## Projectile Lab - Advanced

You will shoot a projectile across the room with a rubber band and calculate how far it flies.

## Part 1 - Preliminary Measurements

## Tasks 1, 2, and 3 can be done at the same time by different members of your group.

## Task 1 - Mass of the Projectile

You will use a simple balance to measure the mass of your projectile.

1. In your lab kit there should be a wooden bar with two ziploc baggies attached. Use this balance as shown in the picture. When the masses in the two baggies are the same, the wooden bar should be horizontal.

You have been provided with pennies and beads to use as mass standards. A useful fact: according to the US mint, a post-1982 penny has mass 2.5 g .
2. Balance the beads against a penny.


1 penny = $\qquad$ beads.

Each bead must have mass $\qquad$ g.
3. Place your projectile (the wooden stick with metal nuts on the end) into one baggie on the balance. Use a combination of pennies and beads to balance the projectile.

1 projectile = $\qquad$ pennies + $\qquad$ beads

The mass of the projectile is $\qquad$ g

## Task 2 - Elasticity of the Rubber Band

You will measure the "elastic constant" of the rubber band which will be used to shoot your projectile. This elastic constant tells you how strong the rubber band is. The bigger the elastic constant, the more force is required to stretch the band and the more energy is stored in the band by stretching it.

If we approximate the rubber band as a spring with elastic constant $\boldsymbol{k}$, then the force exerted by the band when it is stretched a distance $\boldsymbol{x}$ is given by

$$
F_{\text {elastic }}=k x
$$

The gravitational force exerted on an object of mass $\boldsymbol{m}$ is given by the following (where $\boldsymbol{g}=980 \mathrm{~cm} / \mathrm{s}^{2}$ is the gravitational acceleration on Earth):

$$
F_{\text {gravity }}=m g
$$

1. Place a rubber band around the basket provided. On one side of the basket, the rubber band should go across the $1^{\text {st }}$ row of holes under the open top. On the other side of the basket, it should go across the $2^{\text {nd }}$ row of holes from the top. The side where it passes the $2^{\text {nd }}$ row of holes is the side you will use to load your projectile.
2. Adjust the rubber band so that it is evenly tight all around the basket. Do not adjust the rubber band after you measure the elasticity - this would throw off your predictions.
3. Place the package of candy from your kit in a ziploc bag. Use a paper-clip to suspend the bag from the rubber-band. The bag should be hanging from the center of the rubber band on the side where it goes across the $2^{\text {nd }}$ row of holes.

The mass in the ziploc bag is $\qquad$ g
(Look for the net weight on the package of candy, and assume that the packaging itself is so light that you can neglect it)
4. Use a ruler to measure the distance from the rubber band (where the paper-clip is attached) to the side of the basket, as illustrated below:

5. The rubber band stretched a total of $\qquad$ cm from the basket. (Round up if it falls between mm marks)
6. While it is hanging from rubber band, the Ziploc bag is not accelerating. This means the total force on it must add up to 0 . That is, the force of the rubber band pulling up exactly cancels out the force of gravity pulling down on the mass. Using the equations given above for these two forces, solve for the elastic constant, $k$, of the rubber band.

The elastic constant of the rubber band is $k=$ $\qquad$ $g / s^{2}$

## Task 3 - Time of flight

You will calculate the time it takes for your projectile to hit the ground after it is launched.
Time of flight depends only on starting height and on the gravitational acceleration, $\mathbf{g}=\mathbf{9 8 0} \mathbf{~ c m} / \mathbf{s}^{\mathbf{2}}$ It does not depend on the mass of the projectile or on its horizontal velocity as it is launched!

1. Use the meter stick to measure the height of the table from which you will be launching the projectile.

The projectile will be launched from height $\qquad$ cm
2. An object that falls at a constant acceleration $\boldsymbol{g}$ for a time $\boldsymbol{t}$ will travel a vertical distance $\boldsymbol{h}$ given by:

$$
h=\frac{1}{2} g t^{2} \quad \text { where } g=980\left(\frac{\mathrm{~cm}}{s^{2}}\right)
$$

3. How long will it take the projectile to hit the floor after it is launched? Solve the equation for time, plugging in the numbers for height $\boldsymbol{h}$ and acceleration $\boldsymbol{g}$.

The time to fall from the top of the table to the ground is $\qquad$ seconds.
4. If you are done and other members of your group are still working, help them with their task.

## Part 2 - Ready, Aim, Fire

To predict how far the projectile will fly, you need to know the velocity $\boldsymbol{v}$ with which it leaves the launcher. To find this velocity, we will use the conservation of energy.

