Note: Figures may not be drawn to scale.

1. **Answer: $(-13, -16, -18)$**

   The normal to the plane is in the direction $< 3, 4, 5 >$ and so the line going through the point perpendicular to the plane is $(11 - 3t, 16 - 4t, 22 - 5t)$ which intersects the plane at $t = 4$ and hence the reflection of the point occurs at $t = 8$, since the original point is at $t = 0$.

2. **Answer: $\frac{\sqrt{7}}{2}$**

   First, use Heron's Formula to find the area. The semiperimeter is $s = \frac{15}{2}$, so the area is $\sqrt{\frac{15}{2} \cdot \frac{7}{2} \cdot \frac{5}{2} \cdot \frac{3}{2}} = \frac{15\sqrt{7}}{4}$. Now the area is equal to the inradius times the semiperimeter, so $r = \frac{A}{s} = \frac{\sqrt{7}}{2}$.

3. **Answer: $\frac{5\sqrt{2}}{3}$**

   The lengths of the sides of the large cube containing the cubeoctahedron are $\sqrt{2}$, so the volume of the containing cube is $2\sqrt{2}$. The volumes of the removed pyramids are $\frac{1}{3}BH = \frac{1}{3}\left(\frac{1}{2}\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\right)\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{24}$. Because there are 8 pyramids removed, the total volume removed is $8\frac{\sqrt{2}}{24} = \frac{\sqrt{2}}{3}$. Thus, the total volume of the cubeoctahedron is $2\sqrt{2} - \frac{\sqrt{2}}{3} = \frac{5\sqrt{2}}{3}$.

4. **Answer: $\sqrt{15}$**

   By angle bisector theorem, $\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{3}{9} = \frac{BD}{8-BD} \Rightarrow 24 - 3BD = 9BD$. This implies $BD = 2$ and $DC = 6$. Now apply the Stewart's theorem

   $$AB^2 \cdot CD + AC^2 \cdot BD = BC \cdot (AD^2 + BD \cdot BC),$$

   then we have $AD^2 = 15$.

5. **Answer: $540°$**

   The path goes around the center three times. At each turning point, the external angle at that point is the amount that you turn around the center. Hence, the external angles add up to $3 \cdot 360° = 1080°$. Hence, the answer we want is $9 \cdot 180° - 1080° = 540°$.

6. **Answer: $\frac{25\sqrt{13}}{3}$**

   Let N be the opposite point of M in the circle. Then $MN = 50$ and $NB = \sqrt{50^2 - 30^2} = 40$ because $\triangle MBN$ is right triangle. Let C be the midpoint of AB, then $\triangle MCB$ and $\triangle MBN$ are similar, so $BC = NB \cdot \frac{MB}{MN} = 24$, $MC = MB \cdot \frac{MB}{MN} = 18$. Let L be the intersection of $\overline{AB}$ and the tangent. Since we have AB and OT parallel, $CL = OT = 25$, so $BL = 1$. Since $\triangle MCB \sim \triangle BLD$, we have $BD = MB \cdot \frac{BL}{MC} = \frac{5}{3}$, so $MD = \sqrt{MB^2 + BD^2} = \frac{25\sqrt{13}}{3}$.

