Fundamentals of Market Making

**Example Order Book**

<table>
<thead>
<tr>
<th>Best Ask Price</th>
<th>Best Bid Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>104</td>
<td>95</td>
</tr>
<tr>
<td>102.5</td>
<td>93</td>
</tr>
<tr>
<td>101</td>
<td>92</td>
</tr>
<tr>
<td>100</td>
<td>90</td>
</tr>
</tbody>
</table>

**Source of profits:**

1. Repeatedly capture bid-ask spread
2. Obtain rebate for providing liquidity

**Risk factors:**

1. Inventory risk
2. Relatively uninformed trading
Model for price

Optimal Bid and Ask Model

We will use the framework developed by Avellaneda and Stoikov (2008), which obtains optimal bid and ask:

- Agent optimizes its value function:
  \[
  v(x, s, q, t) = E_t[-e^{-\gamma(x+qS_T)}]
  \]

  to obtain the market-maker's indifference price:
  \[
  r(s, t) = s - q\gamma\sigma^2(T - t)
  \]

- Which allows the market-maker to obtain the optimal spread:
  \[
  \delta^a + \delta^b = \gamma\sigma^2(T - t) + \frac{2}{\gamma}\ln\left(1 + \frac{\gamma}{\kappa}\right)
  \]

**Problem:** Does not address inventory risk properly. Symmetric spread is an issue.
Model for inventory control

Dynamic Order Size

We will use a decaying function to model the size of our orders, unlike Guéant, Lehalle and Fernandez-Tapia (2012), who cap trading at a maximum inventory level.

This allows us to keep trading and profit from rebates.
Preliminary results

We simulated our strategy on AAPL on 2/24/2014 following a previous group’s report of parameters. Start trading at 9:30am and end at 4:00pm.

1. Negative P&L $\Rightarrow -12,908.762$ at 4pm
2. Inventory risk is controlled
1. We buy high and sell low ⇒ Negative spread
2. The optimal spread is a function of time

\[
\delta^a + \delta^b = \gamma \sigma^2 \left( T - t \right) + \frac{2}{\gamma} \ln \left( 1 + \frac{\gamma}{\kappa} \right)
\]

Ideally, the spread changes in a more dynamic fashion, depending on other parameters such as time-varying volatility.
Future Work

1. Dynamic spread
   - Volatility modeling
   - Asymmetric spread

2. Parameter estimation
   - Estimate $\sigma$ and $\kappa$ from historical data
   - Calibrate $\gamma$

3. A different pricing model
References