A rubber band with elastic constant $\boldsymbol{k}$, stretched a distance $\boldsymbol{x}$ stores an energy of:

$$
\text { Potential Energy }=\frac{1}{2} k x^{2}
$$

A object of mass $\boldsymbol{m}$, moving at speed $\boldsymbol{v}$ has a kinetic energy of:

$$
\text { Kinetic Energy }=\frac{1}{2} m v^{2}
$$

If all the energy stored in the rubber band is converted into the kinetic energy of the projectile (Potential Energy=Kinetic Energy), solve for the velocity $\boldsymbol{v}$ of the projectile in terms of the variables $\boldsymbol{k}, \boldsymbol{m}$, and $\boldsymbol{x}$

$$
v=
$$

$\qquad$

1. Plug in the values you measured in Part 1 for the mass of the projectile ( $\boldsymbol{m}$ ) [Task 1] and the elastic constant of the rubber band (k) [Task 2].

In the table below, fill in the projectile velocity for different values of $\boldsymbol{x}$

| Distance rubber band <br> is stretched ( $\boldsymbol{x}$ ) | Velocity of <br> projectile ( $\boldsymbol{v}$ ) | Predicted <br> Flight Distance | Measured Flight <br> Distance | \% Error |
| :---: | ---: | ---: | ---: | ---: |
| 2 cm | $\mathrm{~cm} / \mathrm{sec}$ | cm | cm |  |
| 3 cm | $\mathrm{~cm} / \mathrm{sec}$ | cm | cm |  |
| 4 cm | $\mathrm{~cm} / \mathrm{sec}$ | cm | cm |  |

2. For each row in the table, find the distance that the projectile should travel. Use the time of flight you found in Task 3. The horizontal distance traveled depends only on the horizontal velocity and the time spent in flight.
3. Place the targets from your kit at the predicted distances from the edge of the table. If the floor is slippery, you may need to tape them in place to prevent them from sliding.
4. The tail edge of the projectile has been marked at 1 cm intervals. Put the projectile in the launcher as illustrated below. Make sure the projectile is horizontal as it is launched.

5. Try launching the projectile by pulling back to the 2,3 , and 4 cm mark. Everyone should have a chance to launch. Have one person from your group stand to the side and watch where the tip of the projectile lands.
6. Measure the actual distance flown by the projectile and fill in the $4^{\text {th }}$ column of the table. If you have time, do several launches and put in the average distance traveled.
7. Calculate the error in your predictions using \% error. Remember to fill in the values in the table above.

$$
\% \text { Error }=\frac{(\text { measured }- \text { predicted })}{\text { predicted }} \times 100
$$

Note: If the value is negative take its absolute value.

## Part 3 -Bonus Challenge

1. Ask a teacher or volunteer to place a target in front of your table. Your goal is to figure out how far back you want to pull the rubber band so as to hit the target.
2. Measure the distance of the target from the table. Look back to Part 2 to figure out how to do the calculations

We are aiming for a flight distance $\qquad$ cm.

To fly this distance, the projectile needs to be launched with speed $\boldsymbol{v}=$ $\qquad$ cm/s

To achieve this speed, the rubber band needs to be pulled back $\boldsymbol{x}=$ $\qquad$ cm
3. Make a mark on your projectile for the length that you want to pull back on the rubber band. Try launching to hit the target! Do several tries. How close were you?
4. If any other groups have also reached this part, have a competition to see who can come closest to the target.

## Post-Lab Questions

1. By what factor will the distance that the projectile flies change, if the following variables are altered:
(a) The projectile is twice as heavy.
(b) The table is twice as high.
(c.) The elastic constant of the rubber band is twice as large.
(d) You pull the rubber band back twice as far
(e) You launch on the moon, where gravity is $1 / 6$ that of earth.
2. Did your calculations tend to overestimate or underestimate the distance which the projectile flew? If you overestimated, where did some of the stored potential energy go? If you underestimated, where did the extra energy come from?
3. Which of the following errors in measurement would have the greatest effect on your predicted flight distance?

$$
v=x \sqrt{\frac{k}{m}} \quad h=\frac{1}{2} g t^{2}
$$

(a) A $10 \%$ error in measuring the mass.
(b) A $10 \%$ error in measuring the elastic constant
(c) A $10 \%$ error in measuring the height of the table
(d) A $10 \%$ error in measuring the distance to which you pull back the rubber band
4. How would the time of flight and the distance traveled by the projectile change if you were to launch at an angle upwards rather than horizontally off the table?
5. Suppose you had as many pennies as you wanted in your kit, but you did not know the mass of a penny. Could you still do this lab with the materials provided? What extra measurement would be required or what measurement would you change and how?

