Algebra Test
Stanford Math Tournament
February 23, 2002

1. Completely factor the polynomial $x^4 - x^3 - 5x^2 + 3x + 6$.

2. Solve for all real x that satisfy the equation $4^x = 2^x + 6$.

3. A clockmaker wants to design a clock such that the area swept by each hand (second, minute, and hour) in one minute is the same (all hands move continuously). What is the length of the hour hand divided by the length of the second hand?

4. Suppose that $n^2 - 2m^2 = m(n+3) - 3$. Find all integers m such that all corresponding solutions for n will *not* be real.

5. Solve for a, b, and c, given that $a \le b \le c$, and

$$\begin{aligned} a+b+c &= -1 \\ ab+bc+ac &= -4 \\ abc &= -2. \end{aligned}$$

6. How many integers x, from 10-99 inclusive, have the property that the remainder of x^2 divided by 100 is equal to the square of the units digit of x?

7. Find x satisfying $x = 1 + \frac{1}{x + \frac{1}{x + \cdots}}$.

8. Let f be a function with $\frac{f(x)f(y) - f(xy)}{3} = x + y + 2$. List all possible values for $f(36)$.

9. Given three numbers x_1, x_2, x_3, let $a = x_1 + x_2 + x_3$, $b = x_1x_2 + x_2x_3 + x_3x_1$, $c = x_1x_2x_3$, and $d = x_1^3 + x_2^3 + x_3^3$. If $a = 3, b = 7, c = 10$, what is the value of d?

10. Write $\sqrt[3]{2 + 5\sqrt{3 + 2\sqrt{2}}}$ in the form $a + b\sqrt{2}$, where a and b are integers.