

# Review

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ENGR108  
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December 4, 2025

# Outline

Course topics

What you can now do

What's next

# Vectors

- ▶ definitions, notation, stacking, slicing
- ▶ addition/subtraction, scalar multiplication
- ▶ inner product, norm, distance, RMS value
- ▶ mean and standard deviation
- ▶ Cauchy-Schwarz inequality, angle
- ▶ correlation coefficient

# Linear independence

- ▶ linear combination of vectors
- ▶ linear dependence and independence
- ▶ independence-dimension inequality
- ▶ basis
- ▶ orthonormal set of vectors, Gram-Schmidt algorithm

# Matrices

- ▶ definitions, notation, block matrices, submatrices
- ▶ addition/subtraction, scalar multiplication
- ▶ matrix-vector multiplication, linear equations
- ▶ matrix multiplication
- ▶ QR factorization
- ▶ left and right inverses, inverse, pseudo-inverse
- ▶ solving linear equations

## Theory (linear algebra)

- ▶ columns of  $A$  are independent  $\iff$  Gram matrix  $A^T A$  is invertible
- ▶  $A$  has a left inverse  $\iff$  columns of  $A$  are linearly independent
- ▶  $A$  has a right inverse  $\iff$  rows of  $A$  are linearly independent
- ▶ (square)  $A$  is invertible  $\iff$ 
  - columns are independent, or
  - rows are independent

## Least-squares

- ▶ basic least squares problem
- ▶ solution via pseudo-inverse, QR factorization
- ▶ multi-objective least squares via weighted sum
- ▶ equality-constrained least squares
- ▶ solution via KKT system
- ▶ least-norm solution of under determined equations

## Fitting models to data

- ▶ least squares data fitting
- ▶ regression model
- ▶ validation on a test set
- ▶ feature engineering
- ▶ regularization

# Classification

- ▶ Boolean classification
- ▶ false positives, false negatives, error rate
- ▶ least squares classifier
- ▶ multi-class classifier

## Computational complexity

- ▶ floating-point operation (flop)
- ▶ vector-vector operations (inner product, norm, . . .)
- ▶ matrix-vector multiplication, matrix-matrix multiplication
- ▶ QR factorization
- ▶ inverse, solving linear equations
- ▶ least squares
- ▶ linearly constrained least squares
- ▶  $2 \times \text{big} \times \text{small}^2$  mnemonic

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## How to impress your friends, employers, co-founders

- ▶ run  $k$ -means on a set of vectors
- ▶ set up and solve a square system of linear equations
- ▶ form and solve a (constrained, multi-objective) least squares problem
- ▶ make a prediction model (or classifier) and validate it

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## Some related courses: Math & computation

- ▶ Math 113 (linear algebra)
- ▶ Math 104 (applied linear algebra beyond ENGR 108)
- ▶ EE 263 (more applied linear algebra)
- ▶ CME 200 (computational linear algebra)
- ▶ CS 109, EE 178, MS&E 120 (probability)
- ▶ MS&E 111 (basic optimization)
- ▶ EE 364A (convex optimization)

## Some related courses: Applications

- ▶ EE 104 (intro. to machine learning)
- ▶ CS 221 (artifical intelligence)
- ▶ CS 229 (machine learning)
- ▶ Stats 101 (data science)
- ▶ ENGR 105 (basic control)
- ▶ MS&E 145 (investment science)
- ▶ ... and many others