Edge detection

- Gradient-based edge operators
  - Prewitt
  - Sobel
  - Roberts
- Laplacian zero-crossings
- Canny edge detector
- Hough transform for detection of straight lines
- Circle Hough Transform

Gradient-based edge detection

- Idea (continous-space): local gradient magnitude indicates edge strength

$$\left|grad\left(f\left(x,y\right)\right)\right| = \sqrt{\left(\frac{\partial f\left(x,y\right)}{\partial x}\right)^2 + \left(\frac{\partial f\left(x,y\right)}{\partial y}\right)^2}$$

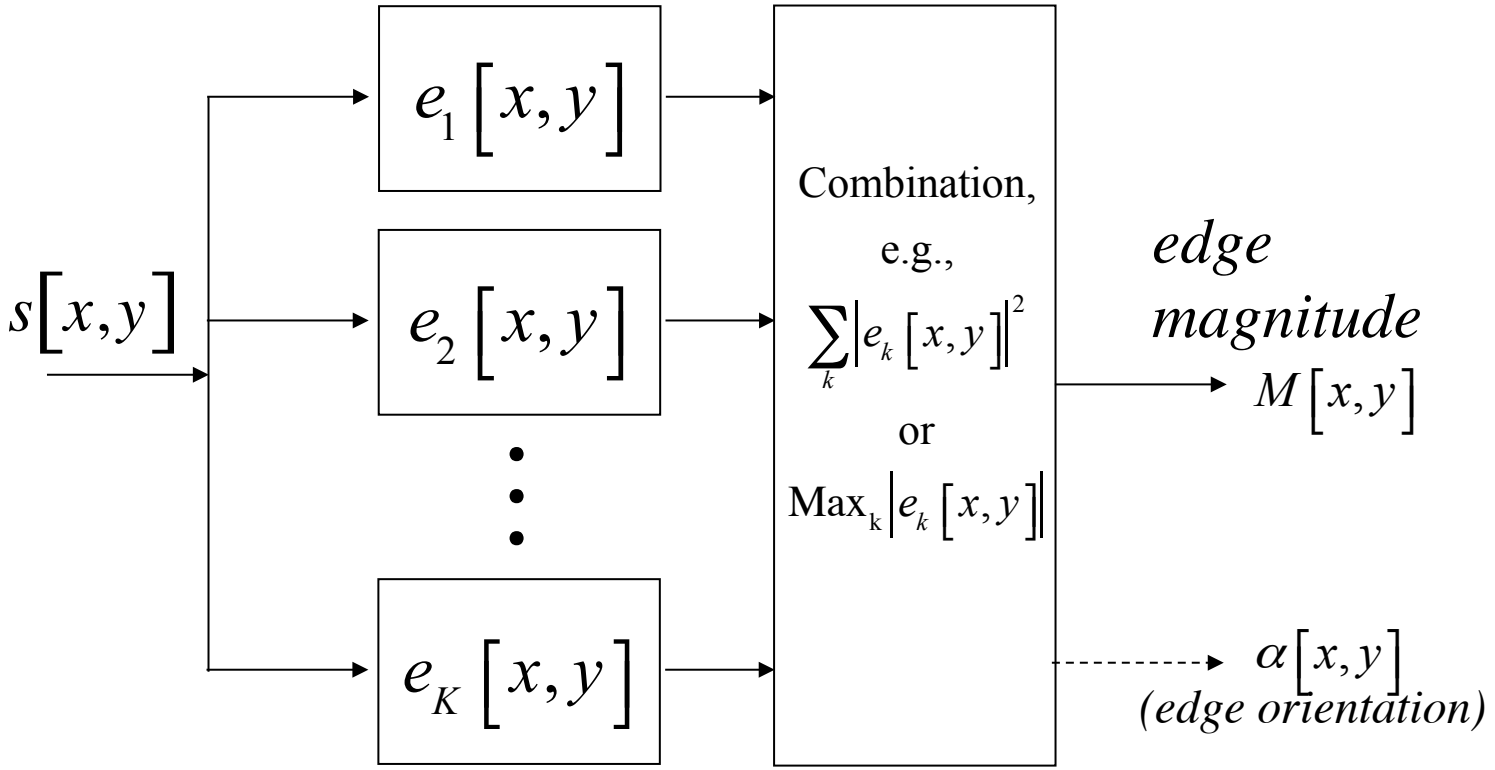
- Digital image: use finite differences to approximate derivatives

- Edge templates

difference	$\begin{pmatrix} -1 & 1 \end{pmatrix}$
central difference	$\begin{pmatrix} -1 & [0] & 1 \end{pmatrix}$
Prewitt	$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$
Sobel	$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix}$

Practical edge detectors

- Edges can have any orientation
- Typical edge detection scheme uses $K=2$ edge templates
- Some use $K>2$



Gradient filters (K=2)

Central Difference

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & [0] & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & [0] & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Roberts

$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$

Prewitt

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Sobel

$$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & -2 & -1 \\ 0 & [0] & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

Kirsch operator (K=8)

Kirsch

$$\begin{pmatrix} +5 & +5 & +5 \\ -3 & [0] & -3 \\ -3 & -3 & -3 \end{pmatrix} \begin{pmatrix} -3 & +5 & +5 \\ -3 & [0] & +5 \\ -3 & -3 & -3 \end{pmatrix} \begin{pmatrix} -3 & -3 & +5 \\ -3 & [0] & +5 \\ -3 & -3 & +5 \end{pmatrix} \begin{pmatrix} -3 & -3 & -3 \\ -3 & [0] & +5 \\ -3 & +5 & +5 \end{pmatrix}$$

$$\begin{pmatrix} -3 & -3 & -3 \\ -3 & [0] & -3 \\ +5 & +5 & +5 \end{pmatrix} \begin{pmatrix} -3 & -3 & -3 \\ +5 & [0] & -3 \\ +5 & +5 & -3 \end{pmatrix} \begin{pmatrix} +5 & -3 & -3 \\ +5 & [0] & -3 \\ +5 & -3 & -3 \end{pmatrix} \begin{pmatrix} +5 & +5 & -3 \\ +5 & [0] & -3 \\ -3 & -3 & -3 \end{pmatrix}$$

Prewitt operator example

Original
1024x710

Magnitude of image filtered with

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

(log display)

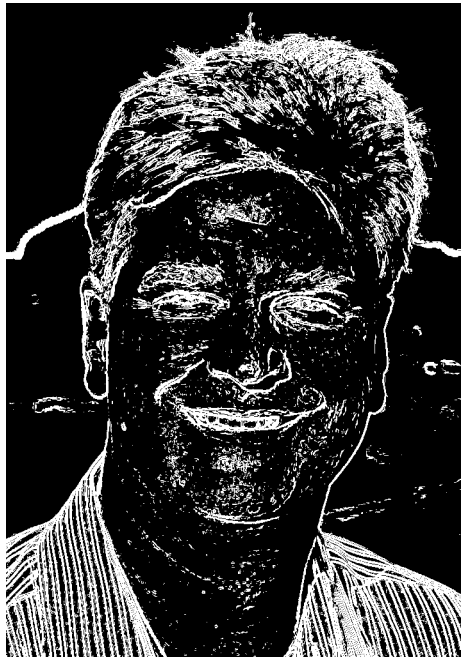
Magnitude of image filtered with

$$\begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

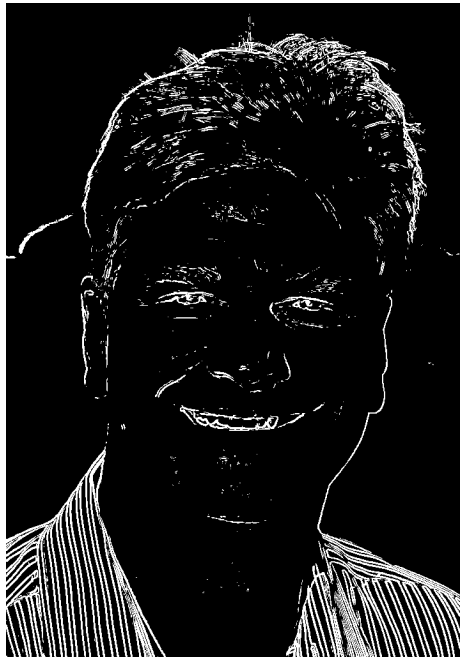
(log display)

Prewitt operator example (cont.)

