\bar{r}_\zeta \bar{y})$$

so

$$-\frac{1}{r_i} \exp(-r_i c_i) = -\frac{\bar{\lambda}_\zeta}{r_i \lambda_i} \exp(-\bar{r}_\zeta \bar{y}).$$

Hence

$$\begin{aligned} \mathbb{E}_s [\zeta_i u_i(c_i(s))] &= -\frac{1}{r_i \lambda_i} \mathbb{E}_s [\zeta_i \bar{\lambda}_\zeta \exp(-\bar{r}_\zeta \bar{y})] \\ &= -\frac{1}{r_i \lambda_i} \mathbb{E}_\zeta \left[\zeta_i \bar{\lambda}_\zeta \exp\left(-\bar{r}_\zeta \bar{\mu} + \frac{\bar{r}_\zeta^2}{2n_\zeta} \bar{\sigma}_\zeta^2\right) \right] \\ &= -\frac{1}{r_i \lambda_i} FC_i. \end{aligned}$$

Moreover

$$\begin{aligned} \mathbb{E}_s [\zeta_i u_i(y_i)] &= -\frac{1}{r_i} \mathbb{E}_s [\exp(-r_i y_i)] \\ &= -\frac{1}{r_i} \mathbb{E}_\zeta \left[\zeta_i \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right) \right] \\ &= -\frac{1}{r_i} \underbrace{\mathbb{P}(\zeta_i = 1)}_{:=p_i} \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right). \end{aligned}$$

This means

$$\mathbb{E}_s \{\zeta_i [u_i(c_i(s)) - u_i(y_i)]\} = \frac{1}{r_i} p_i \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right) - \frac{1}{r_i \lambda_i} FC_i$$

and so

$$\lambda_i = \frac{\alpha_i r_i}{\frac{\lambda_i}{\lambda_i} p_i \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right) - \frac{1}{\lambda_i} FC_i}$$

if and only if

$$\lambda_i = \frac{\lambda_i \alpha_i r_i}{\lambda_i p_i \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right) - FC_i} \iff \lambda_i p_i \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right) - FC_i = \alpha_i r_i$$

if and only if

$$\lambda_i = \frac{\alpha_i r_i + FC_i(\lambda)}{p_i \exp\left(-r_i \mu_i + \frac{r_i^2}{2} \sigma_i^2\right)}$$

as we wanted to show. \square

Proof of Proposition 6.4. The Lagrangian is

$$\mathcal{L} = \alpha + \sum \mu_i \left\{ \mathbb{E}_{y,\zeta} [\zeta_i u_i(c_i) + (1 - \zeta_i) u_i(y_i)] - \alpha \bar{U}_i - (1 - \alpha) \underline{U}_i \right\} + \sum_{\zeta,y} q(y, \zeta) \zeta_i (y_i - c_i) P(y, \zeta)$$

with multipliers $(\mu_i)_{i=1:n}$ and $(q(y, \zeta) P(y, \zeta))_{y,\zeta}$. First order conditions are

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 1 - \mu_i (\bar{U}_i - \underline{U}_i)$$

since $\alpha \in (0, 1)$ (the bliss point cannot be feasible) then, to get an interior solution, we must have $\frac{\partial \mathcal{L}}{\partial \alpha} = 0 \iff \mu_i = 1 / (\bar{U}_i - \underline{U}_i)$. The first order conditions with respect to consumption are

$$\frac{\partial \mathcal{L}}{\partial c_i(y, \zeta)} \Big|_{\zeta_i=1} = 0 \iff \mu_i u'_i(c_i) P(y, \zeta) = q(y, \zeta) P(y, \zeta)$$

therefore, in the planer representation, this equivalent to the Pareto weights being

$$\lambda_i = \mu_i = \frac{1}{\bar{U}_i - \underline{U}_i}.$$

In the CARA-Normal model, let $Y := \sum \zeta_j y_j$. Since $y \sim \mathcal{N}(\mu, \Sigma)$, we have $Y \mid \zeta \sim \mathcal{N}\left(\sum \zeta_j \mu_j, \sum_{i,j} \zeta_i \zeta_j \sigma_{ij}\right)$. Therefore

$$\mathbb{E}_y \left[u_i \left(\sum_i \zeta_j y_j \right) \mid \zeta_i = 1 \right] = -\frac{1}{r} \mathbb{E} [\exp(-rY)] = -\frac{1}{r} M_Y(-r)$$

and $M_Y(t) = \exp\left(\mu_Y t + \frac{t^2}{2} \sigma_Y^2\right) = \exp\left(-r \mu_Y + \frac{r^2}{2} \sigma^2\right) = \exp\left(-r \times n_\zeta \bar{\mu}_\zeta + \frac{r^2}{2} \sum \sigma_{ij}\right)$. In the i.i.d. case, $M_Y(t) = \exp\left(-r n_\zeta \mu + \frac{r^2}{2} \sigma^2 n_\zeta\right) = \exp[n_\zeta (-r \mu + \gamma)]$ and the autarky value is $\mathbb{E}_y [u(y_i)] = -\frac{1}{r} M_{y_i}(-r) = \exp\left(-r \mu + \frac{r^2}{2} \sigma^2\right)$

Therefore

$$\begin{aligned} \bar{U}_i - \underline{U}_i &= \mathbb{E} \left\{ \zeta_i \left[u_i \left(\sum_{j:\zeta_j=1} y_j \right) - u_i(y_i) \right] \right\} = p_i \mathbb{E}_{y,\zeta} \left\{ u_i \left(\sum_{j:\zeta_j=1} y_j \right) - u_i(y_i) \mid \zeta_i = 1 \right\} \\ &= p_i \times \left\{ \exp \left[n_\zeta \left(-r \mu + \frac{r^2}{2} \sigma^2 \right) \right] - \exp \left(-r \mu + \frac{r^2}{2} \sigma^2 \right) \right\} \end{aligned}$$

proving the desired result. \square

Proof of Proposition 8.1. We first need to show the existence of the equilibrium $\bar{\zeta}_i(k_i)$ such that (a) $\zeta_i(k_i) = 0$ for all k_i is the lowest participation equilibrium and (b) there exist thresholds $\bar{k} = (\bar{k}_i)_{i \in [n]}$ and an equilibrium $\bar{\zeta}(k)$ such that $\bar{\zeta}_i(k_i) = 1 \iff k_i \leq \bar{k}_i$, and moreover, $\bar{\zeta}_i(k_i) \geq \zeta_i^*(k_i)$ for all $k \in K^n$, all agents $i \in [n]$, and for any other equilibrium participation $\zeta^*(k)$.

