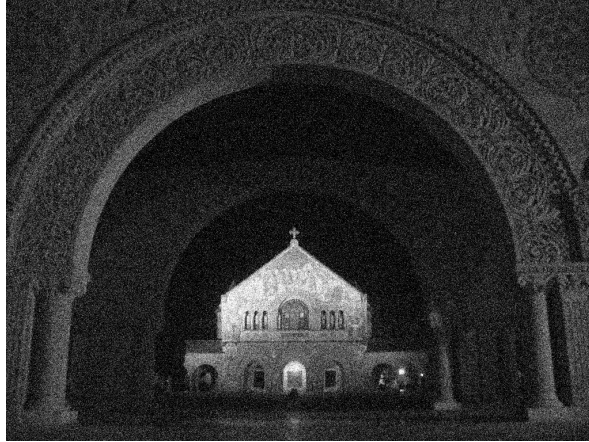


Point operations for combining images

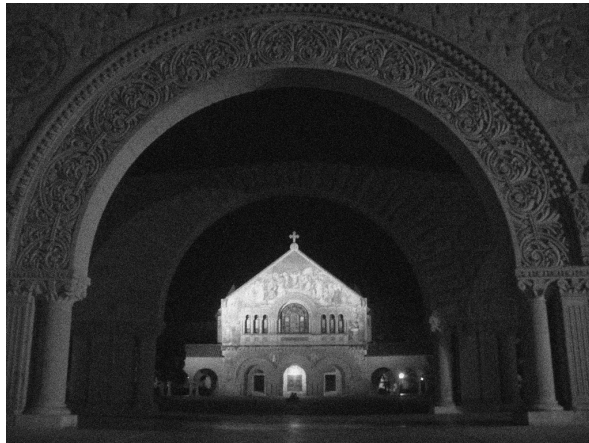
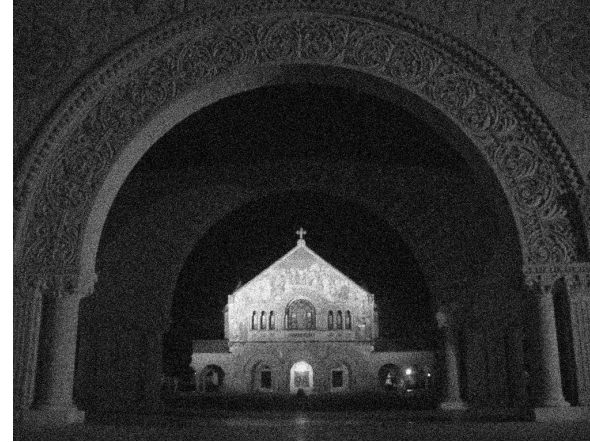
- Image averaging for noise reduction
- Combination of different exposures for high-dynamic range imaging
- Image subtraction for change detection
- Need for accurate alignment
- Displacement estimation