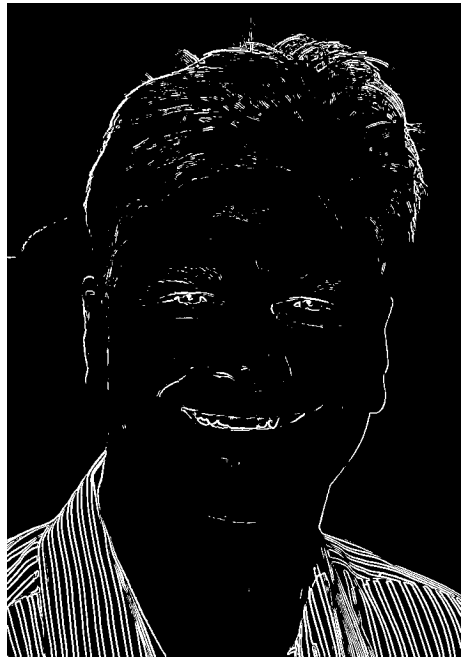
Sum of squared horizontal and vertical gradients (log display)



threshold = 900



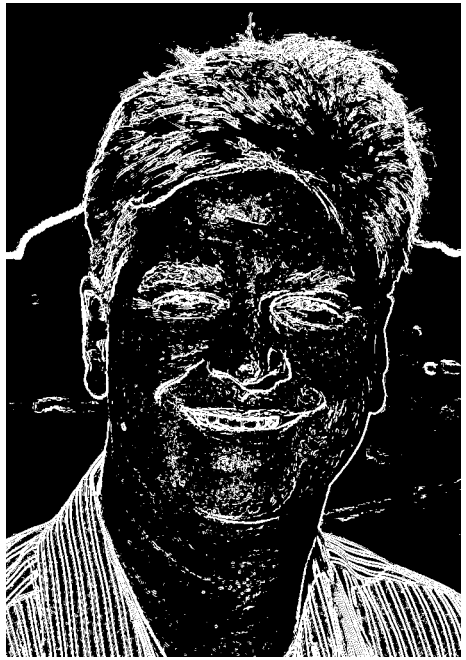
threshold = 4500



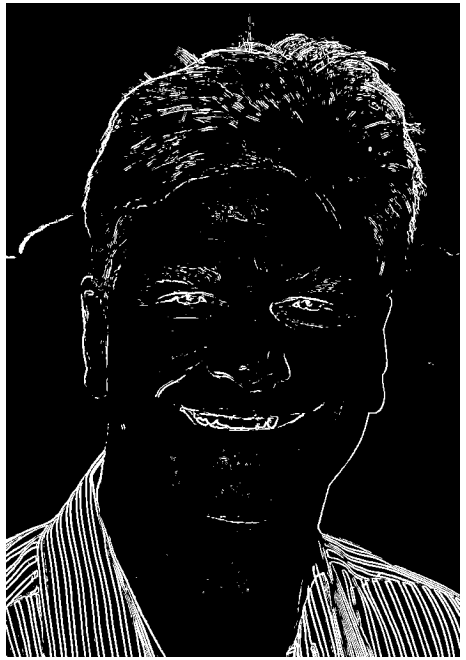
threshold = 7200

Sobel operator example

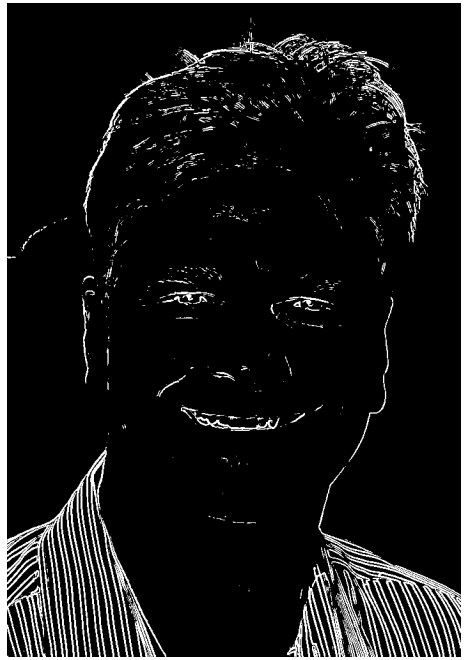
Sum of squared horizontal and vertical gradients (log display)



threshold = 1600



threshold = 8000



threshold = 12800

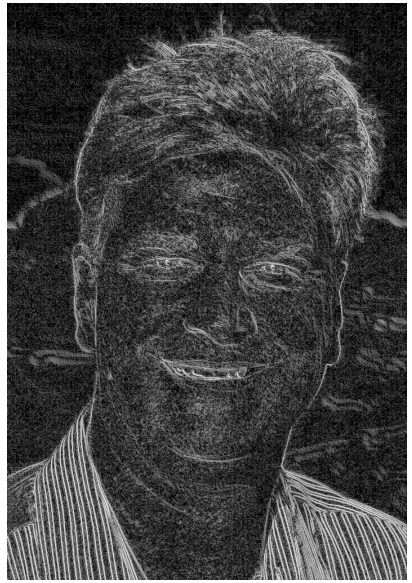
Roberts operator example

Original
1024x710

Magnitude of
image filtered with

$$\begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$

(log display)



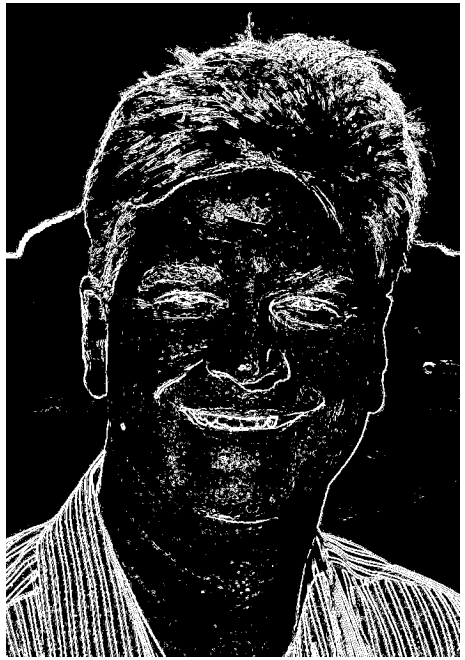
Magnitude of
image filtered with

$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix}$$

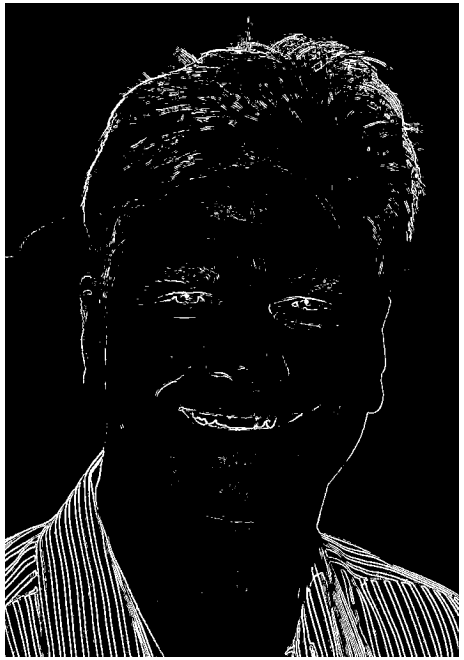
(log display)

Roberts operator example (cont.)

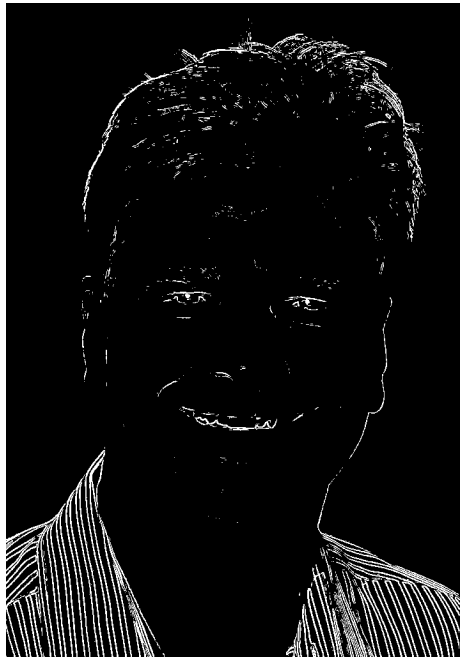
Sum of squared diagonal gradients (log display)



