1. **Answer: 0**

   By L'Hopital's Rule,

$$\lim_{x\to 0} \frac{\tan x - x - \frac{x^3}{3}}{x} = \lim_{x\to 0} \frac{\sec^2 x - 1 - x^2}{1} = 1^2 - 1 - 0 = 0$$

2. **Answer: 3 values**

   If $\frac{n}{20-n} = k^2$ then $n = k^2(20-n)$ so $n = \frac{20k^2}{1+k^2}$. However, since $1+k^2$ is always coprime to k^2, $1+k^2$ must divide 20. Therefore, the only possible values of k are 1, 2, and 3, and the only possible values of n are the corresponding values 10, 16, and 18.

3. **Answer: $x_1 = x_2 = x_3 = \ldots = x_n = 2010$**

   Notice that

$$\begin{aligned}
x_1 &= \frac{1}{2}\left(x_n + \frac{x_{n-1}^2}{x_n}\right) \Rightarrow 2x_1x_n = x_n^2 + x_{n-1}^2 \\
x_2 &= \frac{1}{2}\left(x_1 + \frac{x_n^2}{x_1}\right) \Rightarrow 2x_1x_2 = x_1^2 + x_n^2 \\
x_3 &= \frac{1}{2}\left(x_2 + \frac{x_1^2}{x_2}\right) \Rightarrow 2x_2x_3 = x_2^2 + x_1^2 \\
x_4 &= \frac{1}{2}\left(x_3 + \frac{x_2^2}{x_3}\right) \Rightarrow 2x_3x_4 = x_3^2 + x_2^2 \\
&\vdots \\
x_n &= \frac{1}{2}\left(x_{n-1} + \frac{x_{n-2}^2}{x_{n-1}}\right) \Rightarrow 2x_{n-1}x_n = x_{n-1}^2 + x_{n-2}^2.
\end{aligned}$$

   Adding all these equations up, subtracting the lefthand side to the righthand side of the equation, and factoring, we get

$$(x_1 - x_n)^2 + (x_1 - x_2)^2 + (x_2 - x_3)^2 + \ldots (x_{n-1} - x_n)^2 = 0.$$

   Since the sum of the squares is zero, each of the terms being added must be zero. Then it follows that $x_1 = x_2 = x_3 = \ldots = x_n = 2010$.

4. **Answer: 3**

   Let O be the intersection of three diagonals. Since AF and CD are parallel, we have $\triangle AOF$ and $\triangle DOC$ similar, so $AO/DO = FO/CO$. Applying this method to other pairs of parallel sides, we have $AO/DO = BO/EO$ and $BO/EO = CO/FO$, so $CO/FO = FO/CO$. Thus $CO = FO$, $\triangle AOF$ and $\triangle DOC$ are congruent. Therefore $CD = AF = 3$.

5. **Answer: $B > A > C > D$**

   It is easy to verify that

$$f(n) = \frac{1^n + 2^n + 3^n}{1^{n-1} + 2^{n-1} + 3^{n-1}}$$

   is a monotonically increasing function.

6. **Answer: 11**

   We know that m must be at least 11 because we have a counterexample: the list of 10 integers $0, 0, 0, 0, 0, 1, 1, 1, 1, 1$ does not satisfy the given property. Now, to see that 11 is sufficient, observe that among any three integers we can find two whose sum is even. So, take three integers from a set of 11, find the two whose sum is even, and replace the other. Now, we have two integers with even sum, and nine others. We can repeat this process to get 5 pairs of integers, each with even sum, and the eleventh element is ignored. Among the pair-sums, we either have 3 numbers that all have the same remainder when divided by 3, or 3 that all have different remainders. Either way, the sum of these is a multiple of three, and then also of six, since we said that the pair-sums are all even.

7. **Answer: 60, 48, 40, 32, 17**

Given a number $n = p_1^{a_1} \cdot p_2^{a_2} \ldots p_k^{a_k}$, it is clear that a number is relatively prime to n if and only if it is not divisible by any of the p_i. A little experimentation with this idea, or previous knowledge of number theory, gives the well-known formula:

$$\phi(n) = p_1^{a_1-1} \cdot (p_1 - 1) \cdot \ldots \cdot p_k^{a_k-1} \cdot (p_k - 1)$$

With this, it is clear that any n satisfying $\phi(n) = 16 = 2^4$ must be a power of two, times at most one 3 and at most one 5, or 17 by itself. Then, we have as possible solutions 60, 48, 40, 32, and 17, for five solutions in total.

8. **Answer: -3439**

The fact that the limit exists implies that

$$\lim_{x\to 0}(f(4x) + af(3x) + bf(2x) + cf(x) + df(0)) = (1 + a + b + c + d)f(0) = 0$$

therefore

$$a + b + c + d = -1.$$

Apply L'Hospital's Rule once, then we have

$$\lim_{x\to 0} \frac{f(4x) + af(3x) + bf(2x) + cf(x) + df(0)}{x^4} = \lim_{x\to 0} \frac{4f'(4x) + 3af'(3x) + 2bf'(2x) + cf'(x)}{4x^3}$$

and for the following limit to exist we also need

$$\lim_{x\to 0}(4f'(4x) + 3af'(3x) + 2bf'(2x) + cf'(x)) = (4 + 3a + 2b + c)f'(0) = 0,$$

therefore

$$3a + 2b + c = -4.$$

Repeat this process twice and get another two equations:

$$9a + 4b + c = -16$$

$$27a + 8b + c = -64$$

Solving these four equations one can get $(a, b, c, d) = (-4, 6, -4, 1)$, giving the answer $1000a + 100b + 10c + d = -3439$.