Image averaging for noise reduction

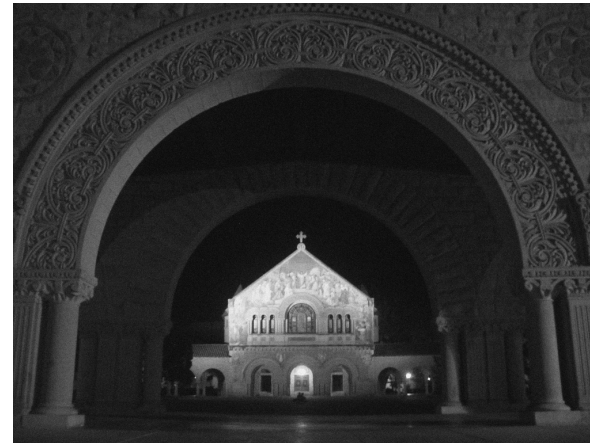
1 image



2 images



8 images



32 images



High-dynamic range imaging



-8 f-stops



-2 f-stops



+2 f-stops



+4 f-stops



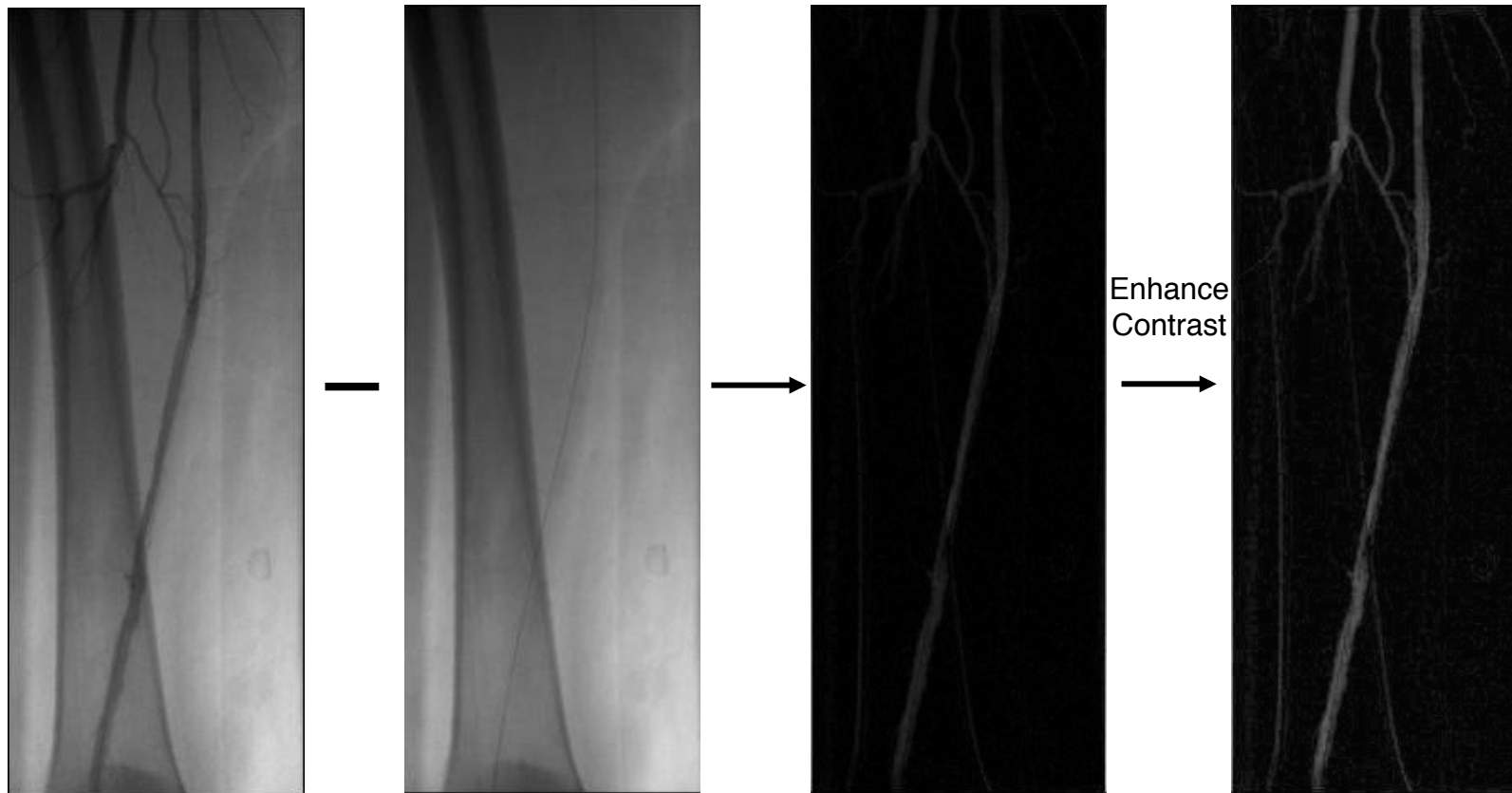
Blended image from
Exposure Fusion

[Tom Mertens et al. 2007]

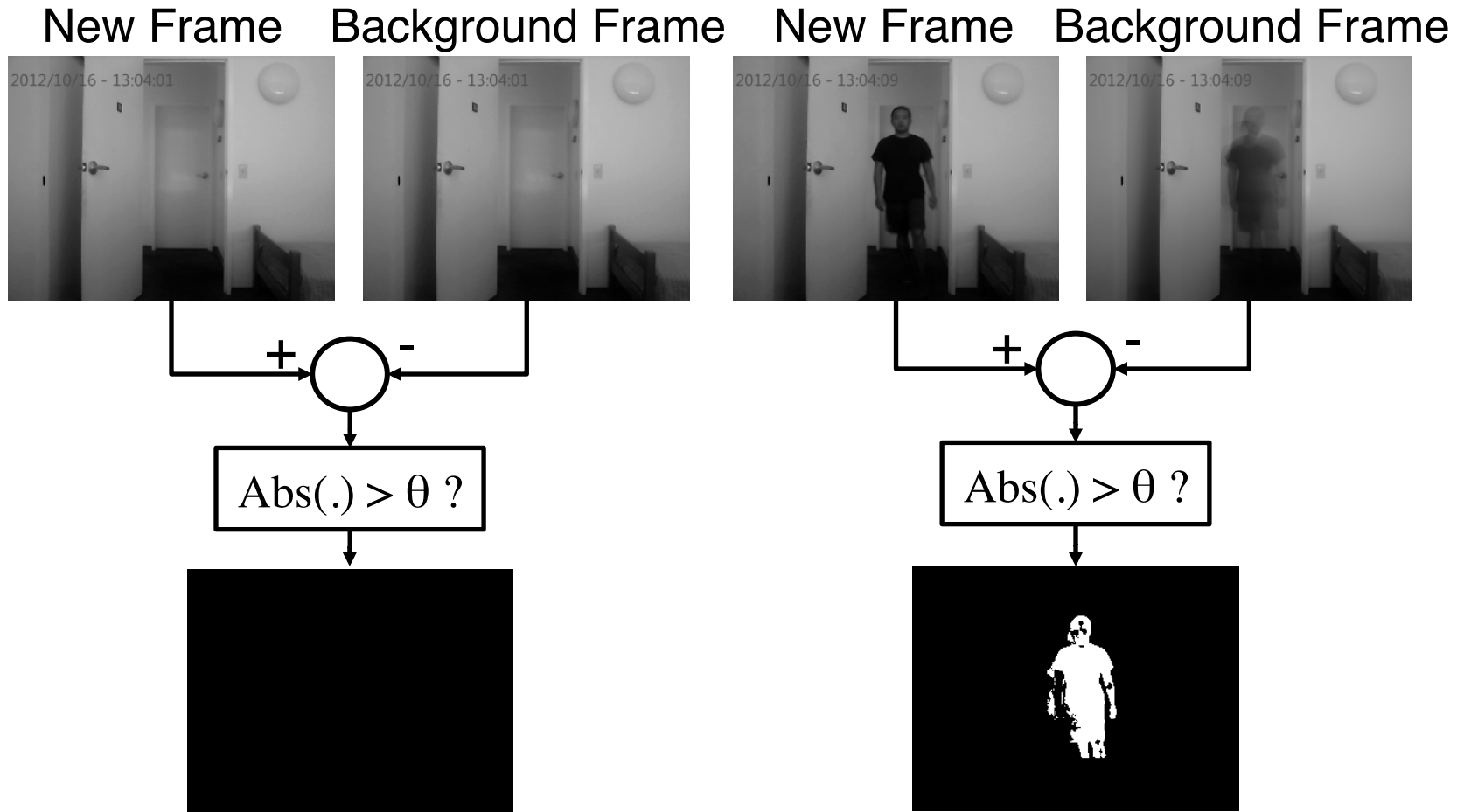


Image subtraction

- Find differences/changes between 2 mostly identical images
- Example: digital subtraction angiography



