The SUMO Speaker Series for Undergraduates

(Pizza Provided)
Wednesday, May 4th
4:15-5:05, room 380C

Rational Points on Elliptic Curves (And a really really difficult way to earn $1,000,000)

Professor Samit Dasgupta (UC Santa Cruz)

ABSTRACT:

The Pythagorean equation $x^2 + y^2 = z^2$ has a well-known parameterization of its integer solutions. A similar parameterization exists for any homogeneous degree-2 polynomial in three variables, provided that there is some solution. The existence of some solution is governed by Hasse's "local to global principle."

The situation is more subtle and interesting when we move from degree-2 equations to cubic equations, which leads to the theory of elliptic curves. First of all, Hasse's local to global principle fails. Next, even if we assume the existence of at least one solution, it is not known in general how to parameterize all solutions. However, the set of solutions has the beautiful structure of an abelian group, and it is known that this group can be generated by finitely many elements. We will conclude by stating the conjecture of Birch and Swinnerton-Dyer, which relates the minimal number of elements needed to generate the group to the number of solutions of the equation modulo prime numbers p; this is another sort of "local to global principle."

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