threshold = 100



threshold = 500



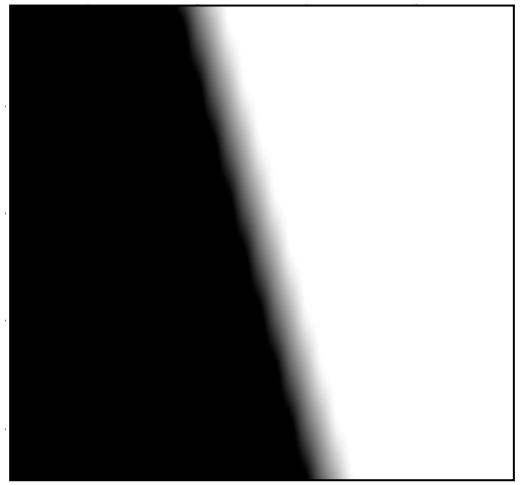
threshold = 800

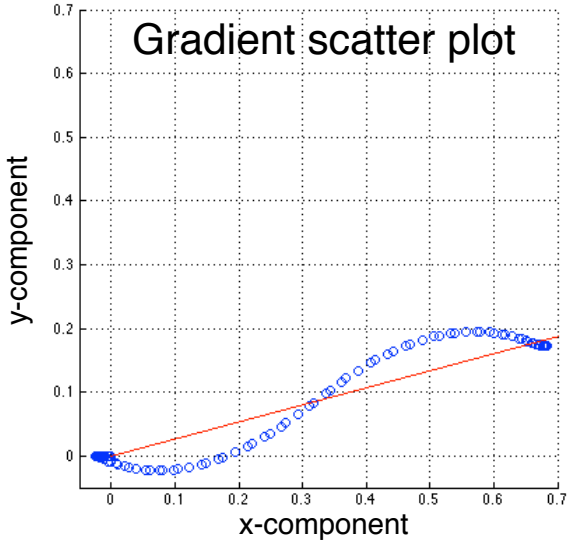
Edge orientation

Central Difference

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & [0] & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 0 & [0] & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

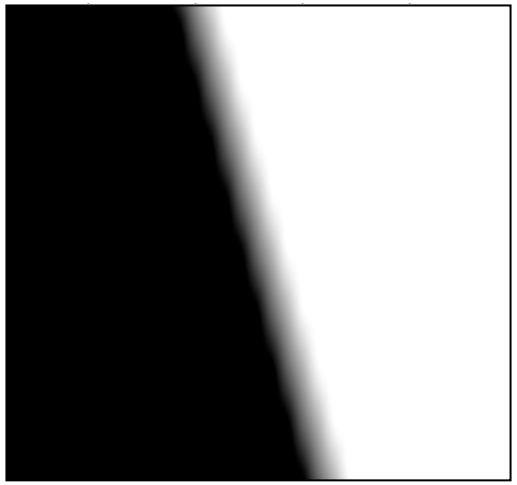


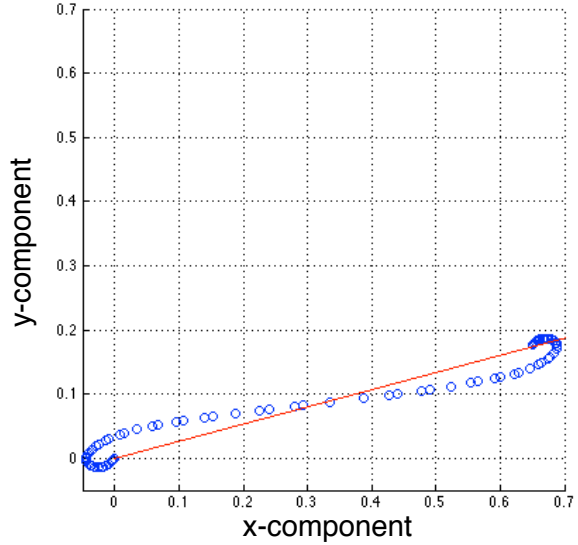


Roberts

$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$



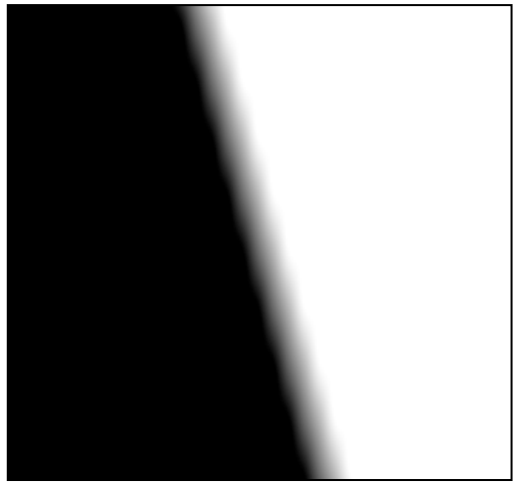


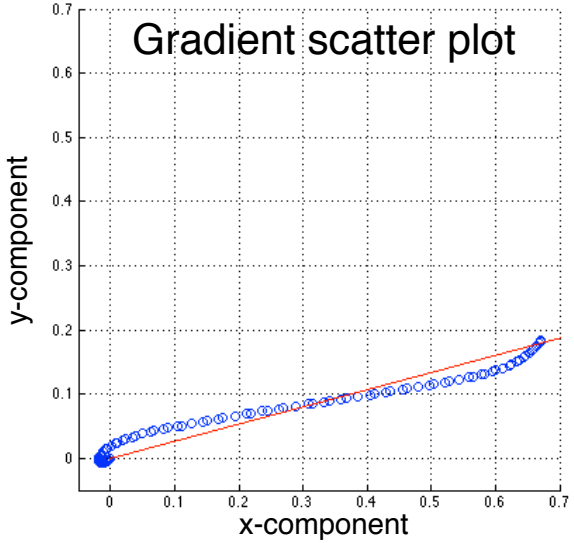
Edge orientation

Prewitt

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

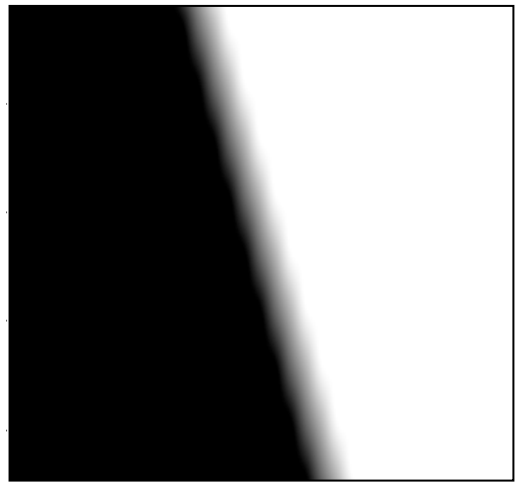


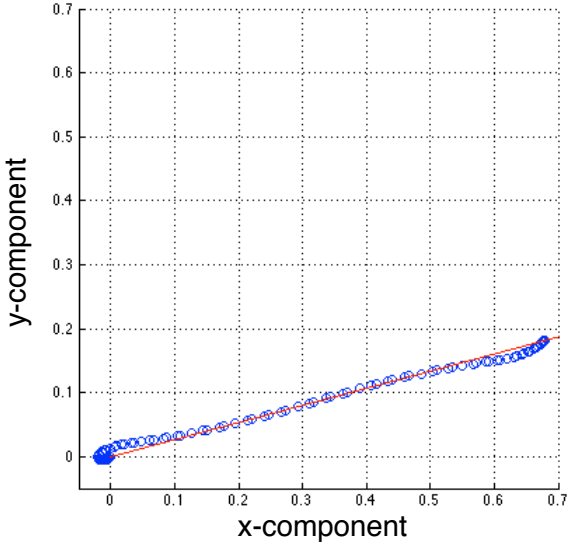


Sobel

$$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

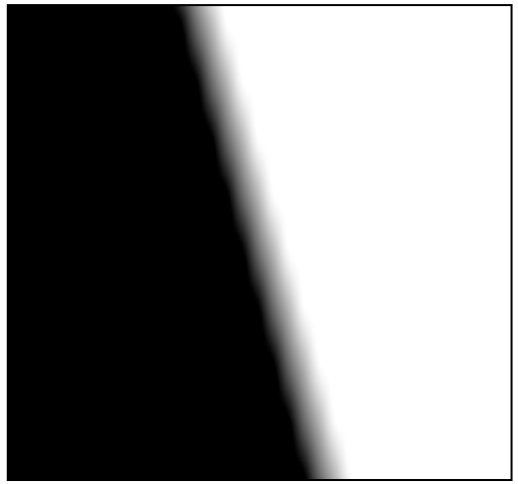
$$\begin{pmatrix} -1 & -2 & -1 \\ 0 & [0] & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

