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RISK AVERSION AND NASH'S SOLUTION FOR BARGAINING GAMES WITH RISKY OUTCOMES

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Recent results have shown that, for bargaining over the distribution of commodities, or other riskless outcomes, Nash's solution predicts that risk aversion is a disadvantage in bargaining. Here we consider bargaining games which may concern risky outcomes as well as riskless outcomes, and we demonstrate that, in such games, risk aversion need not always be a disadvantage in bargaining. Intuitively, for bargaining games in which potential agreements involve lotteries which have a positive probability of leaving one of the players worse off than if a disagreement had occurred, the more risk averse a player, the better the terms of the agreement which he must be offered in order to induce him to reach an agreement, and to compensate him for the risk involved. For bargaining games whose disagreement outcome involves no uncertainty, we characterize when risk aversion is advantageous, disadvantageous, or irrelevant from the point of view of Nash's solution.

1. INTRODUCTION

SEVERAL INVESTIGATORS have considered how risk aversion influences the outcome of bargaining, as modelled by Nash's model of bargaining, and related models. Loosely speaking, Kannai **[3]** noted that when bargaining concerns distribution of a divisible commodity between two risk averse individuals, then Nash's solution assigns a larger share of the commodity to a bargainer as his utility function becomes less risk averse. Thus, risk aversion is a disadvantage in this situation, according to Nash's model. Kihlstrom, Roth, and Schmeidler **[5]** and Roth **[11]** generalized this observation to the case where the bargaining concerns selecting a single outcome from a set of riskless outcomes on which the two bargainers each have concave utility functions. Risk aversion is again a disadvantage. This has been elaborated by Sobel **[14]**, who considers the case of bargaining over the distribution of several divisible commodities. Thomson **[15]** has independently reported related results. All these results find risk aversion to be a disadvantage in bargaining over a set of riskless outcomes. This intuitively plausible relationship between an individual's bargaining ability and his propensity for risk-taking has been established only for bargaining situations whose potential outcomes involve no risk.

This paper concerns the more general case, in which bargaining may be over risky as well as riskless outcomes (however, we consider only the case in which the disagreement outcome is riskless). In some cases, risk aversion continues to be a disadvantage in bargaining; in some cases, it has no influence; and in some cases, risk aversion turns out to be an advantage. Intuitively, for bargaining games in which potential agreements involve lotteries having a positive probabil-

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ity of leaving one of the bargainers worse off than if a disagreement had occurred, the more risk averse a player, the better the terms of the agreement which he must be offered in order to induce him to reach an agreement.

Nash's model of bargaining is reviewed in Section 2, together with the previous results concerning risk aversion. Section 3 studies a class of games introduced in Roth and Malouf [**12**], from which examples can be drawn in which risk aversion is disadvantageous, irrelevant, or advantageous. The general case of games whose disagreement outcome is certain is then considered, and the effect of risk aversion in arbitrary games is characterized.

2. PREVIOUS RESULTS

Following Nash, we consider two-player bargaining games defined by a pair (S,d) where d is a point in the plane, and S is a compact convex subset of the plane containing d and at least one point x such that $x > d$.[2] The interpretation is that S is the set of feasible expected utility payoffs to the players, any one of which will result if agreed to by both players. If no agreement is reached, the disagreement point d results. Let $P(S)$ be the Pareto optimal subset of S.

Nash proposed that bargaining between rational players be modelled by a function called a *solution*, which selects a feasible outcome for every bargaining game. If B denotes the class of all two-player bargaining games, a solution is a function $f: B \to R^2$ such that $f(S,d)$ is in S. Nash also proposed that a solution should have the following properties: Pareto optimality, symmetry, independence of irrelevant alternatives, and independence of equivalent utility representations.[3] Nash proved the following.

THEOREM 1: *There is a unique solution which possesses these four properties. It is the solution F defined by* $F(S,d) = x$ *such that* $x > d$ *and*

$$(x_1 - d_1)(x_2 - d_2) > (y_1 - d_1)(y_2 - d_2)$$

for all y *in* S *such that* $y \neq x$ *and* $y > d$.