9. **Answer: $11\sqrt[11]{\frac{27}{4}}$**

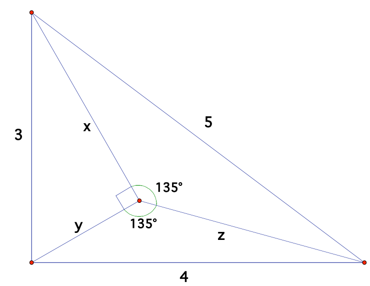
This is an application of the arithmetic-geometric mean inequality:

$$\frac{2 \cdot \frac{x^3}{2} + 3 \cdot \frac{4y^2}{3} + 6 \cdot \frac{9z}{6}}{11} \geq \sqrt[11]{\left(\frac{x^3}{2}\right)^2 \cdot \left(\frac{4y^2}{3}\right)^3 \cdot \left(\frac{9z}{6}\right)^6}$$

When we use the fact that $x^6y^6z^6 = 1$ and multiply both sides by eleven, we get the answer. (Recall that the bound given by the AM-GM inequality is always achievable).

10. **Answer: $12\sqrt{2}$**

Note that these equations come from applying the Law of Cosines to the triangle in the figure. The desired value is simply $2\sqrt{2}A = 12\sqrt{2}$, where A is the area of the large triangle.



11. **Answer: $\frac{20}{3}$**

Let
$$F(x, y, z) = |x + y + z| + |x + y - z| + |x - y + z| + | - x + y + z|.$$
Note that
$$F(x, y, z) = F(-x, y, z) = F(x, -y, z) = F(x, y, -z)$$
so the region is symmetric with respect to all xy, yz, zx-planes. Thus one can only consider the first octant part (where $x, y, z \geq 0$) and reflect it to get a full figure.
Assume that x is the largest. Then $x-y+z, x+y-z \geq 0$. Since the equation of the form $A+|B| \leq C$ implies both $A + B \leq C$ and $A - B \leq C$, we have
$$(x + y + z) + (x + y - z) + (x - y + z) + (-x + y + z) = 2(x + y + z) \leq 4$$
and
$$(x + y + z) + (x + y - z) + (x - y + z) - (-x + y + z) = 4x \leq 4.$$
Thus $x + y + z \leq 2$, and $x, y, z \leq 1$. We can now see that the region is cube $0 \leq x, y, z \leq 1$ cut by the plane $x + y + z \leq 2$. Since the plane goes through three vertices $(1, 1, 0), (1, 0, 1), (0, 1, 1)$ of the cube, it cuts out a prism of volume $\frac{1}{6}$ out of the cube. So the volume is $1 - \frac{1}{6} = \frac{5}{6}$. We multiply 8 to get the volume of whole region, so the answer is $\frac{5}{6} \cdot 8 = \frac{20}{3}$.

12. **Answer: 24**

Let v, e, t, q be the number of vertices, edges, triangular faces, and quadrilateral faces respectively. Note that each vertex is shared by exactly one quadrilateral, and a quadrilateral provides four vertices. By simple counting we get $v = 4q$. Apply the same thing to triangular face, then we have $4v = 3t$. Meanwhile from each vertex we have 5 edges coming out, so $5v = 2e$. Thus we have
$$q = 1/4v, t = 4/3v, e = 5/2v.$$
And from the Euler's formula $v - e + (t + q) = 2$, we have $(1 - 5/2 + 1/4 + 4/3)v = 1/12v = 2$, $v = 24$.

13. **Answer: $A < C < B < D < E$**

These are approximations to the integral
$$\int_{2009}^{2011} \frac{1}{2}\sqrt{x}\, dx$$
A is a trapezoid approximation, B is the integral itself, C is a trapezoid approximation with a finer partition (including the middle of the segment), D approximates the function by its value at 2010, and then integrates, and E is an integral of the function approximated by its tangent line at 2011. Simply drawing the areas represented by these approximations, and keeping in mind the concavity of $\sqrt{x}$, makes it clear how these compare. Note that one should not try to simply write out decimal approximations to these values, as they first differ in the 6th place after the decimal point.

14. **Answer:** $\boldsymbol{e^{4020}}$

Let $H(x) = f(x)^2 + g(x)^2$, then $H(x+y) = H(x)H(y)$. We can prove that $H(x) = e^{ax}$ for some a, since H is continuously differentiable. To find a, consider $H'(0)$. This is $2f'(0)f(0) + 2g'(0)g(0)$.

We claim $f(0) = 1$, $g(0) = 0$. From $H(0) = 1$ we have $f(0)^2 + g(0)^2 = 1$, so we can let $f(0) = \cos\theta$, $g(0) = \sin\theta$. Substituting $x = y = 0$ in both equations, it is easy to see that we get $\cos\theta = \cos(2\theta)$, $\sin\theta = \sin(2\theta)$. So $\theta = 2n\pi$ for some n, $f(0) = 1$, $g(0) = 0$. Then $2f'(0)f(0) + 2g'(0)g(0) = 2 \cdot 1 \cdot 1 + 2 \cdot 2 \cdot 0 = 2$, so $a = 2$, $f(2010)^2 + g(2010)^2 = e^{2010a} = e^{4020}$.

15. **Answer:** $\boldsymbol{m}$

Notice that if $n \geq 5$, we can strictly increase that sum by replacing a_{n-3} with $a_{n-3} + a_n$, and omitting a_n. Hence, we have either $n = 3$, and all three a_i are equal to m, or we have $n = 4$, and then $a_1a_2a_3 + a_2a_3a_4 = (a_1 + a_4)a_2a_3$, so $a_2 = a_3 = a_1 + a_4 = m$, and there are $m - 1$ ways to make this happen, for a total of m possibilities.