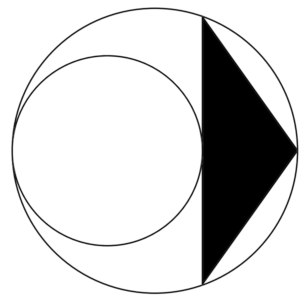
Stanford Mathematics Tournament

February 28, 2004

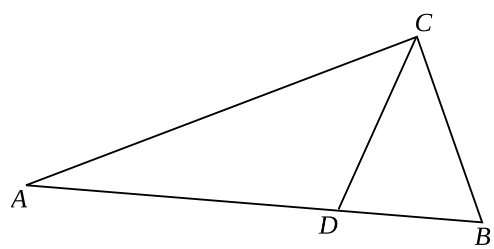
Individual Round: Geometry Subject Test — Problems

1. In the diagram below, the outer circle has radius 3, and the inner circle has radius 2. What is the area of shaded region?



2. A parallelogram has 3 of its vertices at (1, 2), (3,8), and (4, 1). Compute the sum of the possible x-coordinates for the 4th vertex.

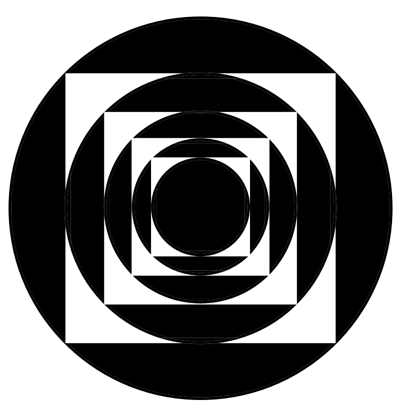
3. AC is 2004. CD bisects angle C. If the perimeter of ABC is 6012, find $(AC \times BC)/(AD \times BD)$.



4. P is inside rectangle $ABCD$. $PA = 2, PB = 3$, and $PC = 10$. Find PD.

5. Find the area of the region of the xy-plane defined by the inequality $|x| + |y| + |x + y| \leq 1$.

6. We inscribe a square in a circle of radius 1 and shade the region between them. Then we incribe another circle in the square and another square in the new circle and shade the region between the new circle and square. After we have repeated this process infinitely many times, what is the area of the shaded region?



7. Yet another trapezoid $ABCD$ has AD parallel to BC. AC and BD intersect at P. If $[ADP]/[BCP] = 1/2$, find $[ADP]/[ABCD]$. (Here the notation $[P_1 \cdots P_n]$ denotes the area of the polygon $P_1 \cdots P_n$.)

8. A triangle has side lengths 18, 24, and 30. Find the area of the triangle whose vertices are the incenter, circumcenter, and centroid of the original triangle.

9. Given is a regular tetrahedron of volume 1. We obtain a second regular tetrahedron by reflecting the given one through its center. What is the volume of their intersection?

10. Right triangle XYZ has right angle at Y and $XY = 228$, $YZ = 2004$. Angle Y is trisected, and the angle trisectors intersect XZ at P and Q so that X, P, Q, Z lie on XZ in that order. Find the value of $(PY + YZ)(QY + XY)$.