1. **Answer: $\frac{-1}{x^2+1}$**

   Notice that as $t \to 0$, both the numerator and the denominator approach 0. Thus, applying L'Hopital's rule on t (keeping x constant):

$$\frac{d}{dt}\left[\tan^{-1}\left(\frac{1}{x+t}\right)\right]_{t=0} = -\frac{1}{1+x^2}$$

2. **Answer: 1**

   Let $f(x) = e^x - x - \frac{x^3}{3}$. Then $f'(x) = e^x - 1 - x^2$. When $x < 0$, $e^x < 1$ and $1 + x^2 > 1$, so $f'(x) = e^x - (1+x^2) < 0$. Thus, f is decreasing on $(-\infty, 0)$. When $x = 0$, $f'(x) = f'(0) = e^0 - 1 - 0^2 = 1 - 1 = 0$. Finally, for $x > 0$, $f'(x) = e^x - 1 - x^2 > 0$ by a Maclaurin series expansion, so f is increasing on $(0, \infty)$. Thus, f must attain its minimum when $x = 0$, at which point f has the value $e^0 - 0 - \frac{0^3}{3} = 1$.

3. **Answer: $\sqrt{2}$**

   Consider:

$$\frac{d}{dt}\sin^{-1}(t - \sqrt{1/2})\Big|_{t=0} = \frac{d}{dt}\int_{-\infty}^{\infty} e^{tx} f(x)\,dx\Big|_{t=0} = \int_{-\infty}^{\infty} xe^{tx} f(x)\,dx\Big|_{t=0} = \int_{-\infty}^{\infty} xf(x)\,dx$$

$$\frac{d}{dt}\sin^{-1}(t - \sqrt{1/2})\Big|_{t=0} = \frac{1}{\sqrt{1 - \left(\sqrt{1/2} - t\right)^2}}\Bigg|_{t=0} = \frac{1}{\sqrt{1 - (1/2)}} = \sqrt{2}.$$

4. **Answer: $x = -\frac{2}{3}$ and $x = 0$**

   Notice that $f(x) \to 0$ as $x \to \pm\infty$. Since $9x^2 + 6x + 2$ has no real roots, the maximum value of $f(x)$ is attained at the maximum of the absolute values of the critical points of $\frac{3x+1}{9x^2+6x+2}$.

   The extrema of $\frac{3x+1}{9x^2+6x+2}$ occur at $x = -\frac{2}{3}$ and $x = 0$. It is easily checked that maxima of $f(x)$ occur at both of these points.

5. **Answer: $\frac{128\sqrt{3}}{27}$**

   Let the circular island be a circle of radius 2 centered at the origin. Without loss of generality, let the length of the rectangular base be from $-x$ to x and the width from $-y$ to y. Notice that by the equation of a circle, $x^2 = 4 - y^2$. Then

$$V = \frac{1}{3}(2x)^2(2y) = \frac{8}{3}x^2y = \frac{8}{3}(4 - y^2)y = \frac{8}{3}(4y - y^3)$$

$$\frac{dV}{dy} = \frac{8}{3}(4 - 3y^2) = 0 \implies y = \sqrt{\frac{4}{3}}$$

$$V = \frac{8}{3}\left(\frac{8}{3}\right)\sqrt{\frac{4}{3}} = \frac{128}{9\sqrt{3}} = \frac{128\sqrt{3}}{27}.$$

6. **Answer: 13**

   This is the evaluation of the mean of a Poisson distribution: for any λ,

$$\sum_{k=0}^{\infty} ke^{-\lambda}\frac{\lambda^k}{k!} = \sum_{k=1}^{\infty} ke^{-\lambda}\frac{\lambda^k}{k!} = \lambda\sum_{k=1}^{\infty} e^{-\lambda}\frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda}\sum_{m=0}^{\infty}\frac{\lambda^m}{m!} = \lambda e^{-\lambda}e^{\lambda} = \lambda.$$