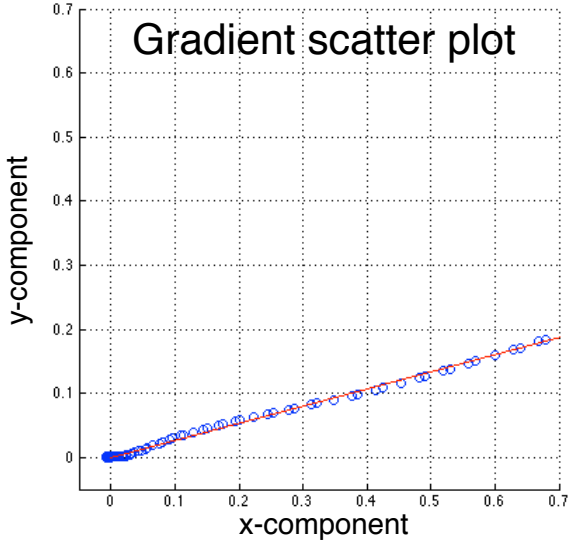




Edge orientation

5x5 "consistent" gradient operator [Ando, 2000]

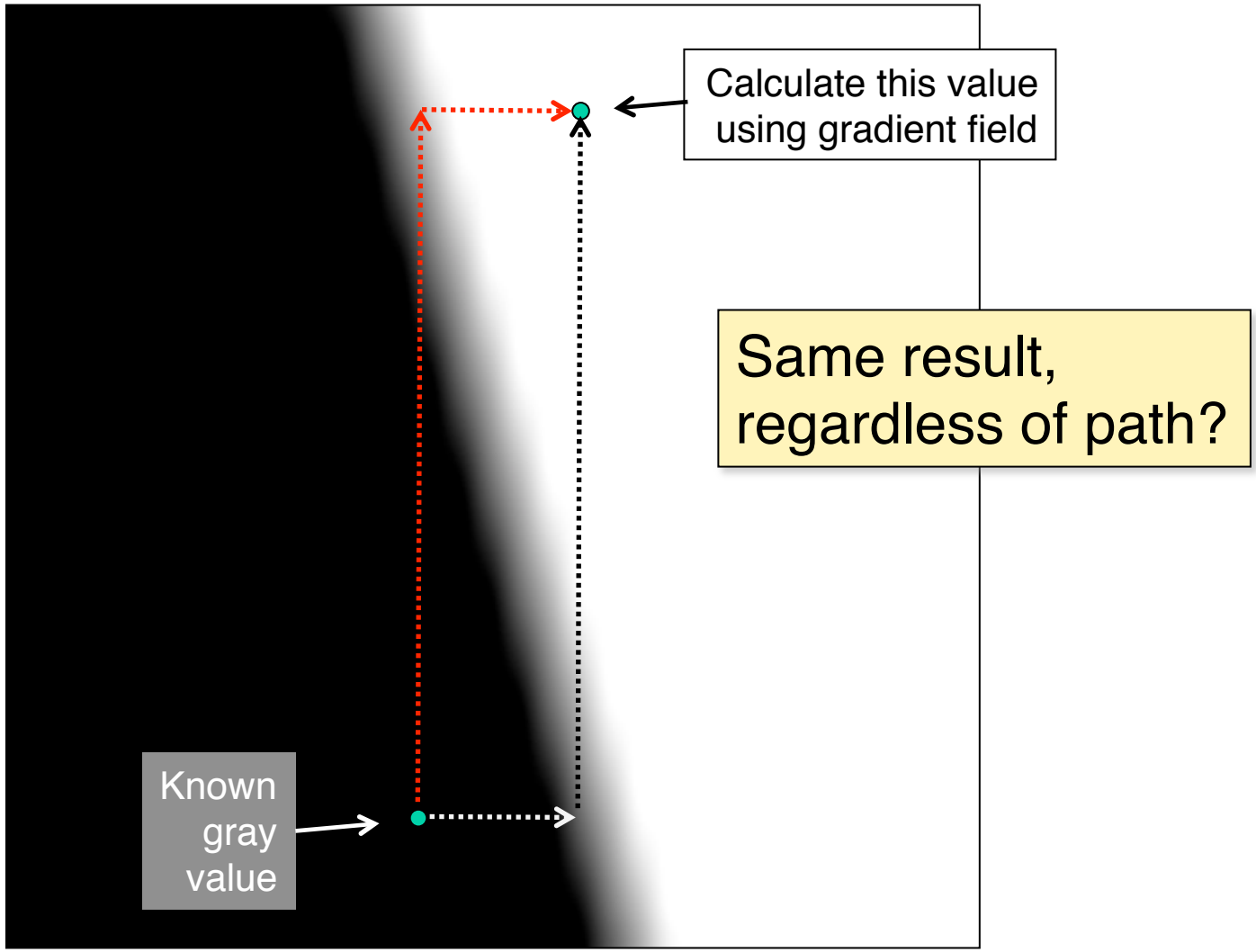




$$\begin{pmatrix} -0.0604 & -0.1632 & 0 & 0.1632 & 0.0604 \\ -0.4286 & -1.1335 & 0 & 1.1335 & 0.4286 \\ -0.7448 & -1.9612 & [0] & 1.9612 & 0.7448 \\ -0.4286 & -1.1335 & 0 & 1.1335 & 0.4286 \\ -0.0604 & -0.1632 & 0 & 0.1632 & 0.0604 \end{pmatrix}$$

$$\begin{pmatrix} -0.0604 & -0.4286 & -0.7448 & -0.4286 & -0.0604 \\ -0.1632 & -1.1335 & -1.9612 & -1.1335 & -0.1632 \\ 0 & 0 & [0] & 0 & 0 \\ 0.1632 & 1.1335 & 1.9612 & 1.1335 & 0.1632 \\ 0.0604 & 0.4286 & 0.7448 & 0.4286 & 0.0604 \end{pmatrix}$$