For this, define the incomplete information game

$$\Gamma = \left\{ A_i = \{\zeta_i \in \{0, 1\}\}, U_i(\zeta, k_i) := \zeta_i \mathbb{E}_y \left\{ u \left(\bar{y}_{1+\sum_{j \neq i} \zeta_j} \right) - u(y_i) - k_i \right\} + \mathbb{E}(u(y_i)) \right\}.$$

Because $y_i \sim \mathcal{N}(\mu, \sigma^2)$ and u is increasing and concave, it is easy to show that Γ is supermodular in ζ_i, ζ_{-i} and supermodular in $\eta = -k_i$ (if k_i were common knowledge). This, together with the FOSD ordering assumption, makes Γ a monotone supermodular game of incomplete information (as in Van Zandt and Vives (2007)), which ensures the existence of monotone BNE $\underline{\zeta}, \bar{\zeta}$ such that for any other equilibria $\zeta^*(k)$, we have $\underline{\zeta}_i(k_i) \leq \zeta_i^*(k_i) \leq \bar{\zeta}_i(k_i)$ for all $i, k_i \in K_i$. Since $\zeta_i \in \{0, 1\}$, both $\underline{\zeta}, \bar{\zeta}$ are threshold strategies, $\bar{\zeta}_i(k_i) = 1 \iff k_i \leq \bar{k}_i$ and $\underline{\zeta}_i(k_i) = 1 \iff k_i < \underline{k}_i$. Since $k_i \geq 0$, it is easy to show that the profile where no one attends the market is a BNE of this game and is clearly the lowest. The highest must prescribe market participation at the threshold, which gives us the fix point equation,

Using the implicit function theorem, we know that if $\det(\mathbf{J}_k) \neq 0$ then any solution to fix point of equation $\bar{k} = \Psi(\bar{k}, \epsilon_i)$ satisfies that

$$\frac{\partial \bar{k}}{\partial \epsilon_i} \Big|_{\epsilon_i=0} = \left(\frac{\partial \bar{k}_j}{\partial \epsilon_i} \Big|_{\epsilon_i=0} \right)_{j \in [n]} = (I - \mathbf{J}_k)^{-1} \times \mathbf{F}^{(i)}$$

where $\mathbf{F}^{(i)} = \left(\frac{\partial \Psi_j}{\partial \epsilon_i} \Big|_{\bar{k}, \epsilon = 0} \right)_{j \in [n]}$. For this, knowing that $u(\cdot)$ is differentiable, we have that

$$u(\bar{y}_m + \zeta_i \epsilon / m) = u(\bar{y}_m) + u'(\bar{y}_m) \zeta_i \frac{\epsilon}{m} + \frac{u''(\xi)}{2} \frac{\epsilon^2}{m^2}$$

for some $\xi \in [0, \frac{\epsilon}{m}]$. This implies that

$$\begin{aligned} k_m^*(\epsilon) &:= \mathbb{E}_y [u(\bar{y}_m + \zeta_i \epsilon / m) - u(y_i)] \\ &= \mathbb{E}_y [u(\bar{y}_m) - u(y_i)] + \mathbb{E}_y \left[\underbrace{\zeta_i u'(\bar{y}_m)}_{=q(s)} \right] \frac{\epsilon}{m} + \mathbb{E} \left[\frac{u''(\xi)}{2} \right] \frac{\epsilon^2}{m^2} \end{aligned}$$

and therefore

$$\frac{\partial k_m^*}{\partial \epsilon} \Big|_{\epsilon=0} = \lim_{\epsilon \rightarrow 0} \frac{k_m^*(\epsilon) - k_m^*}{\epsilon} = \mathbb{E}_y \left[\zeta_i \frac{q(s)}{m} \right].$$

Thus,

$$\begin{aligned} \mathbf{F}_{ij} &:= \mathbf{F}_j^{(i)} = \frac{\partial}{\partial \epsilon} \left[\sum_{m \leq n} \mathbb{E}_y [k_m^*(\epsilon)] \times \pi(m, \bar{k} | \bar{k}_j) \right] = \sum_{m \leq n} \mathbb{E}_y \left\{ \frac{\partial k_m^*(\epsilon)}{\partial \epsilon} \Big|_{\epsilon=0} \right\} \times \pi(m, \bar{k} | \bar{k}_j) \\ &= \sum_{m \leq n} \mathbb{E}_y \left[\zeta_i \frac{q(s)}{m} \right] \pi(m, \bar{k} | \bar{k}_j) = \mathbb{E} \left[\zeta_i \zeta_j \frac{q(s)}{n \zeta} \Big| \bar{k}_j \right]. \end{aligned}$$

Finally, the participation effect in this model is the

$$PE = \sum_{m=1}^n mk_m^* \sum_{j=1}^n \frac{\partial \pi(m, \bar{k})}{\partial \bar{k}_j} \times \frac{\partial \bar{k}_j}{\partial \epsilon_i} = \sum_{j=1}^n \frac{\partial \bar{k}_j}{\partial \epsilon_i} \times \underbrace{\left(\sum_{m=1}^n mk_m^* \frac{\partial \pi(m, \bar{k})}{\partial \bar{k}_j} \right)}_{:= \Lambda_j \geq 0 \text{ from FOSD assumption}}$$

proving the desired result. Moreover,

$$\frac{\partial \Psi_j}{\partial k_h} \Big|_{\bar{k}, \epsilon=0} = \frac{\partial}{\partial k_h} \left[\sum_{m \leq n} \mathbb{E}_y k_m^* \times \pi(m, \bar{k} \mid \bar{k}_j) \right] = \sum_{m \leq n} \mathbb{E}_y k_m^* \times \frac{\partial \pi(m, \bar{k} \mid \bar{k}_h)}{\partial k_h} \geq 0$$

and $\frac{\partial \Psi_j}{\partial k_j} \Big|_{\bar{k}, \epsilon=0} = 0$, again using the fact that $k_{-h} \mid k_h$ FOSDs $k_{-h} \mid k'_h$ whenever $k_h \geq k'_h$. \square

Proof of Proposition 9.1. Since $V(t)$ is concave, this program is convex, and satisfies Slater's condition if $T > 0$, and hence the Kuhn-Tucker conditions of this program are both necessary and sufficient. The Lagrangian of program 9.1 is $\mathcal{L}(t, \eta, \nu) = V(t) + \eta(T - \sum_{j \in J} t_j) + \nu_j t_j$. Kuhn-tucker conditions are

- (1) $V_j(t) = \eta - \nu_j$ for all $j \in J$, where $V_j = \partial V / \partial t_j$
- (2) $\nu_j t_j = 0$ for all $j \in J$
- (3) $\nu_j \geq 0$ for all $j \in J$
- (4) $\eta(T - \sum_{j \in J} t_j) = 0$ and $\eta \geq 0$

If at an optimum t^* we have that $t_i^* > 0$ then $V_i(t^*) = \nu$. If $V_j(t^*) < V_i(t^*) = \nu$ then we must have

$$\nu_j = V_i(t^*) - V_j(t^*) > 0$$

implying that $t_j^* = 0$.