[2]We use the notation $x > d$ to mean that $x_1 > d_1$ and $x_2 > d_2$. Similarly, $x \geq d$ will mean $x_1 \geq d_1$ and $x_2 \geq d_2$.

[3]PROPERTY 1 (Pareto Optimality): If $f(S,d) = x$ and $y \geq x$, then either $y = x$ or $y \notin S$.

PROPERTY 2 (Symmetry): If (S,d) is a symmetric game (i.e., if $(x_1,x_2) \in S$ implies $(x_2,x_1) \in S$ and if $d_1 = d_2$), then $f_1(S,d) = f_2(S,d)$.

PROPERTY 3 (Independence of Irrelevant Alternatives): If (S,d) and (T,d) are bargaining games such that T contains S, and if $f(T,d) \in S$, then $f(S,d) = f(T,d)$.

PROPERTY 4 (Independence of Equivalent Utility Representations): If (S,d) and $(\hat{S},\hat{d})$ are bargaining games such that

$$\hat{S} = \{(a_1x_1 + b_1, a_2x_2 + b_2) \mid (x_1,x_2) \in S\} \quad \text{and} \quad \hat{d} = (a_1d_1 + b_1, a_2d_2 + b_2)$$

where a_1, a_2, b_1, and b_2 are numbers such that a_1 and $a_2 > 0$, then

$$f(S,d) = (a_1 f_1(S,d) + b_1, a_2 f_2(S,d) + b_2).$$

These properties have been discussed amply elsewhere (cf. Nash [**7**], Luce and Raiffa [**6**], Harsanyi [**2**], Roth [**9**, **11**]).

The above description follows the usual custom in describing bargaining games solely in terms of the feasible utility payoffs available to the players, without specifying the particular bargains which yield those utilities. To consider the effects of risk aversion, we need to consider the alternatives over which bargaining is conducted.

One approach is to consider each game (S, d) as arising from bargaining over the set L of all finite lotteries over some set of certain alternatives C contained in R^n, by individuals with (arbitrary) utility functions u_1 and u_2. (Denote $u = (u_1, u_2)$.) Then the feasible set of utility payoffs is the concave set

(1) $$S = \{(u_1(l), u_2(l)) \mid l \text{ is an element of } L\},$$

and the disagreement point d is the point

(2) $$d = (u_1(\bar{c}), u_2(\bar{c}))$$

where $\bar{c} \in C$ is the (deterministic) alternative which results in the case of disagreement. An (extended) *bargaining model* is a quintuplet $(S, d, C, \bar{c}, u)$ where S and d are defined by (1) and (2) and $(S, d) \in B$. Such a bargaining model is *deterministic* if $S = \{u(c) \mid c \in C\}$, i.e., if every payoff can be achieved by a deterministic outcome.

Now consider the effect of replacing one of the players, say player 2, in a bargaining model $(S, d, C, \bar{c}, u)$ with a more risk averse player. Let $u_2 = w$, and let $\hat{w}$ be a more risk averse utility function than w, i.e., $\hat{w}(c) = k(w(c))$ for all c in C, where k is an increasing,[4] concave function (c.f. Arrow [**1**], Pratt [**8**], or Kihlstrom and Mirman [**4**]). Consider the bargaining model $(\hat{S}, \hat{d}, C, \bar{c}, u)$ derived from the original one by replacing individual[5] w with the more risk averse individual $\hat{w}$. We can state the following theorem (c.f. Roth [**11**, Theorem 5], or Kihlstrom, Roth, and Schmeidler [**5**]).