Gradient consistency problem

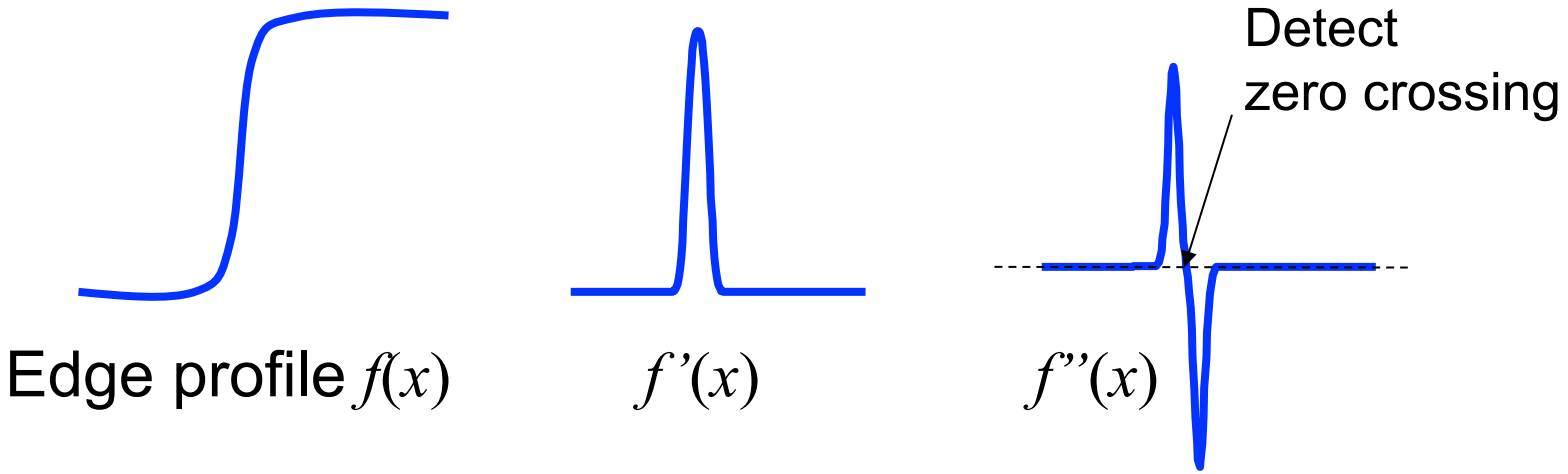


Laplacian operator

- Detect edges by considering second derivative

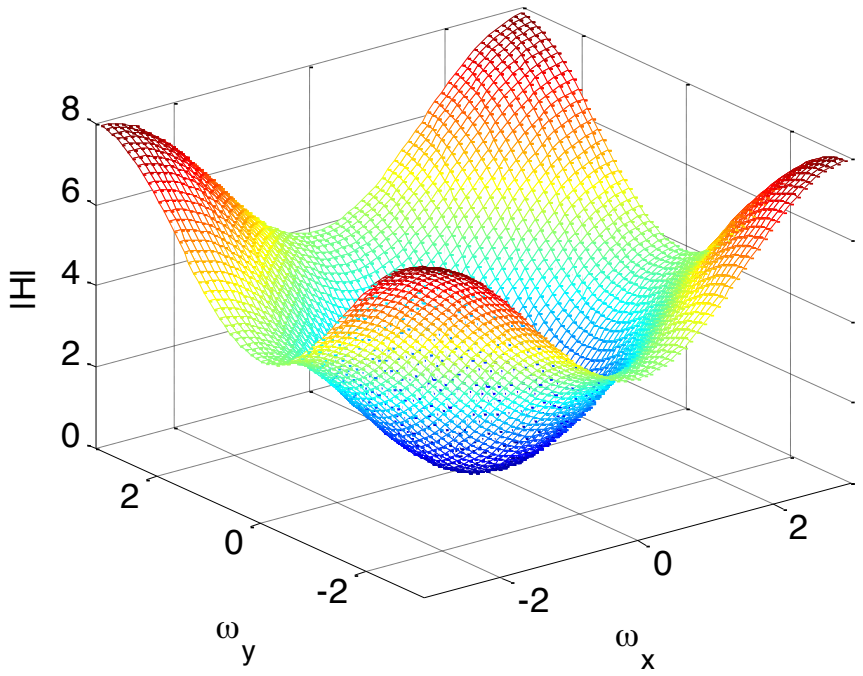
$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

- Isotropic (rotationally invariant) operator
- Zero-crossings mark edge location

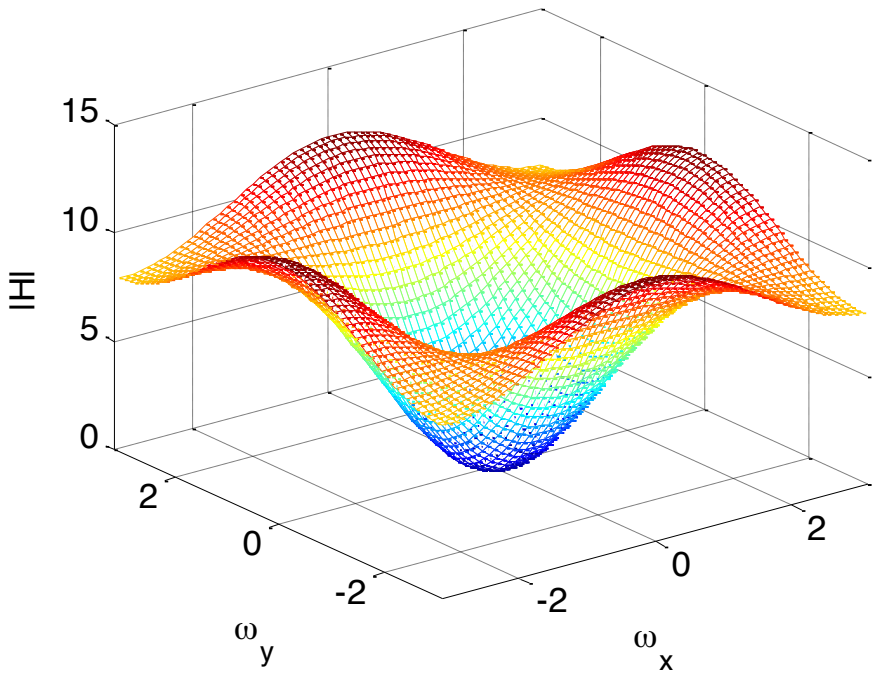


Approximations of Laplacian operator by 3x3 filter

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & [-4] & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & [-8] & 1 \\ 1 & 1 & 1 \end{pmatrix}$$



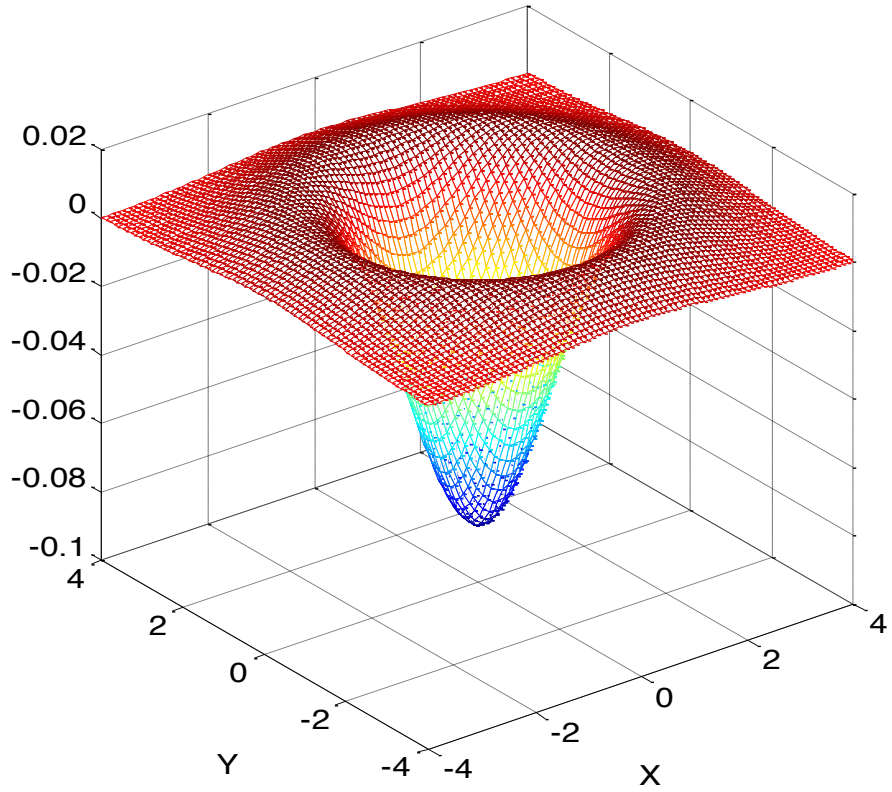
Zero crossings of Laplacian

- Sensitive to very fine detail and noise ➔ blur image first
- Responds equally to strong and weak edges
  ➔ suppress zero-crossings with low gradient magnitude

Laplacian of Gaussian

- Filtering of image with Gaussian and Laplacian operators can be combined into convolution with Laplacian of Gaussian (LoG) operator

$$LoG(x,y) = -\frac{1}{\pi\sigma^4}\left(1-\frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