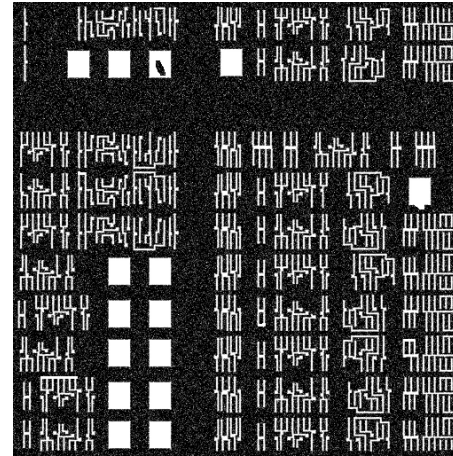
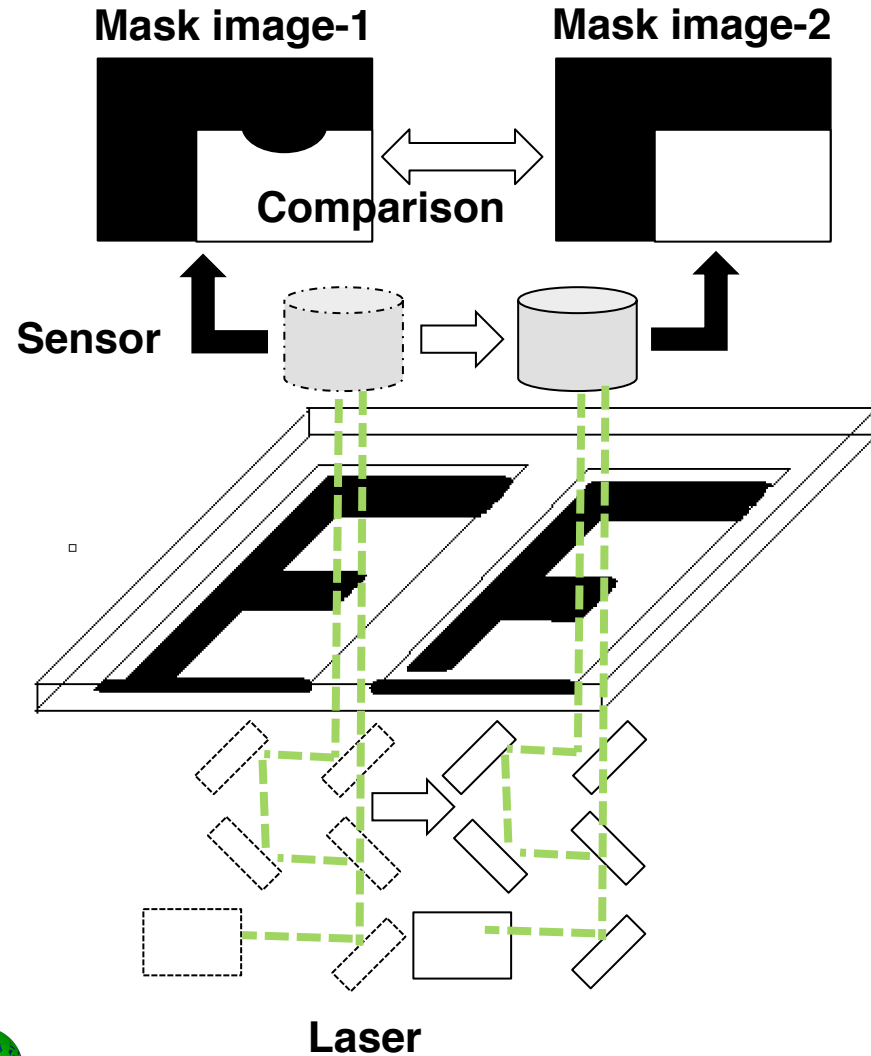
Video background subtraction



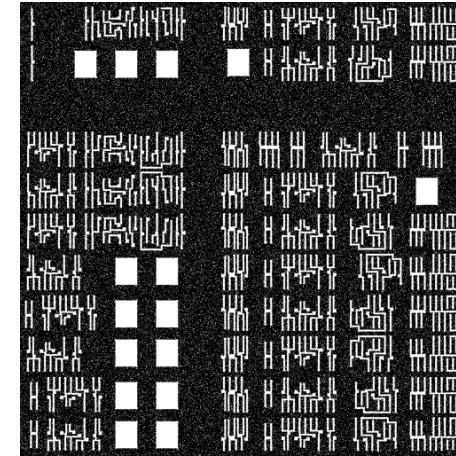
Update:
 $\text{Background}[t] := \alpha \text{Background}[t-1] + (1 - \alpha) \text{New}[t]$



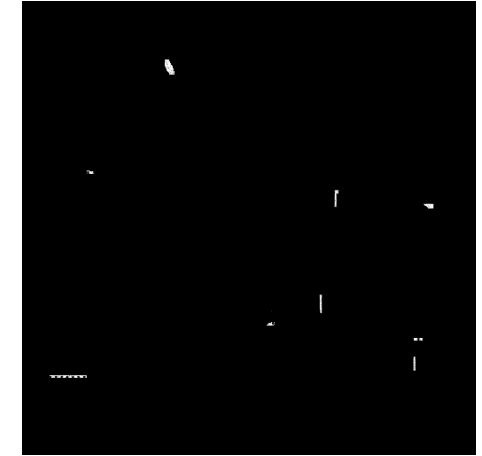
Image subtraction in IC manufacturing: inspection of photomasks



Mask image-1



Mask image-2



Difference image



Where is the defect?

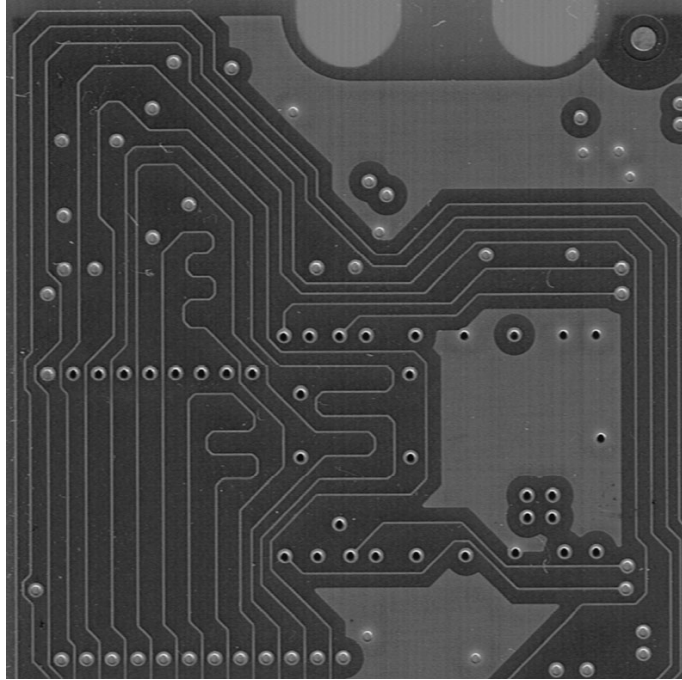


Image $g[x,y]$ (no defect)