THEOREM 2: *In deterministic bargaining models, the utility which Nash's solution assigns to a player increases as his opponent becomes more risk averse. That is,* $F_1(\hat{S}, \hat{d}) \geq F_1(S, d)$, *where* $(\hat{S}, \hat{d})$ *is obtained from* (S, d) *by replacing player* 2 *with a more risk averse player.*

In Roth [**10**] it was shown the Nash's solution could be interpreted as the utility function for a certain kind of individual, reflecting his preferences for bargaining in different games. Interpreted in this way, Theorem 2 states that such a player prefers to bargain against the more risk averse of any pair of possible

[4]We use the word "increasing" to denote a function such that $k(a) > k(b)$ if $a > b$. If the first inequality need not be strict, the function will be called nondecreasing.

[5]Since an individual is represented in this model only by his utility function, an individual whose utility function is w will sometimes be referred to as individual w.

opponents. Another interpretation of this result can be obtained by looking at the second player's utility. When bargaining against a given player 1 over a fixed set of outcomes, Nash's solution predicts that a less risk averse bargainer w obtains a more desirable outcome than does a more risk averse bargainer $\hat{w}$, in terms of the common preferences of both w and $\hat{w}$.

Theorem 2 states that Nash's solution F has a plausible sensitivity to risk posture, in deterministic models. We call an arbitrary solution f *risk sensitive* if it satisfies the conclusion of Theorem 2 for all deterministic bargaining models. It has been established (c.f. Roth [**12**, Theorem 6]) that if a solution f is both Pareto optimal and risk sensitive, then f is independent of equivalent utility representations. Thus risk sensitivity can replace independence of equivalent utility representations in a characterization of Nash's solution, or of any solution which is both risk sensitive and Pareto optimal. Theorem 2 can in fact be proved for any bargaining games in which the disagreement point and all of the Pareto optimal payoffs can be achieved by riskless events.

In the following sections we will see that this simple relationship between risk aversion and Nash's solution fails to carry over to the case of games with Pareto optimal payoffs which can only be achieved by lotteries.

3. RISK AVERSION IN A SIMPLE FAMILY OF GAMES WITH RISKY OUTCOMES

Consider the family of bargaining models $(S, d, C, \bar{c}, u)$ where C contains exactly three-elements, a^1, a^2, $\bar{c}$, where $\bar{c}$ is the disagreement outcome, and a^i is the outcome most preferred by player i. Since $(S, d) \in B$, some lottery between a^1 and a^2 is preferred by both players to the disagreement outcome $\bar{c}$. The set S equals the convex hull of the three points $d = u(\bar{c})$, $u(a^1)$, and $u(a^2)$, and the Pareto set $P(S)$ equals the line segment joining the latter two points. Only the endpoints of $P(S)$ can be achieved by riskless outcomes; all other Pareto optimal points are achieved only by lotteries.

The effect of risk aversion in games of this type depends on the position of the disagreement point. Let (S, d) be a game derived from a three-element set C as described above, with $u_2 = w$, and let $(\hat{S}, \hat{d})$ be a game obtained by replacing player 2 with a more risk averse player, with utility function $u_2 = \hat{w}$ such that $\hat{w}(c) = k(w(c))$ for all c in C, where k is an increasing concave function. Then we have the following parallel to Theorem 2.

THEOREM 3: (i) *If* $w(\bar{c}) \le w(a^1)$, *then* $F_1(\hat{S}, \hat{d}) \ge F_1(S, d)$. (ii) *If* $w(\bar{c}) \ge w(a^1)$, *then* $F_1(\hat{S}, \hat{d}) \le F_1(S, d)$.

PROOF: Since F is independent of equivalent utility representations, it is sufficient to prove the theorem for games with u_1, w, and $\hat{w}$ normalized so $u_1(a^1) = w(a^2) = \hat{w}(a^2) = 1$, and $u_1(a^2) = w(a^1) = \hat{w}(a^1) = 0$. For an arbitrary game (T, d) normalized in this way,

$$F(T,d)=\begin{cases}(0,1) & \text{if } d_1-d_2\leq -1,\\ \left(\dfrac{1+d_1-d_2}{2},\dfrac{1-d_1+d_2}{2}\right) & \text{if } -1\leq d_1-d_2\leq 1,\\ (1,0) & \text{if } d_1-d_2\geq 1.\end{cases} \tag{3}$$

