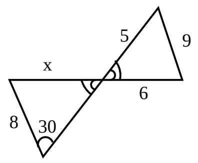
Advanced Topics Test
2004 Stanford Mathematics Tournament
February 28, 2004

1. Find $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix}\cdots\begin{pmatrix} 1 & 99 \\ 0 & 1 \end{pmatrix}$.

2. Write $-.1_{10} + .1_9 - .1_8 + .1_7$ as a repeating decimal in base 6.

3. Express $2\cos(10°) + \sin(100°) + \sin(1000°) + \sin(10000°)$ as $\sin(x°)$ where $-90 \leq x \leq 90$.

4. Find x in the diagram below.



5. How many rearrangements of the letters "STANFORD" contain the letters D, A, F, T in that order (although not necessarily consecutively)?

6. Write $F_{2006} + F_{2002} + F_{2000} + F_{1998} + \cdots + F_2 + F_0$ as aF_b where a and b are positive integers and where $F_0 = 1$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.

7. Adam, Bill, and Chris are playing a game of King of the Hill. Given who is king one day, the probability of who is king the next day is given by the chart below. If Adam is king on day 1, which day is Chris expected to first become king? (Leave your answer as a simplified fraction.)

$n+1$: / Day n:	Adam	Bill	Chris
Adam	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Bill	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$
Chris	0	0	1

8. Compute $(i+1)^{2004} - (i-1)^{2004}$, where $i = \sqrt{-1}$

9. How many solutions does $2004\cos x = x^2$ have?

10. Allen and Zorro decide to play a game using a deck of cards. They alternate turns picking a card from the deck with Allen going first. The first person to draw an ace wins. What is the probability that Allen wins?