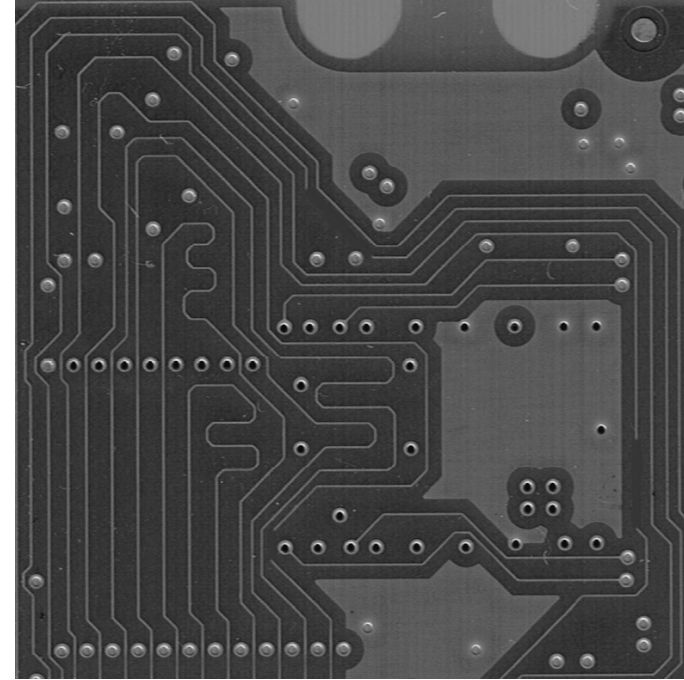
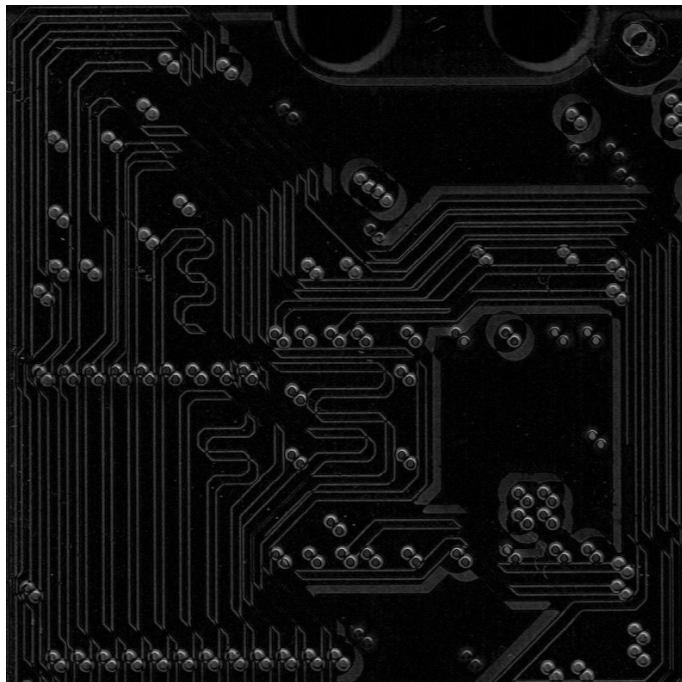


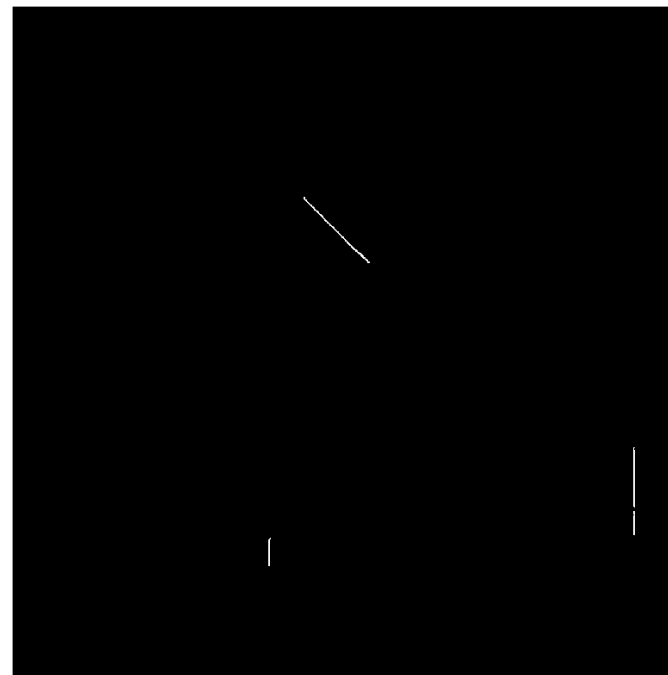
Image $f[x,y]$ (w/ defect)



Absolute difference between two images



$|f-g|$ w/o alignment



$|f-g|$ w/ alignment



Displacement estimation by block matching

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

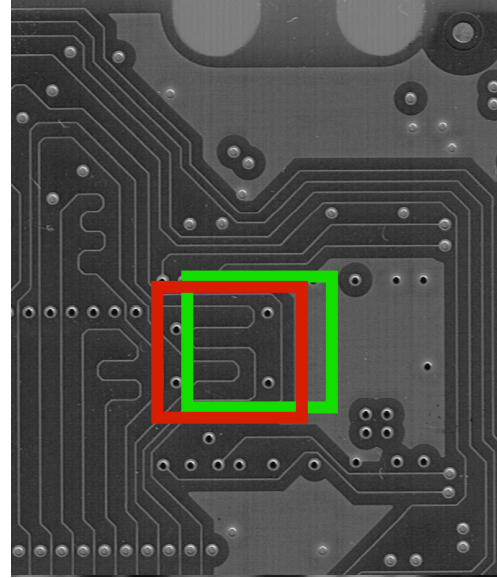


Image $g[x,y]$

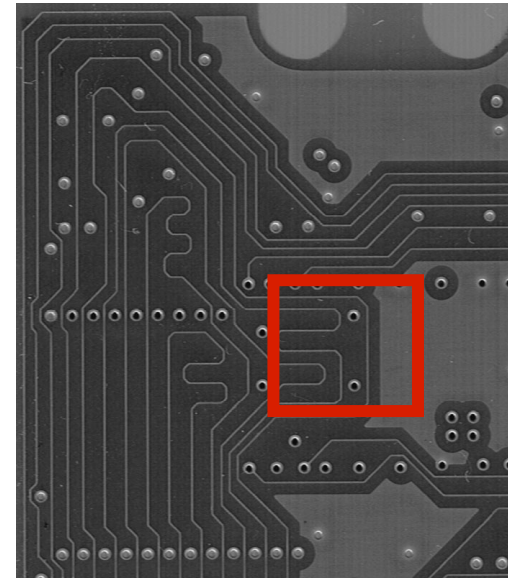


Image $f[x,y]$

Rectangular array of pixels is selected as a measurement window

Displacement estimation by block matching

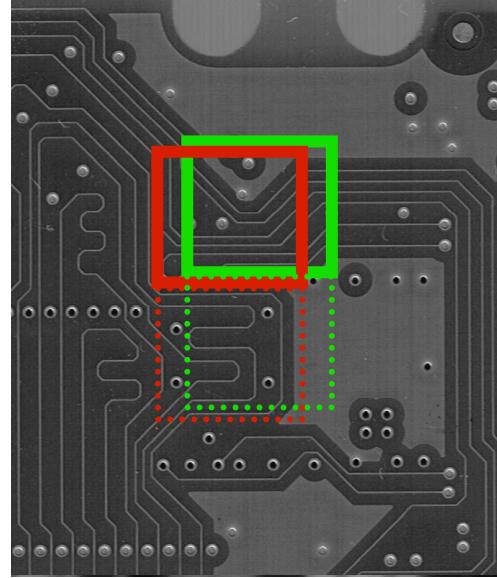


Image $g[x,y]$

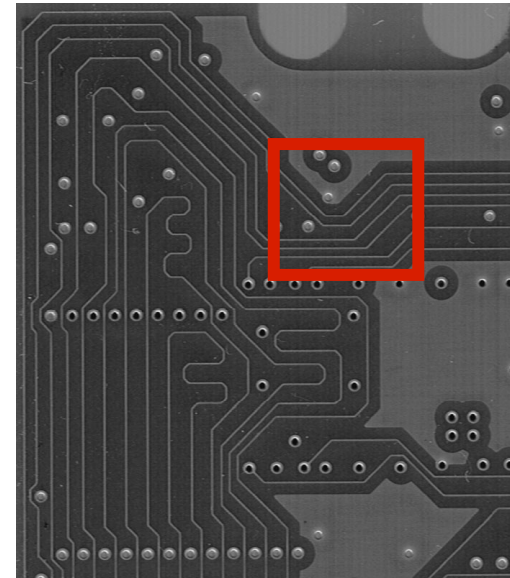


Image $f[x,y]$

... process repeated for another measurement window position.