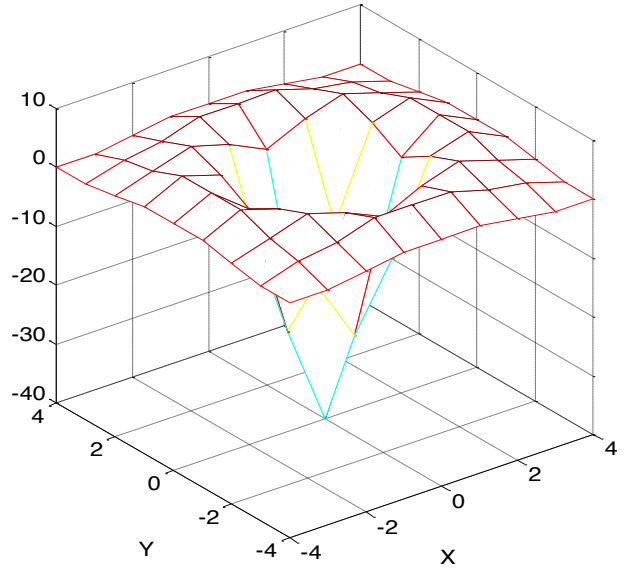


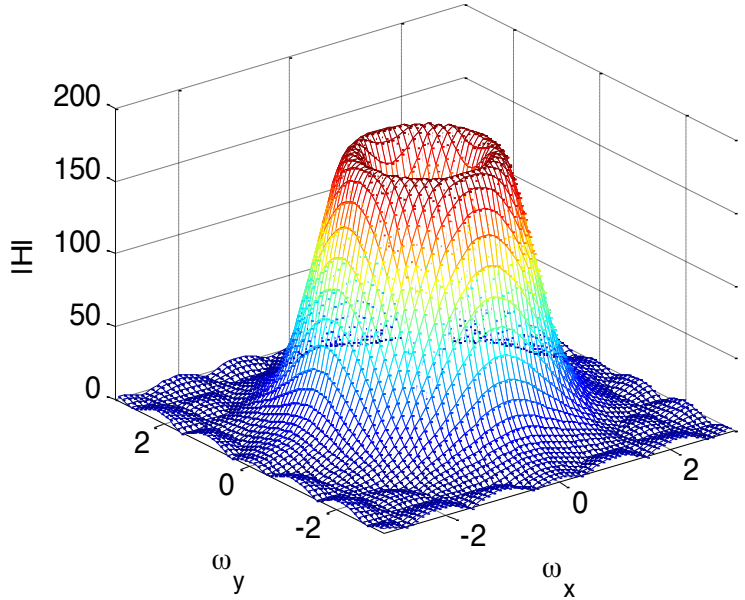
$\sigma = \sqrt{2}$

Discrete approximation of Laplacian of Gaussian

$$\sigma = \sqrt{2}$$

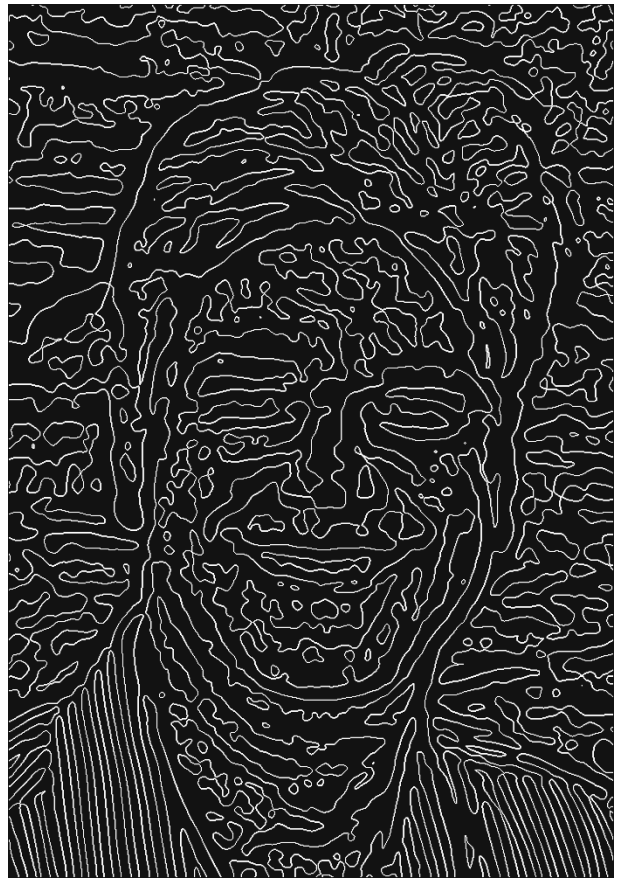
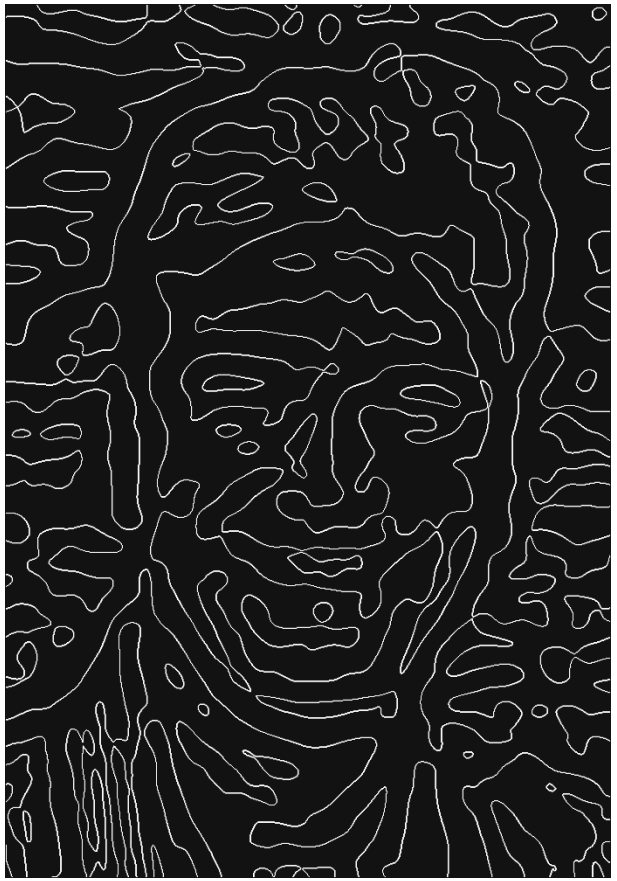
0	0	1	2	2	2	1	0	0
0	2	3	5	5	5	3	2	0
1	3	5	3	0	3	5	3	1
2	5	3	-12	-23	-12	3	5	2
2	5	0	-23	-40	-23	0	5	2
2	5	3	-12	-23	-12	3	5	2
1	3	5	3	0	3	5	3	1
0	2	3	5	5	5	3	2	0
0	0	1	2	2	2	1	0	0





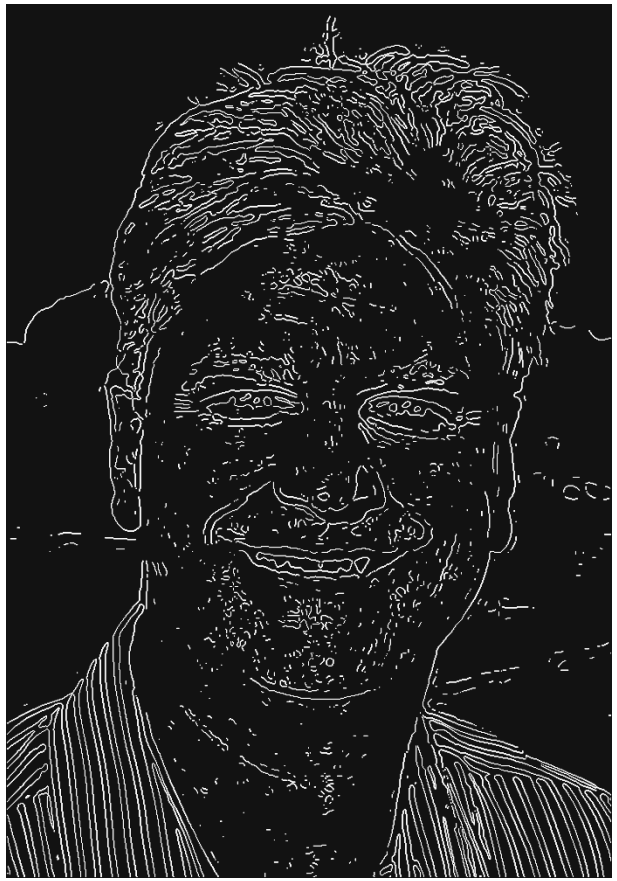
Zero crossings of LoG

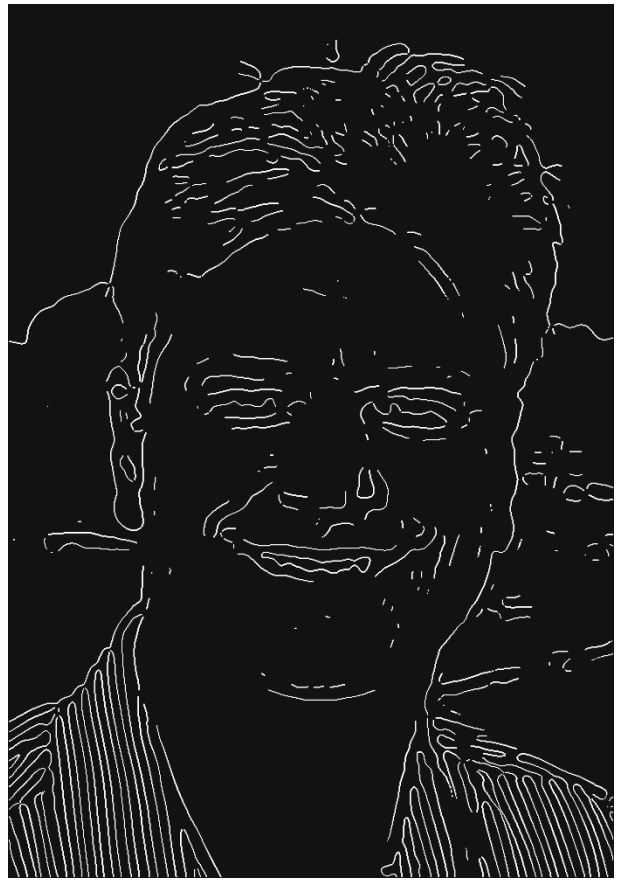


$\sigma = \sqrt{2}$ $\sigma = 2\sqrt{2}$ $\sigma = 4\sqrt{2}$ $\sigma = 8\sqrt{2}$