In case (i), $d_2 = w(\bar{c}) \leq w(a^1) = 0$, and since $\hat{w}$ is a concave transformation of w which keeps $\hat{w}(a^1) = w(a^1) = 0$ and $\hat{w}(a^2) = w(a^2) = 1$, it follows that $\hat{d}_2 = \hat{w}(\bar{c}) \leq w(\bar{c}) = d_2$. Equation (3) therefore implies that $F_1(\hat{S}, \hat{d}) \geq F_1(S, d)$ in this case. In case (ii), $d_2 = w(\bar{c}) \geq w(a^1) = \hat{w}(a^1) = 0$ and $d_2 < w(a^2) = \hat{w}(a^2) = 1$. Since $\hat{w}$ is a concave transformation of w, $\hat{d}_2 = \hat{w}(\bar{c}) \geq w(\bar{c}) = d_2$, and so equation (3) implies $F_1(\hat{S}, \hat{d}) \leq F_1(S, d)$. Note that when $|d_1 - d_2| < 1$, the inequality in the conclusion of the theorem is strict. This completes the proof.

Note that any Pareto optimal payoff can be identified with a lottery between a^1 and a^2. Theorem 3 could be reformulated in terms of these lotteries. Part (i) of the Theorem states that the probability which Nash's solution assigns to a^1 is higher in $(\hat{S}, \hat{d})$ than in (S, d), and the reverse holds in part (ii). Thus, according to Nash's solution, risk aversion is a disadvantage to a player in games where he prefers his opponent's favorite outcome to the disagreement outcome (case (i)). The reverse is true in case (ii): risk aversion is an advantage to a player in games where he prefers the disagreement outcome to his opponent's favorite outcome. In games where he is indifferent between these two outcomes, a player's risk aversion has no influence. (This last property made such games appropriate for the experimental study of bargaining reported in Roth and Malouf **[12]** since the risk aversion of the players need not be controlled for when such games are used to test predictions of Nash's solution.)

4. GENERAL GAMES WITH CERTAIN DISAGREEMENT OUTCOMES

Here we consider arbitrary bargaining models $(S, d, C, \bar{c}, u)$. That is, we no longer restrict the set C to be finite, as in the previous section. Let $P(L)$ denote the Pareto optimal subset of lotteries. We say $x \in S$ is *u-supported* by $c^1, c^2 \in C$ if for some $p \in (0,1)$, $x = pu(c^1) + (1-p)u(c^2)$ and if there is no other point $c^3 \in C$, distinct from c^1 and c^2, such that $c^3 = qu(c^1) + (1-q)u(c^2)$ for $q \in (0,1)$. So x is u-supported by c^1 and c^2 if it can be achieved by a lottery between them, and if they are the "closest" certain outcomes by which x can be achieved. (If $x = u(c)$ for $c \in C$, then x is u-supported by $c^1 = c^2 = c$.)

If $x \in S$ is u-supported by c^1, c^2, then it is *favorably* u-supported for player i if $u_i(c^j) \geq u_i(\bar{c})$ for $j = 1, 2$, and *unfavorably* u-supported for player i if either $u_i(c^1) < u_i(\bar{c})$ or $u_i(c^2) < u_i(\bar{c})$. Thus, x is favorably u-supported for player i if it is u-supported by outcomes c^1 and c^2 both of which player i likes at least as well as the disagreement outcome, and unfavorably u-supported if player i prefers the disagreement point $\bar{c}$ to at least one of the supports c^1 and c^2. Note that every Pareto optimal point in S can be achieved by a lottery between no more than two certain Pareto optimal outcomes. Hence a point $x \in P(S)$ is unfavorably u-supported for player i if and only if it is not favorably u-supported.

We can now consider the effects of risk aversion in games of this form. As before, results are phrased in terms of games (S,d) and $(\hat{S},\hat{d})$, where the latter game is derived from the former by replacing player 2, whose utility function is u_2, with a more risk averse player whose utility function is $\hat{u}_2$. (Denote $\hat{u} = (u_1,\hat{u}_2)$.)