To show (2), Propose the following solution: $t_i^* = T$, $t_j^* = 0$ for all $j \neq i$, $\eta = V_i(t^*)$ and $\nu_j = \eta - V_j(t^*)$. Since V is differentiable, its partial derivatives are continuous around $t = 0$. Therefore, $\exists \hat{T}_J > 0$ such that for all $t \in \tau = \{t : \sum_{j \in J} t_j < \hat{T}_J \text{ and } t_j \geq 0 \text{ for all } j \in J\}$ we have $V_i(t) \geq V_j(t)$ for all $j \in J \sim \{i\}$ (since $FC_i \geq FC_j$). Therefore, if $T < \hat{T}$, a solution $t^* \in \tau$, and therefore we have $V_i(t^*) > V_j(t^*)$ for all j , and hence $\nu_j = \eta - V_j(t^*) = V_i(t^*) - V_j(t^*) > 0$; i.e. t^* satisfies the Kuhn-Tucker conditions. To prove uniqueness, suppose there exists another solution $\hat{t} : \sum_{j \in J} \hat{t}_j < \hat{T}$ and $\exists k \neq i$ with $\hat{t}_k > 0$. If that was the case, then $\eta = V_k(\hat{t})$. But because $\sum_{j \in J} \hat{t}_j < \hat{T}$ we also have that $V_i(\hat{t}) > V_k(\hat{t})$. Therefore, $V_i(\hat{t}) + \nu_i \geq V_i(\hat{t}) > V_k(\hat{t}) = \eta$, violating condition (1). Therefore, the only solution to 9.1 is $t = t^*$. \square

Proof of Proposition 9.2. For part 1, the Lagrangian for this problem (given a vector of transfers $t^* \in \mathbb{R}_+^n$, and assuming $c_i(s) > 0$ in the optimum) is

$$\mathcal{L} = \mathbb{E}_s \left\{ \sum_i \lambda_i u_i(c_i) + q(s) \left[\sum_i \zeta_i (y_i + t_i - c_i) \right] \right\}$$

The first order conditions under the assumption that $y \perp \zeta$, homogeneous CARA preferences and Gaussian income draws are the same as before, with $c_i = r^{-1} \ln(\lambda_i) - r^{-1} \ln[q(s)]$, but now $q(s)$ satisfies

$$r^{-1} \sum_{i=1}^n \zeta_i \{\ln(\lambda_i) - \ln[q(s)]\} = \sum_{i=1}^n \zeta_i (y_i + t_i) \iff$$

$$\ln(\bar{\lambda}_\zeta) - \ln[q(s)] = \bar{y}_\zeta + \bar{t}_\zeta \iff q(s) = \bar{\lambda}_\zeta \exp(-r\bar{y}_\zeta) \exp(-r\bar{t}_\zeta)$$

which then implies that $\mathbb{E}_y[q(s) | \zeta] = \mathbb{E}[\bar{\lambda}_\zeta \exp(-r\bar{y}_\zeta) \exp(-r\bar{t}_\zeta)] = h(\zeta) \exp(-r\bar{t}_\zeta)$, where $h(\zeta) = \bar{\lambda}_\zeta \exp(-r\bar{\mu}_\zeta) \exp(r^2\bar{\sigma}_\zeta^2/n_\zeta)$. Therefore,

$$V_i(t) = \frac{\partial \mathcal{L}}{\partial t_i} |_{t=t^*} = \mathbb{E}_s \{\zeta_i q(s)\} = \mathbb{E}_\zeta [\zeta_i h(\zeta) \exp(-r\bar{t}_\zeta)].$$

For part 2, suppose, by contradiction, that $t_j^* > t_i^*$. Based on Proposition 9.1, since $t_i^* > 0$ then $V_i(t^*) = V_j(t^*) = v^*$. See we can rewrite $V_i(t^*)$ as

$$V_i(t^*) = \sum_{A \subseteq I \setminus \{i,j\}} \mathbb{P}(\zeta^{i,A}) h(\zeta^{i,A}) \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t_k^*\right) \exp\left(-\frac{r}{1+|A|} t_i^*\right)$$

$$+ \sum_{B \supseteq \{i,j\}} \mathbb{P}(\zeta^{i,A}) h(\zeta^{i,A}) \exp(-r\bar{t}_\zeta)$$

and analogously

$$V_j(t^*) = \sum_{A \subseteq I \setminus \{i,j\}} \mathbb{P}(\zeta^{j,A}) h(\zeta^{j,A}) \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t_k^*\right) \exp\left(-\frac{r}{1+|A|} t_j^*\right)$$

$$+ \sum_{B \supseteq \{i,j\}} \mathbb{P}(\zeta^{i,A}) h(\zeta^{i,A}) \exp(-r\bar{t}_\zeta).$$

Therefore

$$V_j(t^*) - V_i(t^*) = \sum_{A \subseteq I \setminus \{i,j\}} \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t_k^*\right) [\mathbb{P}(\zeta^{j,A}) h(\zeta^{j,A}) \exp\left(-\frac{r}{1+|A|} t_j^*\right)$$

$$- \mathbb{P}(\zeta^{i,A}) h(\zeta^{i,A}) \exp\left(-\frac{r}{1+|A|} t_i^*\right)]$$

$$\stackrel{(i)}{\leq} \sum_{A \subseteq I \setminus \{i,j\}} \exp\left(-\frac{r}{1+|A|} \sum_{k \in A} t_k^*\right) \mathbb{P}(\zeta^{i,A}) h(\zeta^{i,A}) \left[\exp\left(-\frac{r}{1+|A|} t_j^*\right) - \exp\left(-\frac{r}{1+|A|} t_i^*\right)\right]$$

$$\stackrel{(ii)}{\leq} 0$$

using in (i) that $\mathbb{P}(\zeta^{i,A}) h(\zeta^{i,A}) > \mathbb{P}(\zeta^{j,A}) h(\zeta^{j,A})$ for all $A \subseteq I \setminus \{i,j\}$ and in (ii) the initial assumption that $t_j^* > t_i^*$. For the second result, if $t_i^* = t_j^*$ then we should have $\mathbb{P}(\zeta^{i,A}) h(\zeta^{i,A}) = \mathbb{P}(\zeta^{j,A}) h(\zeta^{j,A})$ for all $A \subseteq I \setminus \{i,j\}$. As long as one such subset exists with strict inequality, gives the desired result that $t_i^* > t_j^*$. \square

Online Appendix: Not for Publication

APPENDIX B. GENERALIZED POISSON MODEL

This section provides a detailed exposition of the generalized Poisson model.