Zero crossings of LoG – gradient-based threshold





$\sigma = \sqrt{2}$ $\sigma = 2\sqrt{2}$ $\sigma = 4\sqrt{2}$ $\sigma = 8\sqrt{2}$

Canny edge detector

1. Smooth image with a Gaussian filter
2. Approximate gradient magnitude and angle (use Sobel, Prewitt . . .)

$$M[x,y] \approx \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

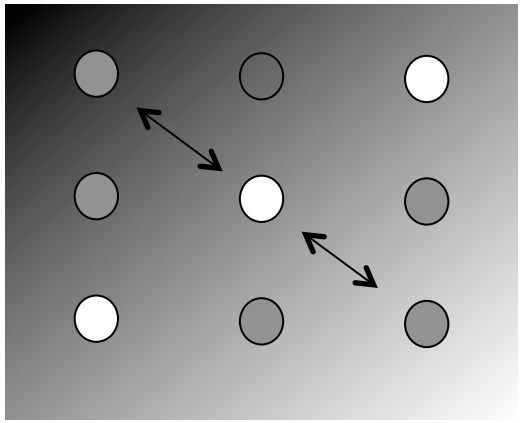
$$\alpha[x,y] \approx \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

3. Apply nonmaxima suppression to gradient magnitude
4. Double thresholding to detect strong and weak edge pixels
5. Reject weak edge pixels not connected with strong edge pixels

[Canny, IEEE Trans. PAMI, 1986]

Canny nonmaxima suppression

- Quantize edge normal to one of four directions:
  horizontal, -45°, vertical, +45°
- If $M[x,y]$ is smaller than either of its neighbors in edge normal direction
  ➔ suppress; else keep.



[Canny, IEEE Trans. PAMI, 1986]

Canny thresholding and suppression of weak edges

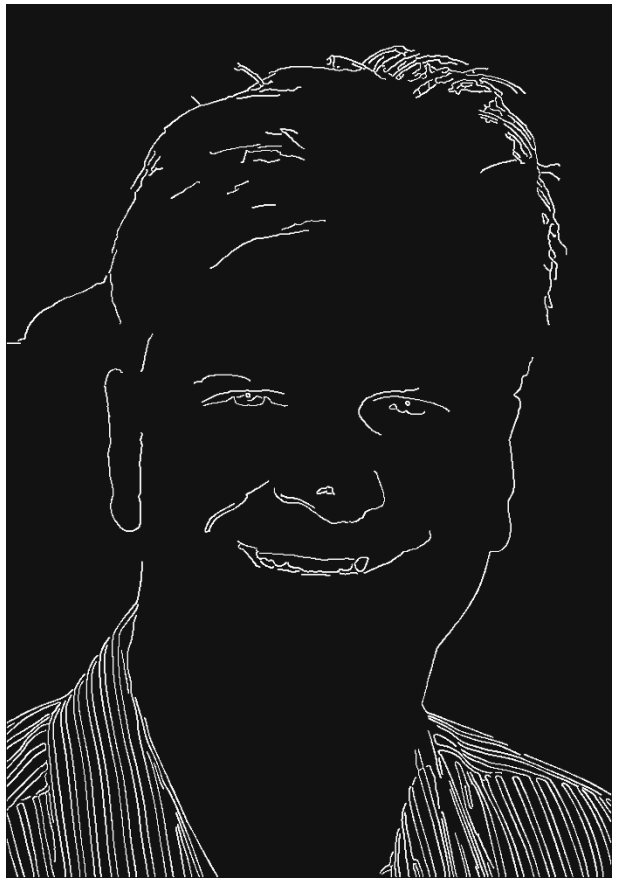
- Double-thresholding of gradient magnitude

Strong edge:	$M[x,y] \geq \theta_{high}$
Weak edge:	$\theta_{high} > M[x,y] \geq \theta_{low}$

- Typical setting: $\theta_{high} / \theta_{low} = 2...3$
- Region labeling of edge pixels
- Reject regions without strong edge pixels

[Canny, IEEE Trans. PAMI, 1986]