THEOREM 4: (i) *If $F(S,d)$ is favorably u-supported for player 2, then* $F_1(\hat{S},\hat{d}) \geq F_1(S,d)$. (ii) *If $F(\hat{S},\hat{d})$ is unfavorably u-supported for player 2, then* $F_1(\hat{S},\hat{d}) < F_1(S,d)$.

Theorem 4 gives sufficient conditions for a bargainer's risk aversion to be advantageous or disadvantageous to his opponent. For small changes in risk aversion, Lemma 2 will show these conditions are necessary as well as sufficient. Part (i) of Theorem 4 generalizes Theorem 2, since for deterministic models, $F(S,d)$ must be favorably u-supported for both players, since F is individually rational.

5. PROOFS

For $\lambda \in [0,1]$ let $u_2^\lambda = (1-\lambda)u_2 + \lambda\hat{u}_2$, and let $u^\lambda = (u_1,u_2^\lambda)$, $d^\lambda = u^\lambda(\bar{c})$, and $S^\lambda = \{u^\lambda(l) \mid l \in L\}$. It is straightforward to verify that u_2^λ is an increasing concave transformation of u_2 on the set C, and that as λ increases, u_2^λ becomes increasingly risk averse (i.e., if $\alpha < \beta$, then u_2^β is a concave transformation of u_2^α on the set C). As λ increases from 0 to 1, the game (S^λ,d^λ) is transformed from $(S^0,d^0) = (S,d)$ to $(S^1,d^1) = (\hat{S},\hat{d})$ in a way which allows the "local" effects of a small change in player 2's risk aversion to be examined. First, we establish that when a player is replaced by a more risk averse player, every certain outcome which was Pareto optimal in the old game is also Pareto optimal in the new game.

LEMMA 1: *Let $P(L)$ and $\hat{P}(L)$ denote the set of Pareto optimal lotteries in the games (S,d) and $(\hat{S},\hat{d})$ respectively. Then $\hat{P}(L) \cap C$ contains $P(L) \cap C$.*

PROOF: We show that if $c \in C$ is not Pareto optimal in $(\hat{S},\hat{d})$, then it is not Pareto optimal in (S,d). If $c \notin \hat{P}(L) \cap C$ then there is an $l \in L$ such that $\hat{u}(l) \geq \hat{u}(c)$ and either $u_1(l) > u_1(c)$ or $\hat{u}_2(l) > \hat{u}_2(c)$. But $\hat{u}_2 = k \circ u_2$ on C,[6] so $\hat{u}_2(l)$ equals the expected value of $k \circ u_2$ on the lottery l; i.e., $\hat{u}_2(l) = E(k \circ u_2)(l) \leq k(E(u_2(l))) = k(u_2(l))$, where the inequality follows from the concavity of k.

[6]That is, $\hat{u}_2$ is equal to the function k composed with the function u_2 (denoted $k \circ u_2$) on the set C.

Thus $k(u_2(l)) \geq \hat{u}_2(l) \geq \hat{u}_2(c) = k(u_2(c))$, and, since k is an increasing function, $u_2(l) \geq u_2(c)$, with a strict inequality if $\hat{u}_2(l) > \hat{u}_2(c)$. This completes the proof.

Lemma 1 establishes that, as λ increases from 0 to 1, the set $P^\lambda(L) \cap C$ of Pareto optimal certain outcomes is nondecreasing. The next part of the proof proceeds by establishing results for games generated by a *finite* set of certain alternatives C, and the proof concludes by establishing the general case.

We use the following proposition whose proof follows from the fact that Nash's solution is continuous in the Hausdorf topology on the (open) set B of bargaining games (see Roth and Rothblum [**13**]).

PROPOSITION 1: *For games generated by a finite set C of certain outcomes, $F(S^\lambda, d^\lambda)$ is a continuous function of λ.*

With Lemma 1, Proposition 1 allows us to prove the following.