Let $z_i \in [0, 1]$ denote the probability that an agent gets selected as the host. Then let \mathbf{p} denote a matrix with entries $\mathbf{p}_{i,j}$ denoting the probability that j is in the market when i is the host, which is independent across j . We set $\mathbf{p}_{i,i} = 1$.

It is useful to define an individual specific parameter, which is the expected number of individuals in the trading room when i is selected as host, ν_i . This can be computed as $\nu_i := \sum_j \mathbf{p}_{ij}$. To characterize financial centrality, we need to know the expected sizes of the trade rooms when i is host and conditional on i being in the room, integrating across the other possible hosts. Two auxiliary random variables will be very useful in the rest of the section: $\mathbf{n}_{-i} = (n_\zeta - 1) \mid i$ is host, and $\mathbf{n}_{-j,i} = (n_\zeta - 2) \mid j$ hosts & $\zeta_i = 1$. This means that whenever i hosts, market size is $n_\zeta = 1 + \mathbf{n}_{-i}$, and whenever j hosts, and we condition on i accessing the market, then market size is $n_\zeta = 2 + \mathbf{n}_{-j,i}$. This auxiliary random variables have range from 0 to $k \in \{n - 1, n - 2\}$, and based on our assumptions, we have

$$\mathbf{n}_{-i} \sim \sum_{k \neq i} \text{Bernoulli}(\mathbf{p}_{i,k}) \quad \text{and} \quad \mathbf{n}_{-ji} \sim \sum_{k \notin \{j,i\}} \text{Bernoulli}(\mathbf{p}_{j,k}),$$

where these Bernoulli distributions are independent, with success probabilities strictly less than 1.²⁴ These distributions are also called *Poisson Binomial distributions* and have been extensively studied in the literature. We will write $X \sim PB(\mathbf{p})$ with $p = (p_1, p_2, \dots, p_k)$ the vector of success probabilities of each Bernoulli trial. This distribution, in some cases, can be well approximated by a Poisson distribution.²⁵ In this model, we have that $\mathbf{n}_{-i} \sim PB(\mathbf{P}_i)$ and $\mathbf{n}_{-ji} \sim PB(\mathbf{P}_{-j,i})$ where $\mathbf{P}_i = (\mathbf{p}_{i,k})_{k \neq i} \in [0, 1]^{n-1}$ and $\mathbf{P}_{-j,i} = (\mathbf{p}_{j,k})_{k \notin \{i,j\}} \in [0, 1]^{n-2}$. These random variables are useful to write our approximation to financial centrality as

$$\begin{aligned} FC_i &\approx \hat{FC}_i := \mathbb{P}(\zeta_i = 1) \times \left[1 + \gamma \mathbb{E} \left(\frac{1}{n_\zeta} \mid \zeta \right) \right] \\ &= p_i \left[1 + \gamma z_i \mathbb{E} \left(\frac{1}{1 + \mathbf{n}_{-i}} \right) + \gamma \sum_{j \neq i} z_j \mathbb{E} \left(\frac{1}{2 + \mathbf{n}_{-ji}} \right) \right], \end{aligned}$$

where $p_i = \mathbb{P}(\zeta_i = 1)$. Therefore, we need to calculate the inverse moments $\mathbb{E}(1/(1 + X))$ and $\mathbb{E}(1/(2 + X))$ for n Poisson Binomial random variables; $X = \mathbf{n}_{-i}$ and $X = \mathbf{n}_{-ji}$ for all $j \neq i$. Hong (2013) provides a general survey on the commonly used methods to calculate explicitly the probability function of Poisson Binomial distributions using either recursive or Discrete Fourier

²⁴Exact and approximation methods for calculating expectations of market sizes are sensitive to the assumption of interior (i.e., in $(0, 1)$) success probabilities. This is the reason for the need to define the random variables \mathbf{n}_{-i} and \mathbf{n}_{-ji} .

²⁵Le Cam (1960) provided bounds on the error of approximation, which were improved by Stein (1986); Chen (1975), and Barbour and Hall (1984); Sason (2013) show that if \hat{X} is the poisson approximation (with mean $\lambda = \sum_i p_i$), then $d_{TV}(X, \hat{X}) \leq (1 - e^{-\lambda}) \sum_i p_i^2 / \lambda$, where $d_{TV}(\cdot)$ denotes the total variation distance. This approximation will then typically be valid when its expected value is not too large.

Transform methods, which are fairly fast even with large n .²⁶ We also survey results (starting with [Le Cam \(1960\)](#)) that show that if the expected number of successes of a Poisson Binomial distribution is sufficiently low (corresponding in this case with low expected market sizes), then it can be well approximated by a Poisson distribution.²⁷ In the context of this model, it means that if $\mathbb{E}(\mathbf{n}_{-i}) = \nu_i - 1$ is small (relative to n), then we can approximate $\mathbf{n}_{-i} \sim \text{Poisson}(\nu_i - 1)$ and $\mathbf{n}_{-ji} \sim \text{Poisson}(\nu_{-ji})$, where $\nu_{-ji} = \mathbb{E}(n_{-ji}) = \sum_{k \notin \{i,j\}} \mathbf{p}_{jk} = \nu_j - \mathbf{p}_{ji} - 1$.

If $X \sim \text{Poisson}(\nu - 1)$, then $\mathbb{E}(1 + X)^{-1} = m_1(\nu) := [1 - \exp(-\nu)] / (\nu - 1)$ and $\mathbb{E}(2 + X)^{-1} = m_2(\nu) := [1 - m_1(\nu)] / (\nu - 1)$, both strictly decreasing functions of $\nu \geq 1$. Using these formulas, we can then approximate \hat{FC}_i by:

$$\hat{FC}_i \approx p_i \times \left\{ 1 + \gamma z_i m_1(\nu_i) + \gamma \sum_{j \neq i} z_j m_2(\nu_j - \mathbf{p}_{j,i}) \right\}.$$

This shows the following. First, nodes with a larger expected reach as measured by ν_i are more central (as long as n is large enough relative to γ). Second, nodes that have larger expected inverse room size when they are hosts are more central. Third, i is more central when $\mathbf{p}_{j,i}$ increases, particularly when ν_j is small. So when j tend to invite small rooms as hosts, but i is likely to be in such a j 's room, then i is more valuable.

A special case are symmetric models, where $z_i = 1/n$ for all i and $\mathbf{p}_{i,j} = \mathbf{p}_{j,i}$ (e.g., the model $\mathbf{p}_{i,j} = \alpha^{\delta(i,j)}$, since distance is symmetric). In this case

$$FC_i = \frac{1}{n} \nu_i \left\{ 1 + \gamma \times \frac{1}{n} m_1(\nu_i) + \gamma \sum_{j \neq i} \frac{1}{n} m_2(\nu_j - \mathbf{p}_{j,i}) \right\}.$$

This has the advantage that the centrality of agent i depends solely on the expected market size of each agent (as a host) that she gets connected to, and the probability that she connects to them. The marginal value of the inverse room size effect when i is the host, proportional to $\nu_i \times m_1(\nu_i)$, declines in ν_i if and only if $\nu_i \geq 2.79$ (there is a positive effect in $\mathbb{P}(\zeta_i = 1)$, but an offsetting negative effect in $m_1(\nu_i)$).

