1. Compute $1^2 - 2^2 + 3^2 - \cdots - 18^2 + 19^2 - 20^2$.

   Answer: $-\mathbf{210}$

   Solution: Note that $(2n+1)^2 - (2n+2)^2 = -4n - 3$. Therefore, the sum is equivalent to $\sum_{i=0}^{9} -2i - 3 = -2(9)(10) - 30 = \boxed{-210}$.

2. Reimu and Marisa are playing a game with 2012 coins. Reimu flips all 2012 coins, and then is permitted to flip any subset of the 2012 coins exactly once more. After this, Reimu pays Marisa \$2 for every head on the table, whereas Marisa pays Reimu \$1 for every tail on the table. Who is more likely to earn a profit, and what is the expected profit for that person, in dollars?

   Answer: Reimu, \$503.

   Solution: We compute the expected profit for a game using 1 coin and then multiply this by 2012. We are allowed to do this by linearity of expectation. If Reimu flips the coin and gets a tail, then she receives \$1 guaranteed. If Reimu flips the coin and gets a head instead, she then reflips the coin and gets \$1 with probability $\frac{1}{2}$ and loses \$2 with probability $\frac{1}{2}$. Therefore, Reimu expects to win $1\left(\frac{1}{2}+\frac{1}{4}\right) - 2\left(\frac{1}{4}\right) = 0.25$ dollars. Multiplying by 2012, Reimu expects to win \$503.

3. We define n to be a squarefree integer if, for every prime p, p^2 does not divide n. Let $f(n)$ be the sum of the reciprocals of all the divisors of n. We define n to be an amazing integer if $f(n) = 2$. How many squarefree amazing integers are there?

   Answer: 1

   Solution: Note that $f(n) = \frac{\sigma(n)}{n} = \prod_{i=1}^{k} \frac{p_i + 1}{p_i}$. In order for $f(n)$ to be integral, the only primes that can divide n are 2 and 3, since those are the only consecutive primes. Note that $f(6) = 2$, but 1, 2, and 3 do not satisfy this constraint. Therefore, there is exactly $\boxed{1}$ squarefree amazing integer.