Algebra Test
2007 Stanford Math Tournament
March 4, 2007

1. Find all real roots of f if $f(x^{1/9}) = x^2 - 3x - 4$.
2. Given that $x_1 > 0$ and $x_2 = 4x_1$ are solutions to $ax^2 + bx + c$ and that $3a = 2(c - b)$, what is x_1?
3. Let a, b, c be the roots of $x^3 - 7x^2 - 6x + 5 = 0$. Compute $(a + b)(a + c)(b + c)$.
4. How many positive integers n, with $n \leq 2007$, yield a solution for x (where x is real) in the equation $\lfloor x \rfloor + \lfloor 2x \rfloor + \lfloor 3x \rfloor = n$?
5. The polynomial $-400x^5 + 2660x^4 - 3602x^3 + 1510x^2 + 18x - 90$ has five rational roots. Suppose you guess a rational number which could possibly be a root (according to the rational root theorem). What is the probability that it actually is a root?
6. What is the largest prime factor of $4^9 + 9^4$?
7. Find the minimum value of $xy + x + y + \frac{1}{xy} + \frac{1}{x} + \frac{1}{y}$ for $x, y > 0$ real.
8. If $r + s + t = 3$, $r^2 + s^2 + t^2 = 1$, and $r^3 + s^3 + t^3 = 3$, compute rst.
9. Find $a^2 + b^2$ given that a, b are real and satisfy

$$a = b + \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{a + \cdots}}}; \qquad b = a - \cfrac{1}{b + \cfrac{1}{a - \cfrac{1}{b + \cdots}}}$$

10. Evaluate

$$\sum_{k=1}^{2007} (-1)^k k^2$$