If we want to calculate centrality exactly, we can still use the calculation of the exact pdf of \mathbf{n}_{-i} and \mathbf{n}_{-ji} to get the exact financial centrality. For example, in the CARA-Normal model with homogeneous preferences and independent and identically distributed income draws, we know that $FC_i = \mathbb{E}_\zeta \{ \zeta_i \exp(\gamma/n_\zeta) \}$, which can be decomposed as

$$FC_i = p_i \left\{ z_i \mathbb{E} \left[\exp \left(\frac{\gamma}{1 + \mathbf{n}_{-i}} \right) \right] + \sum_{j \neq i} z_j \mathbb{E} \left[\exp \left(\frac{\gamma}{2 + \mathbf{n}_{-ji}} \right) \right] \right\}$$

and then be calculated explicitly using the distributions for \mathbf{n}_{-i} and \mathbf{n}_{-ji} .

²⁶[Chen and Liu \(1997\)](#) show stable (i.e. non-alternating) methods are $O(n^2)$, which would make the calculation of financial centrality of a given agent be $O(n^3)$. Discrete Fourier Methods are usually much faster ([Fernández and Williams \(2010\)](#)). See [Hong \(2013\)](#) for a general survey on the existing exact and approximating methods.

²⁷This is not the only approximation studied in the literature. In models where the expected market size is high, Gaussian approximations behave rather well (see [Volkova \(1996\)](#), [Hong \(2013\)](#)). If success probabilities are similar (i.e., the variance $\sigma_p^2 := n^{-1} \sum_i (p_i - \bar{p})^2$ is small enough) then approximation to a Binomial distribution is fairly accurate ([Ehm \(1991\)](#); [Barbour et al. \(1992\)](#)).

APPENDIX C. EXTENSIONS

In this section we study two extensions that depart from the class of environments above. One of the most seemingly important restrictions on the models studied so far is the existence of *centralized markets*. That is, agents either are in autarky or have market access and can trade with any other agent that also has market access. While the bilateral trading chains introduced above relaxes this interpretation, it maintains the possibility that any agent is reachable by any other through a finite sequence of trades, as long as both have market access. In Section C.1 we introduce a generalization of the basic environment, allowing for the existence of several *segmented markets* working in parallel, where agents can only trade among a subset of all agents who have market access. That is, a draw from the stochastic financial network consists of a collection of subgraphs (cliques). For example $ijkl$ and $mnop$ may be two cliques of four who can exchange with each other in some state of the world. But in another state of the world, perhaps the cliques are ij , kl , mno , and p (a singleton). Each clique is a segmented market. We show that the basic definitions and formulas of financial centrality still hold, if we reinterpret having “market access” to be present in the market where the agent being injected with liquidity is trading at.

Another important assumption maintained throughout this paper is that the social planner evaluating the marginal value of injected liquidity also is able to implement the allocation $c(\cdot)$ that maximizes her expected utility. However, a relevant case is one where the planner can only influence the economy by the liquidity injection policies introduced in proposed in Section 3.1 and cannot directly choose the allocation herself. This would be the case when the allocation is chosen according to some other solution concept, like Walrasian Equilibrium, multi-player bargaining games, and so on. In such situations, the social planner would have to take the consumption allocation as given when measuring the marginal effects of injecting liquidity in this economy. In Section C.2 we study financial centrality under the assumption that the consumption allocation is Pareto optimal, which implies that there exist some representing social preferences (i.e., Pareto weights) for which it would be optimal. We then obtain similar expressions for financial centrality, which now incorporates a term relating the Pareto weights of the social planner with the representative Pareto weights of the allocation.

C.1. Segmented Markets. We consider an environment with the same income shocks and preferences, but one where agents may gain access to random, segmented markets. Formally, a *market segmentation* is a partition $\pi = \{m_1, m_2, \dots, m_r\}$ over the set of agents I ; i.e., $\cup_{m \in \pi} m = I$ and $m \cap m' = \emptyset$ for all $m \neq m'$. In this alternative environment, the relevant state of nature is now $s = (y, \pi)$, where π is the *market segmentation state*, with probability distribution $P(s)$. We refer to each $m \in \pi$ as a *market* at state s . Let \mathcal{P} be the set of all partitions of I that have positive probability under $P(s)$. We denote $m(i, \pi) \in \pi$ to be the market (at segmentation π) where i is able to trade. If $m(i, \pi) = \{i\}$ we say i is in autarky at π , and otherwise we say i has *market access* at π .

Segmented markets now modify the definition of feasibility of allocations. We say that an allocation $c = (c_i(s))_{i \in I}$ is feasible if and only if, for all $s = (y, \pi)$ and all $m \in \pi$ we have $\sum_{i \in m} c_i(s) \leq \sum_{i \in m} y_i$. Clearly, the class of environments embeds the single market environments

studied before—i.e., markets where any partition π in the support is made up of a single multi-agent market $m_u(\pi) \subseteq I$ with $\#m_u(\pi) \geq 1$, and everyone else being in autarky. Hence we can summarize the state by $s = (y, \zeta)$ where $\zeta_i = 1$ if and only if $i \in m_u(\pi)$. In general, for a given partition π we write $\zeta_i^m \in \{0, 1\}$ for the indicator of whether i has access to market m .

Given Pareto weights $\lambda \in \Delta$ and agent $i \in I$, the planner's problem value function of injecting liquidity $t_i \geq 0$ to agent i is $V^s(t_i) := \max_{(c_j(y, \pi))_{j \in I}} \mathbb{E}_s \left\{ \sum_{j \in I} \lambda_j u_j [c_j(s)] \right\}$ subject to $\sum_{j \in m} c_j(s) \leq \sum_{j \in m} y_j + t_i \zeta_i^m$ for all $s = (y, \pi)$ and all $m \in \pi$. Financial centrality is now defined as before. Intuitively, a planner needs to integrate also over all possible market segmentations in order to assess the marginal value of the liquidity injection policy for agent i , since the shadow value of the injection will depend on the market agent i is trading at. We show that the financial centrality measure follows the same formula as in the centralized markets environments, in a “virtual single market economy” where having market access is understood as being able to trade with the agent of interest.