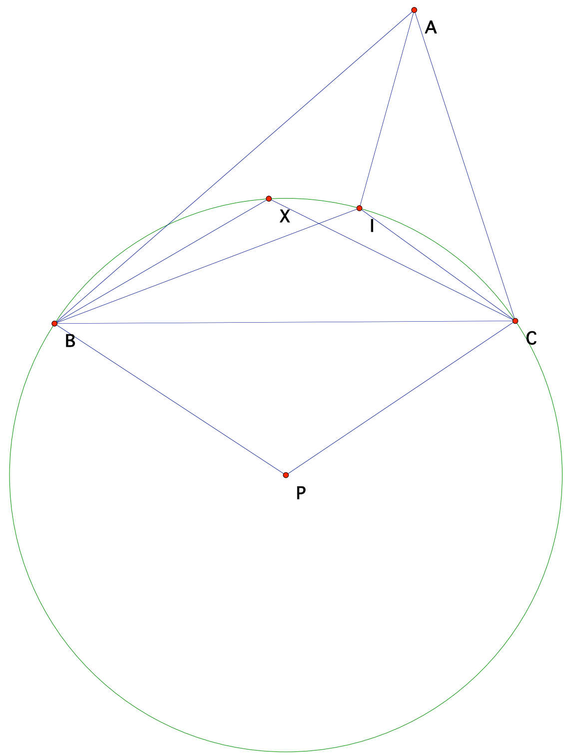
7. **Answer: $110 + 24\sqrt{6}$**

   We can calculate the area of the middle triangle using Heron's formula. Hence, we can calculate the semiperimeter of the triangle, 9, and then calculate the area as $\sqrt{9 \times (9-5) \times (9-6) \times (9-7)} = 6\sqrt{6}$. Notice that $\angle BCA$ and $\angle ECD$ are supplementary. Hence, we can rotate $\triangle ECD$ about point C so that segments $\overline{AC}$ and $\overline{CD}$ overlap and the resulting figure BAE will be a triangle. In this position, we can see that $\triangle BAC$ and $\triangle DCE$ have the same altitude, and since they have the same base (length 7) they must have the same area. By the same reasoning, all the triangles must have the same area. Hence, the total area of the figure is simply the areas of the squares plus four times the area of the middle triangle $25 + 36 + 49 + 4 \times 6 \times \sqrt{6} = 110 + 24\sqrt{6}$.

8. **Answer: $8\sqrt{3}$**

   The center of the sphere is located at the centroid of the tetrahedron, which is located $\frac{1}{4}$ of the way up the altitude from a face to the opposite vertex. In other words, the tetrahedron has height 4. Let its edge length be s. Then the altitude of a face is $s\frac{\sqrt{3}}{2}$, and the distance from the centroid of a face to a vertex is $\frac{2}{3}$ of that, which is $\frac{\sqrt{3}}{3}$. This length and the height of the tetrahedron form a right triangle, with an edge as the hypotenuse. That is, $\frac{1}{3}s^2 + 16 = s^2$. Thus $s^2 = 24$, and so the area of a face is $6\sqrt{3}$. The volume is $\frac{1}{3} \cdot 6\sqrt{3} \cdot 4 = 8\sqrt{3}$.

9. **Answer: $\sqrt{65}$**



Since $\angle ABX + \angle ACX$ and $\angle CBX + \angle BCX$ add up to $\angle B + \angle C$, we can see that $\angle CBX + \angle BCX = \frac{\angle B + \angle C}{2}$, so $\angle BXC = 90 + \frac{\angle A}{2}$. Thus X should lie on a fixed circle, which also goes through I, the incenter of the triangle.
Let P be the center of that circle. We have $\angle BPC = 2(180 - \angle BXC) = 180 - \angle A$, so P lies on the circumcircle of $\triangle ABC$. And from $BP = CP$ we have $\angle BAP = \angle CAP$, so A, I, P lie on the same line. Thus $X = I$ minimizes AI.
Then it is the problem of finding AI. First we find the radius r of incircle, which is $\frac{2S}{a+b+c} = \frac{2 \cdot 84}{13+14+15} = 4$. Suppose AB and the incircle meets at D. Then $AD = (AB + AC - BC)/2 = 7$, $AI = \sqrt{AD^2 + r^2} = \sqrt{7^2 + 4^2} = \sqrt{65}$.

10. **Answer: 6**

Notice that $ABCD$ is a cyclic quadrilateral. Set $AB = x$ and $AE = y$, so that $CE = 11 - y$. We apply the Power of a Point Theorem at the point E to get $y(11 - y) = 4 \cdot 5$, so that $11y - y^2 = 20$.

We have two pairs of similar triangles: $\triangle ABE \sim \triangle DCE$ and $\triangle ADE \sim \triangle BCE$; these yield $AD = \frac{3}{2}y$ and $CD = \frac{5x}{y}$. Applying Ptolemy's Theorem to quadrilateral $ABCD$ then yields

$$x \cdot \frac{5x}{y} + 6 \cdot \frac{3}{2}y = 9 \cdot 11$$

Clearly the denominators and rearranging, we see that the y terms are precisely those given by the Power of a Point Theorem:

$$5x^2 = 99y - 9y^2 = 9(11y - y^2) = 9 \cdot 20 = 180,$$

and therefore $x = 6$.