Integer pixel shifts

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

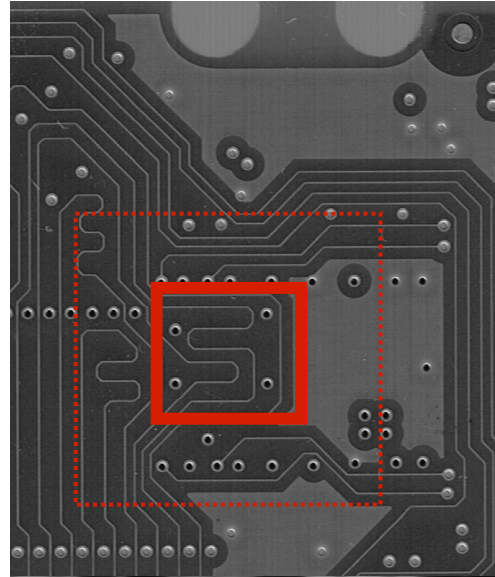


Image $g[x,y]$

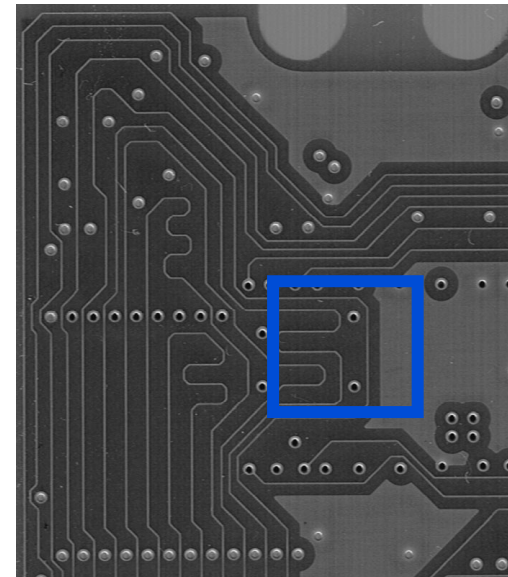


Image $f[x,y]$

Rectangular array of pixels is selected as a measurement window

Integer pixel shifts

| | | | | | | | | | | |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 28 | 42 | 42 | 43 | 44 | 40 | 32 | 20 | 29 | 32 | 22 |
| 30 | 44 | 45 | 45 | 45 | 42 | 30 | 21 | 26 | 27 | 18 |
| 35 | 54 | 54 | 53 | 52 | 52 | 52 | 51 | 49 | 41 | 45 |
| 40 | 63 | 62 | 63 | 62 | 59 | 59 | 59 | 55 | 51 | 45 |
| 74 | 121 | 120 | 120 | 119 | 118 | 118 | 118 | 118 | 112 | 117 |
| 79 | 127 | 130 | 128 | 128 | 126 | 125 | 125 | 125 | 119 | 118 |
| 80 | 129 | 131 | 129 | 129 | 127 | 126 | 126 | 126 | 120 | 119 |
| 50 | 78 | 77 | 71 | 73 | 73 | 73 | 73 | 67 | 65 | 62 |
| 22 | 37 | 37 | 37 | 39 | 40 | 40 | 41 | 41 | 38 | 25 |

| | | | | | |
|-----|-----|-----|-----|----|----|
| 54 | 53 | 52 | 49 | 31 | 21 |
| 62 | 63 | 59 | 60 | 44 | 33 |
| 120 | 114 | 112 | 111 | 80 | 32 |
| 130 | 128 | 124 | 125 | 88 | 24 |
| 131 | 124 | 127 | 127 | 96 | 42 |
| 77 | 71 | 73 | 75 | 63 | 52 |

Rectangular array of pixels is selected as a measurement window

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

Error metric

- *Sum of Squared Differences*

Sum all values in measurement window

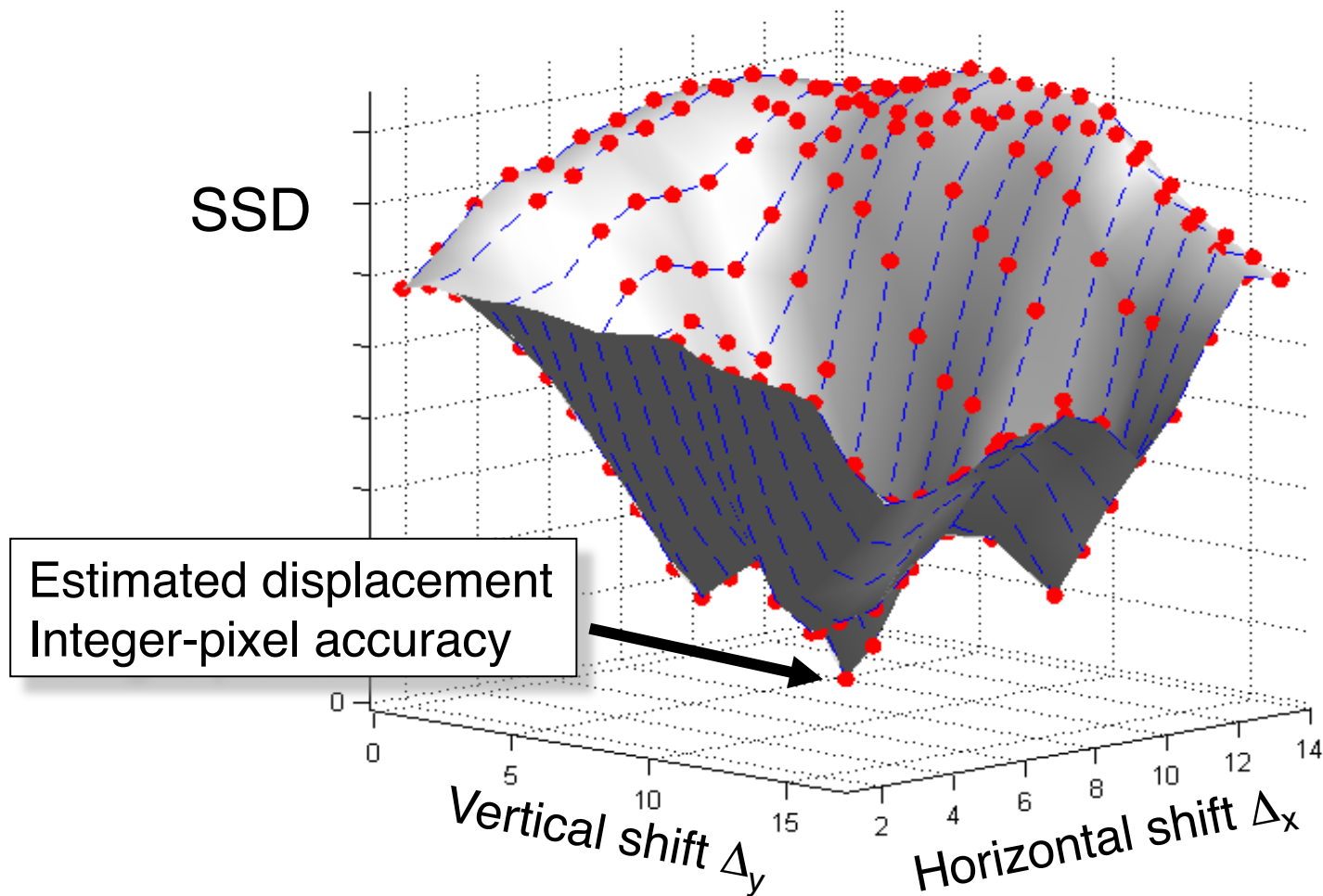
$$SSD[\Delta_x, \Delta_y] = \sum_{[x,y] \in \text{msmnt window}} \left(f[x,y] - g[x + \Delta_x, y + \Delta_y] \right)^2$$