Definition C.1. Take a segmented market economy \mathcal{E} , with distribution over states $\mathbf{P}(y, \pi)$. Define \mathcal{E}_i to be a *virtual single market economy* where all agents have identical preferences over consumption, and the distribution over outcomes $\tilde{\mathbf{P}}(y, \zeta)$ is given by:

$$(C.1) \quad \tilde{\mathbf{P}}(y, \zeta) = \mathbf{P} \left((y, \pi) \in Y \times \mathcal{P} : \begin{cases} (1) : j \in m(i, \pi) \text{ for all } j : \zeta_j = 1 & \text{and} \\ (2) : \#m(i, \pi) > 1 \end{cases} \right),$$

i.e., an agent $j \neq i$ has market access on economy \mathcal{E}_i only when she is able to trade (i.e., in the same market) with agent i in \mathcal{E} .

Proposition C.1 asserts that financial centrality in a segmented markets economy follows the same “asset pricing formula” we had in Proposition 3.3, but on the virtual single market economy \mathcal{E}_i . The proof is quite straightforward, and simply generalizes the proof of Proposition 3.3 and is therefore omitted.

Proposition C.1. *Suppose $y \perp \pi$. Let \mathcal{E} be a segmented markets economy and $i \in I$. Then, for any $\lambda \in \Delta$, the financial centrality for agent i coincides with the financial centrality of agent i in the virtual single market economy \mathcal{E}_i . That is,*

$$FC_i := \frac{\partial V^s(t)}{\partial t_i} \Big|_{t=0} = \mathbb{E}_{s=(y, \zeta)}^{\tilde{\mathbf{P}}} \{ \zeta_i q(s) \}$$

where $\mathbb{E}^{\tilde{\mathbf{P}}}(\cdot)$ is the expectation taken w.r.t measure $\tilde{\mathbf{P}}$ defined in C.1.

Intuitively, financial centrality only deals with the effect of the increase in agent's i endowment, which can only impact those agents who can trade with her. Because of separability of the planner's preferences over different agents consumptions, the marginal welfare effect on the segmented markets i is trading on have no effect on the welfare evaluation of other segmented markets at the same time. Therefore, whether agents not trading with i are either trading among themselves, or in autarky, is irrelevant when evaluating the policy. Moreover, any two states with generate the same

segmented market for agent i are equivalent from the point of view of the planner when evaluating this policy. This result is easily generalized for endogenous market participation economies.

C.2. Passive Planners. In this section, we consider the original environment, but assume the consumption allocation is a primitive of the model (e.g., being determined by a Walrasian Equilibrium or a bargaining protocol). In this setup, the social planner can only influence the allocation by making the proposed liquidity injections of Section 3.1. If the social planner has preferences given by $V = \mathbb{E} [\sum \lambda_i u_i(c_i)]$, and agents consume according to a (differentiable) allocation $c(\cdot)$, financial centrality is defined as

$$FC_i = \mathbb{E}_s \left\{ \zeta_i \sum_{j:\zeta_j=1} \lambda_j u'_j [c_j(s)] \frac{\partial c_j}{\partial y_i}(s) \right\}.$$

An important case is where $c(\cdot)$ is a (constrained) Pareto optimal allocation; i.e., there exists a *representing Pareto weight* vector φ such that $c(\cdot)$ solves problem 2.1 with φ instead of λ . Also, let $q(s)$ be the usual normalized Lagrange multiplier of the resource constraint at state s , for this φ - planner problem. It is easy to show (see below) that financial centrality in this setting is

$$(C.2) \quad FC_i = \mathbb{E}_s \left\{ \zeta_i q(s) \left[\sum_{j:\zeta_j=1} \rho_j \frac{\partial c_j(s)}{\partial y_i} \right] \right\},$$

where $\rho_j := \lambda_j / \varphi_j$ ²⁸.

A special case is when the consumption allocation satisfies $\partial c_j / \partial y_i = n_\zeta^{-1}$ whenever $\zeta_j = \zeta_i = 1$. This is the case in the CARA model with homogeneous preferences, even if income draws are not normal (see below). Whenever this happens, equation C.2 can be simplified to

$$FC_i = \mathbb{E}_s \{ \zeta_i q(s) \times \bar{\rho}_\zeta \},$$

where $\bar{\rho}_\zeta = n_\zeta^{-1} \sum \zeta_j \rho_j$ is the arithmetic mean of the Pareto weights ratio, and $q(s)$ is the Lagrange multiplier in the Pareto problem with weights φ . In the CARA-Normal model this then translates into

$$FC_i = \mathbb{E}_\zeta \left\{ \zeta_i \bar{\rho}_\zeta \exp(-r\bar{\mu}_\zeta) \exp\left(\frac{r^2 \bar{\sigma}_\zeta^2}{2 n_\zeta}\right) \times \bar{\rho}_\zeta \right\},$$

which is the same formula as in Section 4.2, but with an extra term, $\bar{\rho}_\zeta := n_\zeta^{-1} \sum \zeta_j (\lambda_j / \varphi_j)$ which is the mean of relative Pareto weights. Another important case where $\partial c_j / \partial y_i = n_\zeta^{-1}$ is an environment where agents are homogeneous preferences and identical and independently distributed random draws. If the allocation comes from a Walrasian equilibrium, we know that the representing Pareto weight is $\varphi_j = 1$ for all j (see Proposition D.2), and therefore $FC_i = \exp(-r\mu) \mathbb{E}_\zeta \left\{ \zeta_i \exp\left(\frac{r^2 \sigma_\zeta^2}{2 n_\zeta}\right) \times \bar{\lambda}_\zeta \right\}$, where $\bar{\lambda}_\zeta := n_\zeta^{-1} \sum \zeta_j \lambda_i$ is now the mean of the Pareto weight of the social planner. In the baseline case of Section 4, with homogeneous preferences, i.i.d. income draws and a representing Pareto weight $\varphi_j = 1$ for all j (so $c_i = \bar{y}$ if $\zeta_i = 1$) we can approximate the centrality measure to $FC_i \approx \mathbb{E}_\zeta \left\{ \zeta_i \left(1 + \gamma \frac{\sigma_\zeta^2}{n_\zeta}\right) \bar{\lambda}_\zeta \right\}$, which resembles the centrality measure obtained in Subsection D for CES and CARA preferences.

²⁸Of course, when $\lambda = \varphi$ we have $\rho_j = 1$ for all j , and since $\sum_{j:\zeta_j=1} \partial c_j(s) / \partial y_i = 1$ for all $s : \zeta_i = 1$, we recover the usual formula in this case.