LEMMA 2: *For games generated by a finite set C of certain outcomes, and for any $\lambda \in [0,1]$, there exists a $\delta > 0$ such that:*

(a) *If $F(S^\lambda, d^\lambda)$ is favorably u^λ-supported for player 2, then for $\alpha \in [\lambda, \lambda + \delta]$, $F_1(S^\alpha, d^\alpha)$ is a nondecreasing function of α, and $F(S^\alpha, d^\alpha)$ is favorably u^α-supported for player 2 in (S^α, d^α).*

(b) *If $F(S^\lambda, d^\lambda)$ is unfavorably u^λ-supported for player 2, then for $\alpha \in [\lambda, \lambda + \delta]$, $F_1(S^\alpha, d^\alpha)$ is a strictly decreasing function of α, and $F(S^\alpha, d^\alpha)$ is unfavorably u^α-supported for player 2 in (S^α, d^α).*

PROOF: First suppose $F(S^\lambda, d^\lambda)$ is u^λ-supported by outcomes $c^1, c^2 \in C$ such that $u(c^1) \neq u(c^2)$. Lemma 1 implies c^1 and c^2 remain Pareto optimal as λ increases, and Proposition 1 therefore implies that there exists a positive δ, sufficiently small so that for $\alpha \in [\lambda, \lambda + \delta]$, $F(S^\alpha, d^\alpha)$ remains u^α-supported by c^1 and c^2. Let $E = \{c^1, c^2, \bar{c}\}$, and let (T^α, d^α) be the game generated by the three element set E and the utility functions u_1 and u_2^α. Then $F(S^\alpha, d^\alpha) \in T^\alpha$ for $\alpha \in [\lambda, \lambda + \delta]$, and so $F(S^\alpha, d^\alpha) = F(T^\alpha, d^\alpha)$ since F is independent of irrelevant alternatives (Property 3). But the behavior of $F_1(T^\alpha, d^\alpha)$ for $\alpha \in [\lambda, \lambda + \delta]$ is given by Theorem 3: that is, if $\alpha < \beta$ for α, $\beta \in [\lambda, \lambda + \delta]$, then (T^α, d^α) and (T^β, d^β) play the roles of (S,d) and $(\hat{S}, \hat{d})$, respectively, in Theorem 3. So Lemma 2 is proved when $u(c^1) \neq u(c^2)$. If, instead, $F(S^\lambda, d^\lambda)$ is u^λ-supported by $c^1 = c^2$, then $F(S^\lambda, d^\lambda) = u^\lambda(c^1)$. In this case $F(S^\lambda, d^\lambda)$ must be favorably supported, since F is individually rational. If there is some $\delta > 0$ such that $F(S^\alpha, d^\alpha) = u^\alpha(c^1)$ for $\alpha \in [\lambda, \lambda + \delta]$, then Lemma 2 follows immediately. Otherwise, Proposition 1 and the finiteness of C imply that there is a $\delta > 0$ and an outcome c^3 such that, for all $\alpha \in [\lambda, \lambda + \delta]$, $F(S^\alpha, d^\alpha)$ is u^α-supported by c^1 and c^3. If we now let $E = \{c^1, c^3, \bar{c}\}$, then the argument of the previous paragraph assures that $F(S^\alpha, d^\alpha) = F(T^\alpha, d^\alpha)$, where (T^α, d^α) is the game generated by E. So Theorem 3 implies that in this case also, $F_1(S^\beta, d^\beta) \geq F_1(S^\alpha, d^\alpha)$ for α, $\beta \in [\lambda, \lambda + \delta]$ such that $\alpha \leq \beta$. This completes the proof of Lemma 2, which shows that a sufficiently small increase in the risk aversion of one of the players in a game (S^λ, d^λ) does not change the nature of the support of $F(S,d)$—i.e., there is an interval in which $F(S,d)$ will remain favorably or unfavorably u^λ-supported. The following lemma establishes a global result.