Horizontal displacement

Vertical displacement

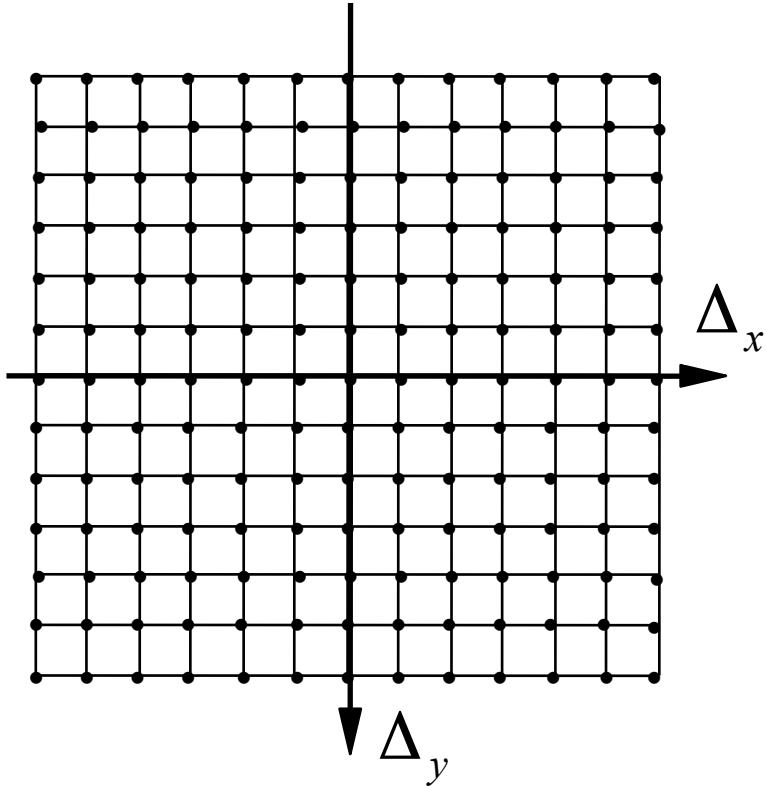
- Alternatives: SAD (*Sum of Absolute Differences*), cross correlation, mutual information . . .
- Robustness against outliers: sum of saturated squared differences, median of squared differences . . .

SSD values resulting from block matching



Block matching: search strategies

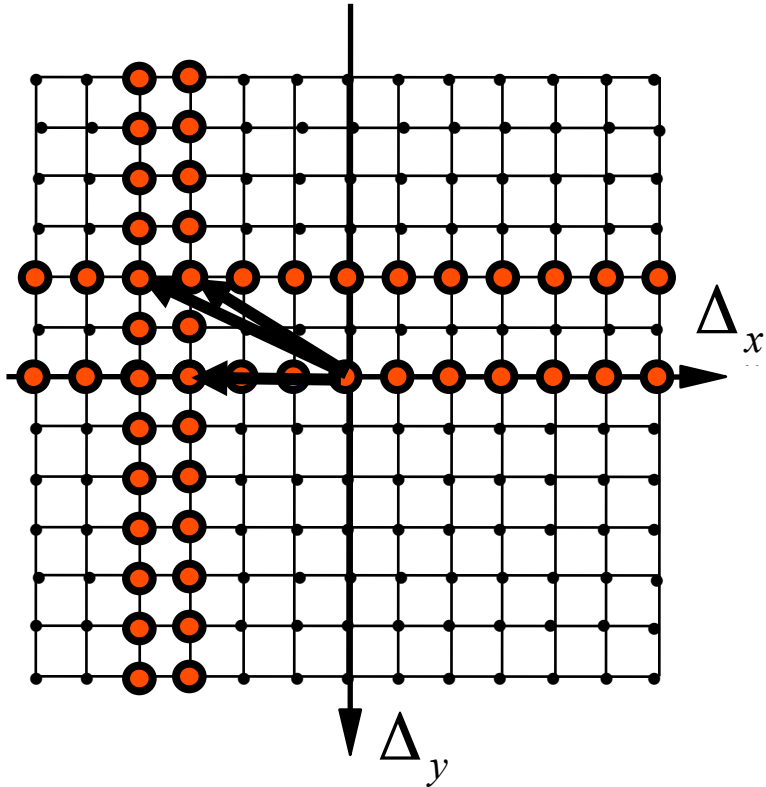
Full search



- All possible displacements within the search range are compared.
- Computationally expensive
- Highly regular, parallelizable

