Vectors in Julia

ENGR108
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September 9, 2021
Vectors in Julia

main topics:

- how to create and manipulate vectors in Julia
- how Julia notation differs from math notation
Outline

Vectors

Vector operations

Norm and distance
Vectors

- Vectors are represented by arrays in Julia.
- To create the 3-vector
  
  \[ x = (8, -4, 3.5) = \begin{bmatrix} 8 \\ -4 \\ 3.5 \end{bmatrix} \]
  
  use
  
  \[ x = [8, -4, 3.5] \]
  
  \((x = [8;-4;3.5] \text{ also works})\)

- Watch out for similar looking expressions
  
  - \((8, -4, 3.5)\) and \([8 \ -4 \ 3.5]\) are not equivalent in Julia

- Length of a vector: \text{length}(x)
Ranges

- to get a range from \( i \) to \( j \) (for \( i \leq j \)), use a colon (\( : \))
  - the range from 1 to 10 is 1:10
  - \texttt{collect(1:10)} returns the array

- the default increment between values is 1. (1:3 is 1, 2, 3)

- to specify an increment size add an additional argument:
  - the range from 1 to 10 with a step size of 0.1 is 1:0.1:10
Indexing and slicing

- Indexes run from 1 to \( n \): \( x_2 \) is \( x[2] \)
- Can also set an element, e.g., \( x[3] = 10.5 \)
- Use a range to select more than one element
  - \( x[2:3] \) selects the second and third elements
- \( x[end] \) selects the last element
- To select every other element use \( x[1:2:end] \)
Block vectors

▶ to form a stacked vector like

\[ a = (b, c) = \begin{bmatrix} b \\ c \end{bmatrix} \]

(with \( b \) and \( c \) vectors)

\[ a = [b; c] \]

(a = \([b, c]\) does NOT work)

▶ can mix vectors and scalars:

\[ a = [b; 2; c; -6] \]
Basic functions for arrays

- sum of (the entries of) a vector: \( \text{sum}(x) \)
- mean of the entries (\( \text{avg}(x) \)): \( \text{mean}(x) \)
- \( 0_n \) is \( \text{zeros}(n) \)
- \( 1_n \) is \( \text{ones}(n) \)
Creating unit vectors

- form $e_3$ with length 10
- create a zero vector of size 10 then set the third element to 1

```matlab
e_3 = zeros(10); e_3[3] = 1;
```
List of vectors

- to form a list with vectors $a$, $b$, and $c$:
  
  ```python
  vector_list = [a,b,c]
  ```

- the second vector in this list is `vector_list[2]`

- to access an element in a vector: `vector_list[2][3]`
Outline

Vectors

Vector operations

Norm and distance
Vector addition and subtraction

- Vector addition uses $+$, for example
  \[
  \begin{bmatrix}
  1 \\
  2 \\
  3 \\
  \end{bmatrix}
  +
  \begin{bmatrix}
  4 \\
  5 \\
  6 \\
  \end{bmatrix}
  \]

  is written
  \[
  [1, 2, 3] + [4, 5, 6]
  \]

- Subtraction uses $-$

- The arrays must have the same length (unless one is scalar)
Scalar-vector addition

- in Julia, a scalar and a vector can be added
- the scalar is added to each entry of the vector

\[ [2, 4, 8] + 3 \]

gives (in mathematical notation)

\[
\begin{bmatrix}
2 \\
4 \\
8 \\
\end{bmatrix} + 3 \mathbf{1} =
\begin{bmatrix}
5 \\
7 \\
11 \\
\end{bmatrix}
\]
Scalar-vector multiplication

- scalar-vector multiplication uses *
- for example,

\[
(-2) \begin{bmatrix}
1 \\
9 \\
6
\end{bmatrix}
\]

is written

\[-2 \times [1, 9, 6]\]

- the other order gives the same result:

\[[1, 9, 6] \times -2\]
Inner product

- inner product $a^T b$ is written as $\text{dot}(a,b)$
- $a$ and $b$ must have the same length
Outline

Vectors

Vector operations

Norm and distance
Norm and distance

- the norm $\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$ is written norm(x)
- $\text{dist}(x, y) = \|x - y\|$ is written norm(x-y)
RMS value

- \( \text{rms}(x) \) is defined as
  
  \[
  \text{rms}(x) = \sqrt{\frac{1}{n} (x_1^2 + \cdots + x_n^2)} = \frac{\|x\|}{\sqrt{n}}.
  \]

- can be expressed as
  
  \[
  \text{rms}_x = \frac{\text{norm}(x)}{\sqrt{\text{length}(x))}}
  \]
standard deviation is defined as

\[
\text{std}(x) = \frac{\| x - \text{avg}(x) \mathbf{1} \|}{\sqrt{n}}
\]

which can be expressed as

\[
\text{std\_of\_x} = \frac{\text{norm}(x - \text{mean}(x))}{\sqrt{\text{length}(x)}}
\]

warning: the Julia function \text{std} does \textit{not} use this definition
the angle between two vectors $a$ and $b$ is

$$\angle(a, b) = \arccos\left(\frac{a^T b}{\|a\| \|b\|}\right)$$

can be expressed as

$$\text{angle}_a_b = \acos\left(\frac{\text{dot}(a,b)}{\text{norm}(a) \times \text{norm}(b)}\right)$$