Proof. First, we want to show equation C.2. For that, we use again the first order conditions of planner's problem 2.1 but representing Pareto weights $\varphi \geq 0$: $\varphi_j u'_j(c_j(s)) = q(s) \iff \lambda_j u'_j(c_j(s)) = \rho_j q(s)$ where $\rho_j = \lambda_j/\varphi_j$. Using this in the original definition of centrality in this setup, we get

$$\begin{aligned} FC_i &= \mathbb{E}_\zeta \left\{ \zeta_i \sum_{j:\zeta_j=1} \lambda_j u'_j(c_j(s)) \frac{\partial c_j(s)}{\partial y_i} \right\} = \mathbb{E}_\zeta \left\{ \zeta_i \sum_{j:\zeta_j=1} \rho_j q(s) \frac{\partial c_j(s)}{\partial y_i} \right\} \\ &= \mathbb{E}_\zeta \left\{ \zeta_i q(s) \sum_{j:\zeta_j=1} \rho_j \frac{\partial c_j(s)}{\partial y_i} \right\}, \end{aligned}$$

showing the desired result. Also, because the resource constraint is always binding at every state s , we have the identity $\sum_{j:\zeta_j=1} c_j(s) = \sum_{j:\zeta_j=1} y_j$, which at states $s : \zeta_i = 1$ implies that $\sum_{j:\zeta_j=1} \partial c_j(s)/\partial y_i = 1$. Therefore, if $\lambda = \varphi$, then $\rho_j = 1 \forall j$, $q(s)$ is the multiplier for the Pareto problem with Pareto weights $\lambda = \varphi$ and hence, $FC_i = \{\zeta_i q(s)\}$, like we had above. \square

We now study the special case of the CARA-Normal model with homogeneous preferences and a representing Pareto weight vector φ . We know (see D) that in this model, $c_j(s) = r^{-1} \ln(\varphi_j/\bar{\varphi}_\zeta) + \bar{y}$, where $\bar{\varphi}_\zeta = \exp(n_\zeta^{-1} \sum \zeta_j \ln \varphi_j)$. This then means that whenever $\zeta_i = \zeta_j = 1$, we have $\partial c_j(s)/\partial y_i = n_\zeta^{-1}$. Moreover, we also showed that in this environment, $q(s) = \bar{\varphi}_\zeta \exp(-r\bar{y})$. Therefore, using C.2 we get $FC_i = \mathbb{E}_s \left\{ \zeta_i \bar{\varphi}_\zeta \exp(-r\bar{y}) \times \bar{\rho}_\zeta \right\}$, where now $\bar{\rho}_\zeta := n_\zeta^{-1} \sum \zeta_j \rho_j$ is the arithmetic mean of relative Pareto weights. Using the assumption $y \perp \zeta$, we can then rewrite it as

$$\mathbb{E}_s \left\{ \zeta_i \bar{\varphi}_\zeta \exp(-r\bar{\mu}_\zeta) \exp\left(\gamma \frac{\bar{\sigma}_\zeta^2}{n_\zeta}\right) \times \bar{\rho}_\zeta \right\}.$$

APPENDIX D. WALRASIAN EQUILIBRIUM WITHOUT TRANSFERS

Following the definitions in Subsection 6.1, and given a (normalized) price function $r \in \Delta(S)$, we can simplify the consumer's problem by just choosing consumption to maximize utility, given only one "expected" budget constraint. Formally, agent $i \in \{1, \dots, n\}$ solves

$$(D.1) \quad \begin{aligned} V_i(q) &:= \max \mathbb{E}_s \{ \zeta_i u_i [c_i(s)] + (1 - \zeta_i) u_i (y_i) \} \\ &\text{subject to: } \mathbb{E}_s [\zeta_i c_i(s) r(s)] \leq \mathbb{E}_s [\zeta_i y_i r(s)]. \end{aligned}$$

As we did when defining the Lagrange multipliers for the planning problem, we normalize the price function as $q(s) P(s) = \hat{q}(s)$, where \hat{q} is the actual price measure. A Walrasian equilibrium is a pair $(c, q) = \left(\{c_i(s)\}_{i \in I, s \in S}, \{q(s)\}_{s \in S} \right)$ such that

- $\{c_i(s)\}_{s \in S}$ solves D.1 given prices $q(s)$
- and markets clear at all states: $\sum_i \zeta_i c_i(s) \leq \sum_i \zeta_i y_i$. for all $s = (y, \zeta)$.

Proposition 6.1 implies there exists a vector λ such that the equilibrium allocation solves the planning problem (2.1), and such that the normalized prices satisfy $r(s) = q(s)$, where $q(s)$ are the normalized Lagrange multipliers of the resource constraint at state s . Following Negishi (1960) and more recently Echenique and Wierman (2012), we can then solve for the equilibrium allocation by finding the Pareto weights that satisfy the budget constraints for all agents. Formally, let $c_i^*(s | \lambda)$ be the optimal consumption allocation in the planning problem with weights λ , and $q^*(s | \lambda)$ the Lagrange multipliers (normalized by the probabilities of each state). Then, a Pareto weight vector λ corresponds to a Walrasian equilibrium allocation if and only if

$$(D.2) \quad \mathbb{E}_s [\zeta_i c_i^*(s | \lambda) q^*(s | \lambda)] = \mathbb{E}_\zeta [\zeta_i y_i q^*(s | \lambda)] \text{ for all } i = 1, 2, \dots, n.$$

The next proposition characterizes the Pareto weights equation for the CARA-Normal case.

Proposition D.1. *Suppose $u_i(c) = -r_i^{-1} \exp(-r_i c)$ and $y \sim \mathcal{N}(\mu, \Sigma)$. Let $\bar{r}_\zeta := \left(\frac{1}{n_\zeta} \sum \zeta_i r_i^{-1} \right)^{-1}$ be the harmonic mean of risk aversion in market ζ , and $\bar{\lambda}_\zeta := \exp \left[\frac{1}{n_\zeta} \sum_i \zeta_i (\bar{r}_\zeta / r_i) \ln(\lambda_i) \right]$ be the average Pareto weight in the market, weighted by the relative risk aversion. Also, let $\Sigma_{i,\zeta} := \sum_j \zeta_j \sigma_{ij}$. Then the Pareto weight vector λ solving (D.2) satisfies*

$$(D.3) \quad \ln(\lambda_i) = \frac{\mathbb{E}_\zeta \left\{ \zeta_i \left[\ln(\bar{\lambda}_\zeta) + \left(r_i \mu_i - \bar{r}_\zeta \bar{\mu}_\zeta \right) - \frac{\bar{r}_\zeta}{n_\zeta} \left(r_i \Sigma_{i,\zeta} - \bar{r}_\zeta \bar{\sigma}_\zeta^2 \right) \right] \eta(s) \right\}}{\mathbb{E}_\zeta \{ \zeta_i \eta(s) \}}$$

for $i = 1, \dots, n$, where $\eta(s) := \bar{\lambda}_\zeta \exp \left(-\bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta^2}{2} \frac{\bar{\sigma}_\zeta^2}{n_\zeta} \right)$.