Block matching: search strategies

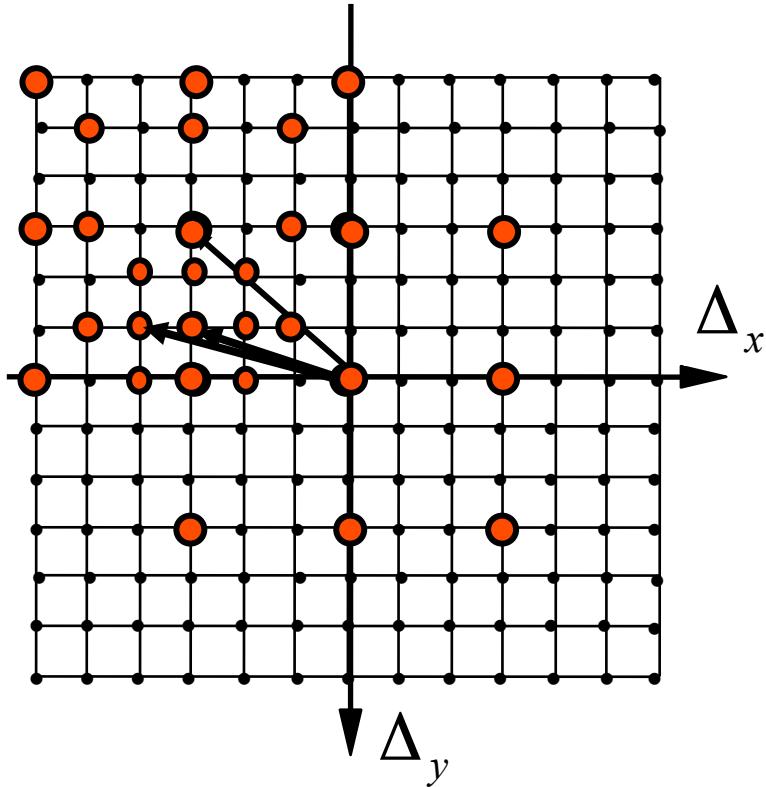
Conjugate direction search



- Alternate search in x and y directions
- Stop when there is no further improvement

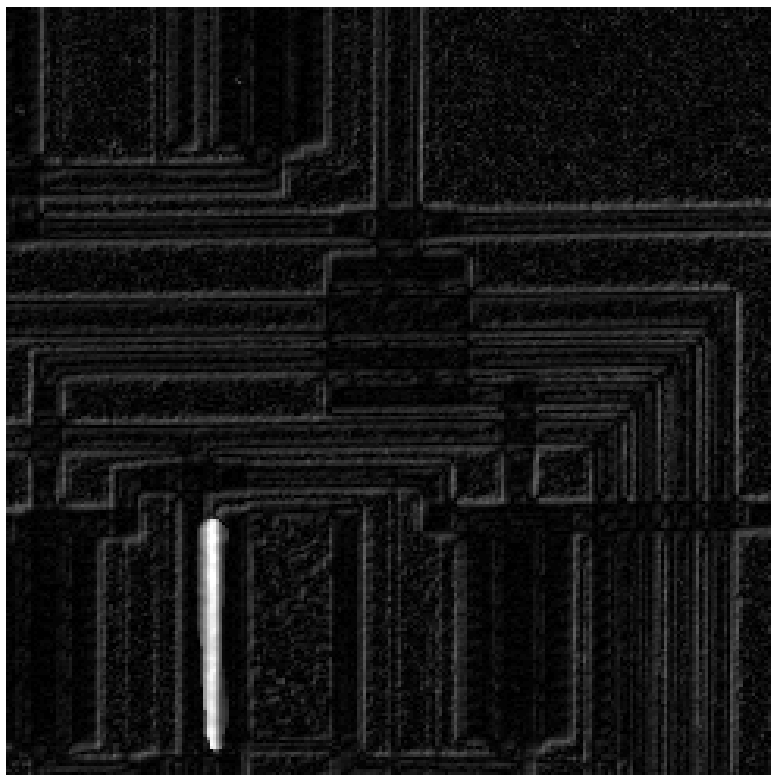
Block matching: search strategies

Coarse-to-fine

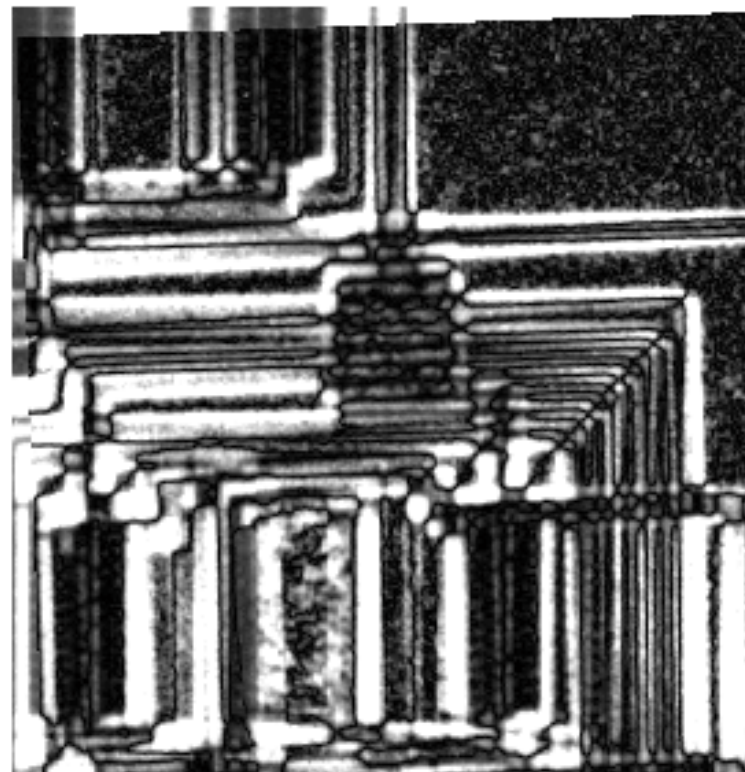


- Start with coarsely spaced candidate displacements
- Smaller pattern when best match is in the middle
- Stop when desired displacement accuracy is reached

