

Matrix inverses in Julia

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Matrix inverses in Julia

- ▶ QR factorization
- ▶ inverse
- ▶ pseudo-inverse
- ▶ backslash operator

QR factorization

- ▶ the `qr` command finds the QR factorization of a matrix

`A = rand(5, 3)`

`Q, R = qr(A)`

- ▶ when columns of $n \times k$ matrix A are independent, `qr` is same as ours
- ▶ when columns are *dependent*, `qr` is *not* same as ours
 - $A = QR$, $Q^T Q = I$, and $R_{ij} = 0$ for $i > j$ always holds
 - R can have zero or negative diagonal entries
 - R is not square when A is wide

Checking linear independence with Julia's QR

- ▶ let's check if columns of A are linearly independent
- ▶ A must be tall or square
- ▶ columns are linearly independent if and only if R has no 0 diagonal entries
- ▶ check if columns of (tall or square) A are linearly independent:

```
a1 = rand(5)
```

```
a2 = rand(5)
```

```
A = [a1 a2 a1+a2] # linearly dependent columns
```

```
Q, R = qr(A)
```

```
# find the entry of diagonal of R closest to 0
```

```
# R can have negative entries
```

```
min(abs(diag(R)))
```

Inverse

- ▶ `inv(A)` returns the inverse matrix A^{-1}
- ▶ Julia will issue an error if
 - A is not square
 - A is not invertible
- ▶ you can solve square set of linear equations $Ax = b$, with invertible A , using

```
b = rand(5,1)
```

```
A = rand(5,5)
```

```
x = inv(A)*b
```

```
norm(A*x-b) # check residual
```

but there is a better way, using backslash

Pseudo-inverse

- ▶ for a $m \times n$ matrix A , $\text{pinv}(A)$ will return the $n \times m$ pseudo-inverse
- ▶ if A is square and invertible
 - $\text{pinv}(A)$ will return the inverse A^{-1}
- ▶ if A is tall with linearly independent columns
 - $\text{pinv}(A)$ will return the left inverse $(A^T A)^{-1} A^T$
- ▶ if A is wide with linearly independent rows
 - $\text{pinv}(A)$ will return the right inverse $A^T (A A^T)^{-1}$
- ▶ in other cases, $\text{pinv}(A)$ returns an $m \times n$ matrix, but
 - it is not a left or right inverse of A
 - what it is is beyond the scope of this class

The backslash operator

- ▶ given A and b , the \backslash operator solves the linear system $Ax = b$ for x
- ▶ for a $m \times n$ matrix A and a m -vector b , $A \backslash b$ returns a n -vector x
- ▶ if A is square and invertible
 - $x = A^{-1}b$
 - the unique solution of $Ax = b$
- ▶ if A is tall with linearly independent columns
 - $x = (A^T A)^{-1} A^T b$
 - the least squares approximate solution of $Ax = b$
- ▶ if A is wide with linearly independent rows
 - $x = A^T (A A^T)^{-1} b$
 - x is the least norm solution of $Ax = b$
- ▶ in other cases, $A \backslash b$ will print an error message
- ▶ uses a factor and solve method similar to QR

Solving matrix systems with backslash

- ▶ solve matrix equation $AX = B$ for X , with A square
- ▶ with $X = [x_1 \cdots x_k]$, $B = [b_1 \cdots b_k]$, same as solving k linear systems

$$Ax_1 = b_1, \dots, Ax_k = b_k$$

- ▶ $X = A \setminus B$ solves the system, doing the right thing:
 - factor A once (order n^3)
 - back substitution to get $x_i = A^{-1}b_i$, $i = 1, \dots, k$ (order kn^2)