LEMMA 3: *For games generated by a finite set C of certain outcomes, if $F(S,d)$ is favorably u-supported for player 2, then $F(S^\lambda, d^\lambda)$ is favorably u^λ-supported for player 2 for any $\lambda \in [0,1]$.*

PROOF: Suppose Lemma 3 is false, and let $\beta = \inf\{\lambda \mid F(S^\lambda, d^\lambda)$ is unfavorably u^λ-supported for player 2$\}$. Then $F(S^\beta, d^\beta)$ is unfavorably supported for player 2, since otherwise Lemma 2 implies there is a $\delta > 0$ such that $F(S^\alpha, d^\alpha)$ is favorably u^α-supported for $\alpha \in [\beta, \beta + \delta]$, contradicting the definition of β. But $\beta > 0$ since $(S^0, d^0) = (S,d)$. Since the set $P^\lambda(L) \cap C$ cannot become larger as λ decreases (Lemma 1), Proposition 1 and the arguments of Lemma 2 imply that, if $F(S^\beta, d^\beta)$ is unfavorably u^β-supported for player 2, there exists a $\delta > 0$ such that for $\alpha \in [\beta - \delta, \beta]$, $F(S^\alpha, d^\alpha)$ is unfavorably u^α-supported for player 2. But this contradicts the definition of β, and completes the proof.

PROOF OF THEOREM 4: First consider games generated by lotteries over a *finite* set C of certain outcomes. If $F(S,d)$ is favorably u-supported for player 2, then by Lemma 3, $F(S^\lambda, d^\lambda)$ is favorably u^λ-supported for every $\lambda \in [0,1]$, and Lemma 2 therefore implies that, for every λ, there exists an interval $[\lambda, \lambda + \delta]$ in which $F_1(S^\alpha, d^\alpha)$ is a nondecreasing function of α. Together with Proposition 1, this implies $F_1(S^\lambda, d^\lambda)$ is a nondecreasing function of λ for all $\lambda \in [0,1]$, completing the proof of part (i) when C is finite. If $F(\hat{S}, \hat{d})$ is unfavorably $\hat{u}$-supported for player 2, then Lemma 3 implies $F(S^\lambda, d^\lambda)$ is unfavorably u^λ-supported for player 2 for every $\lambda \in [0,1]$. Using Lemma 2 and Proposition 1 as above, it follows that $F_1(S^\lambda, d^\lambda)$ is a decreasing function of λ for $\lambda \in [0,1]$, completing the proof of part (ii) when C is finite.

For an arbitrary (possibly infinite) set C of certain outcomes, let $F(S,d)$ be u-supported by $c^1, c^2 \in C$, let $F(\hat{S}, \hat{d})$ be $\hat{u}$-supported by $c^3, c^4 \in C$, and let $E = \{c^1, c^2, c^3, c^4, \bar{c}\}$. Let (T,d) and $(\hat{T}, \hat{d})$ be the games generated by the finite set E as in equations (1) and (2), using utility functions $u = (u_1, u_2)$ and $\hat{u} = (u_1, \hat{u}_2)$, respectively. Then, since Nash's solution possesses the property of independence of irrelevant alternatives, $F(T,d) = F(S,d)$ and $F(\hat{T}, \hat{d}) = F(\hat{S}, \hat{d})$. But Theorem 4 has already been proved for the finitely generated games (T,d) and $(\hat{T}, \hat{d})$, and so it holds for (S,d) and $(\hat{S}, \hat{d})$ as well, completing the proof.

Note that, when a given bargainer is replaced by a more risk averse individual, the set of Pareto optimal certain events may grow in such a way that Nash's solution will become favorably supported, even if it were unfavorably supported in the original game. When this happens, a bargainer's risk aversion stops being disadvantageous to his opponent, and starts being advantageous. This is why part (ii) of Theorem 4 is only able to compare games whose Nash solution remains unfavorably supported. Lemma 3, however, shows that a favorably supported solution remains favorably supported when a bargainer is replaced by a more risk averse individual. So part (i) of Theorem 4 establishes that, when Nash's solution is favorably supported, an individual's risk aversion is always advantageous to his opponent.

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