Proof. From the first order conditions under CARA preferences, we get

$$(D.4) \quad \lambda_i \exp(-r_i c_i) = q(s) \iff c_i = \frac{1}{r_i} \ln(\lambda_i) - \frac{1}{r_i} \ln(q(s))$$

and that

$$q(s) = \bar{\lambda}_\zeta \exp(-\bar{r}_\zeta \bar{y}) = \bar{\lambda}_\zeta \exp(-\bar{r}_\zeta \bar{y}).$$

Using the first order conditions again, whenever $\zeta_i = 1$ we get

$$(D.5) \quad c_i(s) = \frac{\ln(\lambda_i/\bar{\lambda}_\zeta)}{r_i} + \frac{\bar{r}_\zeta}{r_i} \bar{y}(s).$$

Then, the value of the consumption allocation, at prices $q(s)$ is

$$\begin{aligned} \mathbb{E}_s \{ \zeta_i c(s) q(s) \} &= \mathbb{E}_s \left[\zeta_i \frac{\ln(\lambda_i)}{r_i} q(s) \right] - \mathbb{E}_s \left\{ \zeta_i \frac{\ln(\bar{\lambda}_\zeta)}{r_i} q(s) \right\} + \mathbb{E}_s \left\{ \zeta_i \frac{\bar{r}_\zeta}{r_i} \bar{y}(s) q(s) \right\} \\ &= \frac{1}{r_i} \ln(\lambda_i) FC_i(\lambda) - \frac{1}{r_i} \mathbb{E}_s \left\{ \zeta_i \bar{\lambda}_\zeta \left[\ln(\bar{\lambda}_\zeta) + \bar{r}_\zeta \bar{y} \right] \exp(-\bar{r}_\zeta \bar{y}) \right\} \end{aligned}$$

where $\mathbb{E}_y(-\bar{r}_\zeta \bar{y}) = \exp(-\bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta \bar{\sigma}_\zeta^2}{2})$ as we have seen before. Moreover

$$\mathbb{E}_s [\bar{r}_\zeta \bar{y} \exp(-\bar{r}_\zeta \bar{y})] = \mathbb{E}_\zeta \left[\left(\bar{r}_\zeta \bar{\mu}_\zeta - \frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{n_\zeta} \right) \exp \left(-\bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{2 n_\zeta} \right) \right].$$

On the other hand, the value of agent i 's income stream is

$$w_i = \mathbb{E}_\zeta [\zeta_i y_i q(s)] = \mathbb{E}_s \left\{ \zeta_i \bar{\lambda}_\zeta y_i \exp[-\bar{r}_\zeta \bar{y}(s)] \right\}.$$

Using the moment generating function $M_y(t) = \mathbb{E}_y[\exp(t'y)] = \exp(t'\mu + \frac{1}{2}t'\Sigma t)$, we get $\mathbb{E} \left[y_i \exp(-\bar{r}_\zeta \bar{y}_\zeta) \right] = \frac{\partial M}{\partial t_i} \Big|_{t = -\frac{\bar{r}_\zeta}{n_\zeta} \mathbf{1}}$ where $\mathbf{1}$ is a vector of 1's, so that $t'y = \bar{r}_\zeta \bar{y}$. This then implies that $\mathbb{E}_y [y_i \exp(-\bar{r}_\zeta \bar{y})] = \left(\mu_i - \frac{\bar{r}_\zeta}{n_\zeta} \Sigma_{i,\zeta} \right) \exp(-\bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta}{2n_\zeta} \bar{\sigma}_\zeta^2)$, where $\Sigma_{i,\zeta} := \sum_j \zeta_j \sigma_{ij}$. Putting all this results together, we can write the budget constraint as

$$\mathbb{E}_s [\zeta_i (c_i - y_i) q(s)] = 0$$

if and only if

$$\begin{aligned} &r_i^{-1} \ln(\lambda_i) FC_i(\lambda) - r_i^{-1} \mathbb{E}_\zeta \left\{ \zeta_i \bar{\lambda}_\zeta \left[\ln(\bar{\lambda}_\zeta) - \bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{n_\zeta} \right] \exp \left(-\bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{2n_\zeta} \right) \right\} \\ &= \mathbb{E}_\zeta \left\{ \zeta_i \bar{\lambda}_\zeta \left(\mu_i - \frac{\bar{r}_\zeta}{n_\zeta} \Sigma_{i,\zeta} \right) \exp \left(-\bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{2n_\zeta} \right) \right\} \end{aligned}$$

and so

$$\ln(\lambda_i) FC_i(\lambda) = \mathbb{E}_\zeta \left\{ \zeta_i \bar{\lambda}_\zeta \left[\ln(\bar{\lambda}_\zeta) + (r_i \mu_i - \bar{r}_\zeta \bar{\mu}_\zeta) - \frac{\bar{r}_\zeta}{n_\zeta} (r_i \Sigma_{i,\zeta} - \bar{r}_\zeta^2 \bar{\sigma}_\zeta^2) \right] \exp \left(-\bar{r}_\zeta \bar{\mu}_\zeta + \frac{\bar{r}_\zeta^2 \bar{\sigma}_\zeta^2}{2n_\zeta} \right) \right\}.$$

□

Observe that the denominator has $\mathbb{E}_\zeta \{ \zeta_i \eta(s) \} = FC_i$. Also, because $\lambda \in \Delta$, we have $\ln(\lambda_i)$ and $\ln(\bar{\lambda}_\zeta) < 0$, which implies that if we could, somehow, increase FC_i without affecting the numerator of the right hand side of (D.3), we would increase λ_i in the fixed point equation. An important corollary of Proposition D.1 is the proof of Proposition 3.1, since we would have $\bar{r}_\zeta = r_i = r$ for all ζ , and the fact that incomes are identically distributed and independent imply $\bar{\sigma}_\zeta^2 = \sigma^2$, $\bar{\mu}_\zeta = \mu$

and $\Sigma_{i,\zeta} = \sigma^2$. This simplifies the fixed point equation as

$$\ln(\lambda_i) FC_i(\lambda) = \mathbb{E}_\zeta \left\{ \zeta_i \bar{\lambda}_\zeta \ln(\bar{\lambda}_\zeta) \exp\left(-r\mu + \frac{r^2 \sigma^2}{2 n_\zeta}\right) \right\}$$

to which a solution is $\lambda_i = 1/n$. We summarize this result in Proposition D.2.

Proposition D.2. *Suppose $u_i(c) = -r^{-1} \exp(-rc)$ and $y_i \sim_{i.i.d.} \mathcal{N}(\mu, \sigma^2)$. Then $\lambda_i = 1/n \forall i$ solves D.2, and hence $FC_i(\lambda) = \exp(-r\mu) \mathbb{E}_\zeta \left\{ \zeta_i \exp\left(\frac{r^2 \sigma^2}{2 n_\zeta}\right) \right\}$.*