7. **Answer: $\frac{-2\cos(t^2)}{t}$**

By the Leibniz integral rule, the above integral becomes

$$\begin{aligned}
&\int_{-\ln 1/t}^{\ln 1/t} -e^x \sin(te^x)\,dx + \cos(te^{\ln(1/t)})(-1/t) - \cos(te^{-\ln(1/t)})(1/t) \\
&= \left.\frac{\cos(te^x)}{t}\right|_{-\ln 1/t}^{\ln 1/t} - \frac{\cos(1)+\cos(t^2)}{t} \\
&= \frac{-2\cos(t^2)}{t}.
\end{aligned}$$

8. **Answer: ln 3**

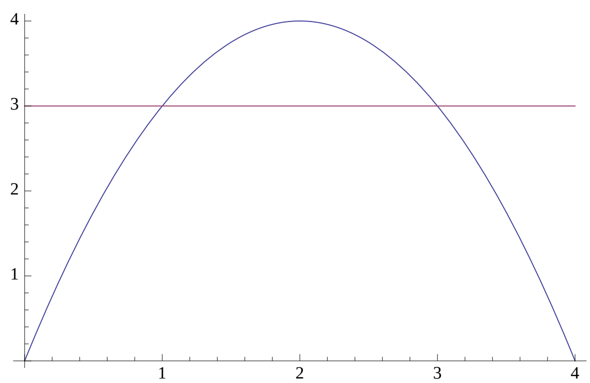
The partial sums of this sum are equal to

$$\begin{aligned}
&\left(\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{3n}\right) - 3\left(\frac{1}{3\cdot 1}+\frac{1}{3\cdot 2}+\cdots+\frac{1}{3\cdot n}\right) \\
&= \frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{3n} = \frac{1}{n}\left(\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\cdots+\frac{1}{1+\frac{2n}{n}}\right)
\end{aligned}$$

This is a Riemann sum, so as $n \to \infty$ the partial sums converge to

$$\int_0^2 \frac{1}{1+x}\,dx = \ln 3.$$

9. **Answer: 3**



As you can see it from this graph, $F(k)$ is the area of region that "lies between" $y = f(x) = x(4-x)$ and $y = k$. Let A be the region below $y = f(x)$ and above $y = k$, and B be the region below $y = k$ and above $y = f(x)$. Then $F(k) = A + B$. Meanwhile, we can find the area of A by integrating with respect to y-variable. Since x belongs to the interval of length $l(t)$ when $y = t$, we can say

$$A = \int_k^4 l(t)\,dt.$$

Apply the same reasoning to B, then we have

$$B = \int_0^k (4 - l(t))\,dt.$$

Thus, by the fundamental theorem of calculus,

$$\begin{aligned}
\frac{d}{dk}F(k) &= \frac{d}{dk}A + \frac{d}{dk}B \\
&= \frac{d}{dk}\left(\int_k^4 l(t)\,dt\right) + \frac{d}{dk}\left(\int_0^k (4-l(t))\,dt\right) \\
&= -l(k) + (4 - l(k)) \\
&= 4 - 2l(k).
\end{aligned}$$

Since $l(k)$ is decreasing by k, $F(k)$ achieves minimum when $\frac{d}{dk}F(k) = 0$. One can easily find that $k = 3$ if $l(k) = 2$, so the answer is 3.

10. **Answer: $y = -4x^2 + 5x - 7$**

Such a parabola intersects $f(x)$ precisely where $f'(x) = 0$. Hence, the value of the intersection points do not change when we replace $f(x)$ by $f(x)+g(x)f'(x)$ for any $g(x)$. Therefore, since $f'(x) = 6x^5-12x+6$, we must have that $f(x) - 1/6xf'(x) = -4x^2 + 5x - 7$ passes through the three critical points. Since three points determines a parabola uniquely, this must be the unique parabola passing through the three critical points.