Absolute difference between images

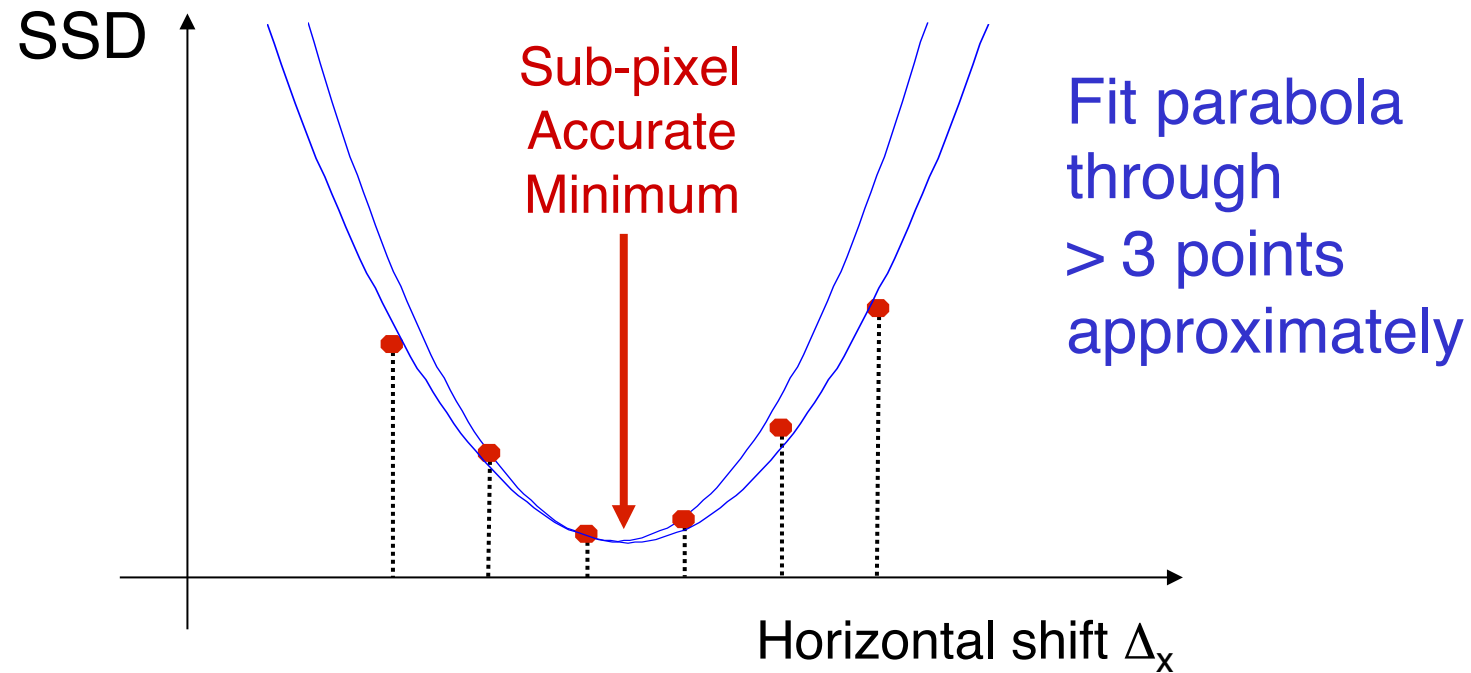


w/ integer-pixel alignment

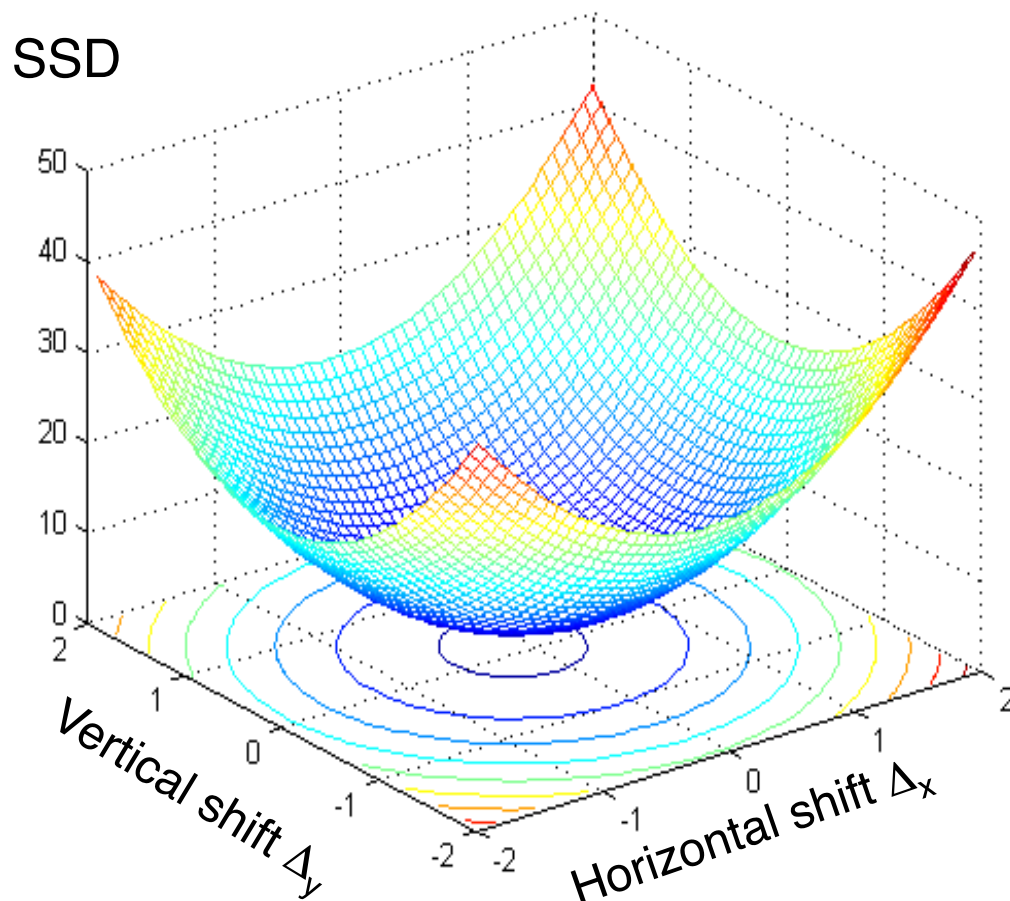


w/o alignment

Interpolation of the SSD Minimum



2-d Interpolation of SSD Minimum

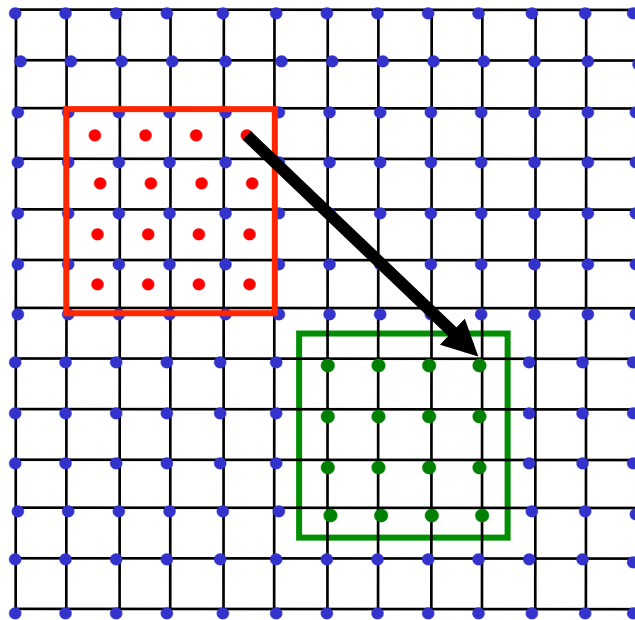


Paraboloid

- Perfect fit through 6 points
- Approximate fit through > 6 points

Sub-pixel accuracy

- Interpolate pixel raster of the reference image to desired sub-pixel accuracy (e.g., by bi-linear or bi-cubic interpolation)
- Straightforward extension of displacement vector search to fractional accuracy
- Example: half-pixel accurate displacements



$$\begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix} = \begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix}$$