1. **Answer:** $\frac{1+\sqrt{5}}{2}$

   Let $x = \sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}$. Then $x^2 = 1+\sqrt{1+\sqrt{1+\ldots}}$. Thus $x^2 = x+1$. The positive root of $x^2 - x - 1 = 0$ is $\frac{1+\sqrt{5}}{2}$.

2. **Answer:** $\frac{334703}{1665000}$

   This is simply
$$\frac{2010}{10000} + \frac{1}{10000} \cdot \frac{228}{999} = \frac{334703}{1665000}.$$

3. **Answer: 19801 and 20201**

   Notice that $4x^4 + 1 = 4x^4 + 4x^2 + 1 - (2x)^2 = (2x^2+2x+1)(2x^2-2x+1)$. Setting $x = 100$, we have that $400000001 = 19801 \cdot 20201$.

4. **Answer:** ± 123

   Note that $\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 9$. Thus, $x + \frac{1}{x} = \pm 3$. Therefore,
$$\begin{aligned} x^5 + \frac{1}{x^5} &= \left(x+\frac{1}{x}\right)^5 - 5\left(x^3+\frac{1}{x^3}\right) - 10\left(x+\frac{1}{x}\right) \\ &= \left(x+\frac{1}{x}\right)^5 - 5\left(x+\frac{1}{x}\right)^3 + 5\left(x+\frac{1}{x}\right) \\ &= \pm 123 \end{aligned}$$

5. **Answer: 17**

   The open lockers will be the ones with an odd number of odd divisors. These numbers are of the form $2^k \cdot n^2$, where n is odd. We can simply check that the open lockers are numbered
$$1, 2, 4, 8, 9, 16, 18, 25, 32, 36, 49, 50, 64, 72, 81, 98, 100.$$

6. **Answer: 1027**

   If $S(n)$ is the nth partial sum, note that if m is the kth triangular number, $S(m) = k^2$. Since $44^2 = 1936$ and $45^2 = 2025$, we want to begin our search at $44(44+1)/2 = 990$. Because $(2010-1936)/2 = 37$, 37 more $2s$ are needed, so the needed term is $n = 990 + 37 = 1027$.

7. **Answer: 21,26,31,36,41,46**
$$\begin{aligned} 6x+5 &\equiv -19 \mod 10 \\ 6x &\equiv -24 \mod 10 \\ x &\equiv -4 \mod \frac{10}{\gcd(10,6)} \\ x &\equiv -4 \mod 5 \\ x &\equiv 1 \mod 5 \end{aligned}$$

   That is, x is in the form $5k+1$ where k is an integer.

8. **Answer:** $3^{n+1} - 2^{n+1}$

   We use the fact that if $P(x)$ is a polynomial of degree n, then $P(x+1) - P(x)$ is a polynomial of degree $n-1$. Define $\Delta P(x) = P(x+1) - P(x)$. By induction on m, it can be easily proved that $\Delta^m P(x)$ is a polynomial of degree $n-m$ such that $\Delta^m P(k) = 2^m \cdot 3^k$ for $0 \le k \le n-m$ when $0 \le m \le n$. Moreover, $\Delta^{n+1} P$ is identically zero, since $\Delta^n P$ is degree zero and applying Δ to constants leaves zero. Thus

$$\begin{aligned}
P(n+1) &= P(n) + (P(n+1) - P(n)) \\
&= P(n) + \Delta P(n) \\
&= P(n) + \Delta P(n-1) + (\Delta P(n) - \Delta P(n-1)) \\
&= P(n) + \Delta P(n-1) + \Delta^2 P(n-1) \\
&= P(n) + \Delta P(n-1) + \Delta^2 P(n-2) + (\Delta^2 P(n-1) - \Delta^2 P(n-2)) \\
&= P(n) + \Delta P(n-1) + \Delta^2 P(n-2) + \Delta^3 P(n-2) \\
&= \cdots \\
&= \sum_{i=0}^{n} \Delta^i P(n-i) + \Delta^{n+1} P(0) \\
&= \sum_{i=0}^{n} 2^i 3^{n-i} \\
&= 3^{n+1} - 2^{n+1}.
\end{aligned}$$

9. **Answer: 31**

   Factor the equation as $(x+2)(y-5)+10 = 30$, or $(x+2)(y-5) = 20$. x must be 2 less than a factor of 20. The solutions for x are thus 2, 3, 8, and 18, which sum to 31.

10. **Answer:** $\mathbf{\frac{2}{1005}}$

    We can rewrite this equation as

$$\begin{aligned}
&\frac{x^2}{x^2-1} + \frac{x^2}{x^2-2} + \frac{x^2}{x^2-3} + \frac{x^2}{x^2-4} = \\
&\frac{1+(x^2-1)}{x^2-1} + \frac{2+(x^2-2)}{x^2-2} + \frac{3+(x^2-3)}{x^2-3} + \frac{4+(x^2-4)}{x^2-4} \\
&= (2010x-4) + 4 = 2010x.
\end{aligned}$$

    We divide by x; this makes us lose the solution $x = 0$, but this does not affect the sum of solutions. Therefore, we have

$$\frac{x}{x^2-1} + \frac{x}{x^2-2} + \frac{x}{x^2-3} + \frac{x}{x^2-4} = 2010$$

    Clearing denominators yields the polynomial equation

$$\begin{aligned}
&x((x^2-2)(x^2-3)(x^2-4) + (x^2-1)(x^2-3)(x^2-4) + \\
&\qquad (x^2-1)(x^2-2)(x^2-4) + (x^2-1)(x^2-2)(x^2-3)) \\
&= 2010(x^2-1)(x^2-2)(x^2-3)(x^2-4)
\end{aligned}$$

    The solutions that we want are therefore the roots of the polynomial

$$2010x^8 - 4x^7 + (\text{lower order terms}) = 0$$

    By Vieta's formulas, the sum of the roots of this polynomial equation is therefore $\frac{4}{2010}$.