Canny edge detector



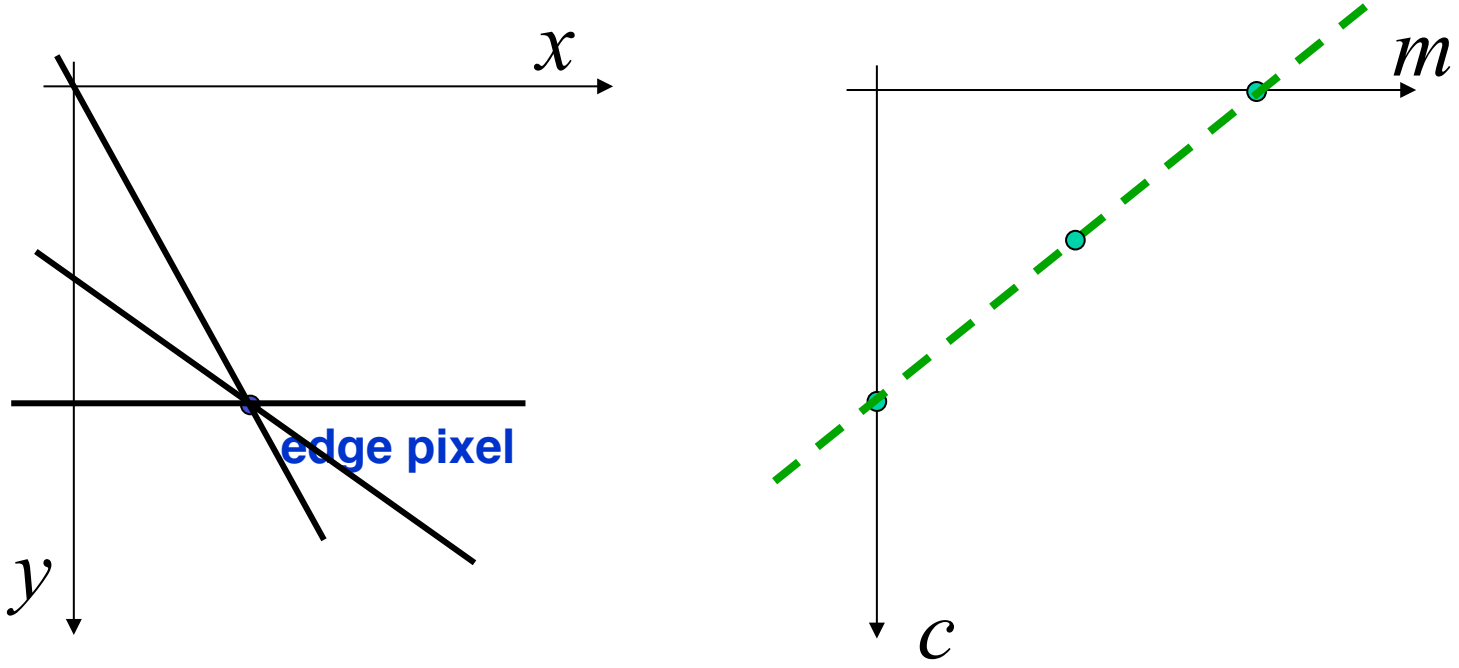
$\sigma = \sqrt{2}$

$\sigma = 2\sqrt{2}$

$\sigma = 4\sqrt{2}$

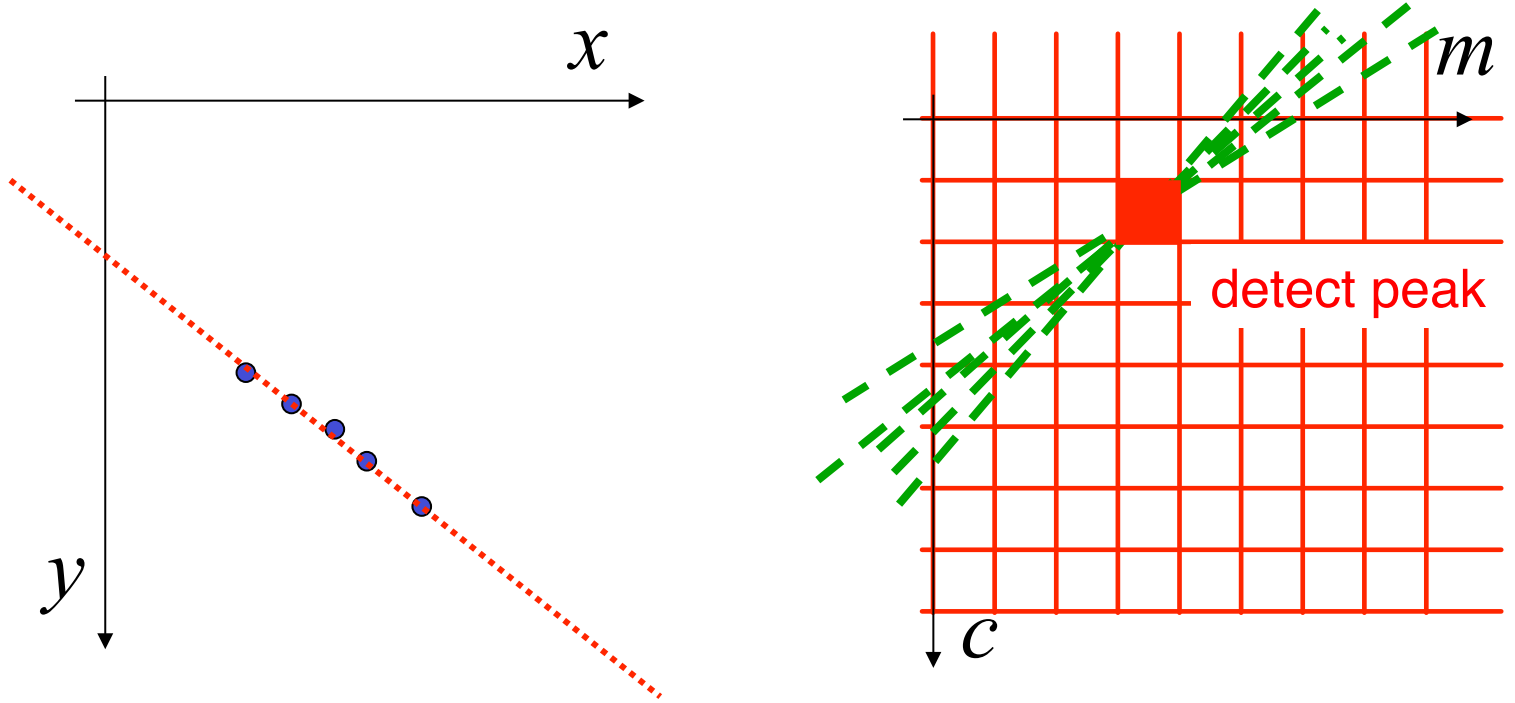
Hough transform

- Problem: fit a straight line (or curve) to a set of edge pixels
- Hough transform (1962): generalized template matching technique
- Consider detection of straight lines $y = mx + c$



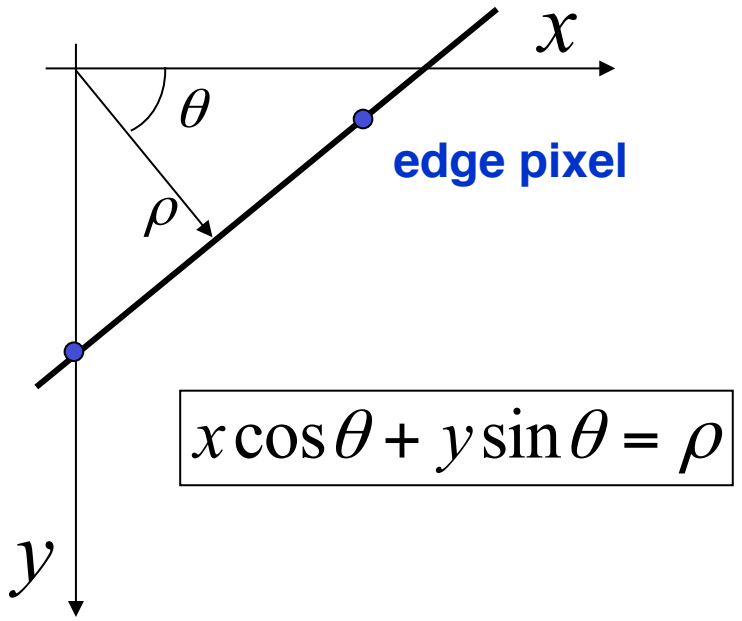
Hough transform (cont.)

- Subdivide (m,c) plane into discrete "bins," initialize all bin counts by 0
- Draw a line in the parameter space $[m,c]$ for each edge pixel $[x,y]$ and increment bin counts along line.
- Detect peak(s) in $[m,c]$ plane

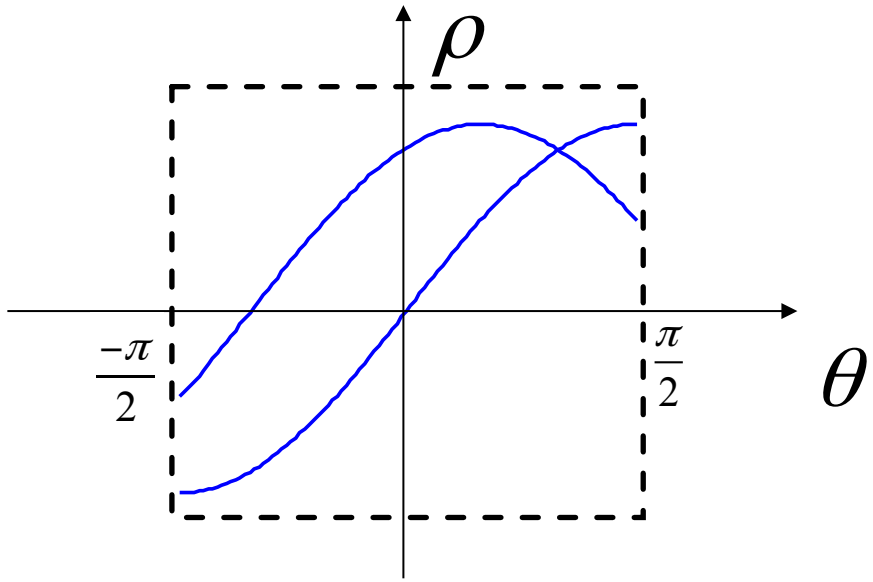


Hough transform (cont.)

- Alternative parameterization avoids infinite-slope problem

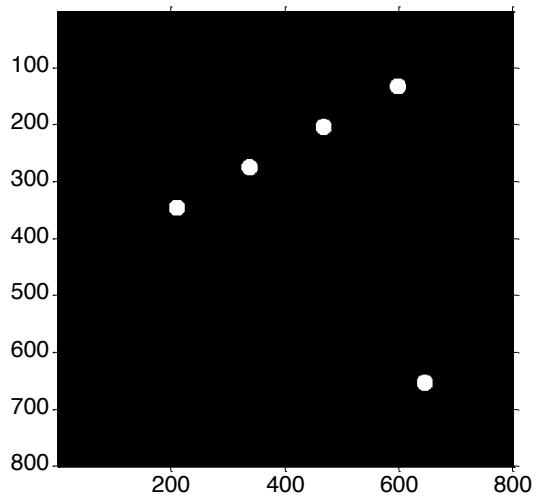


$$x\cos\theta + y\sin\theta = \rho$$

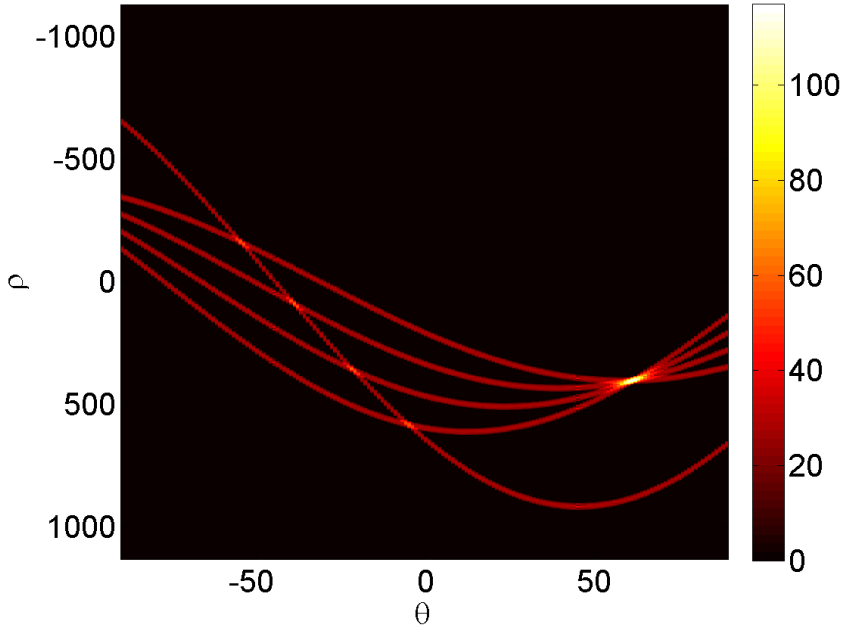


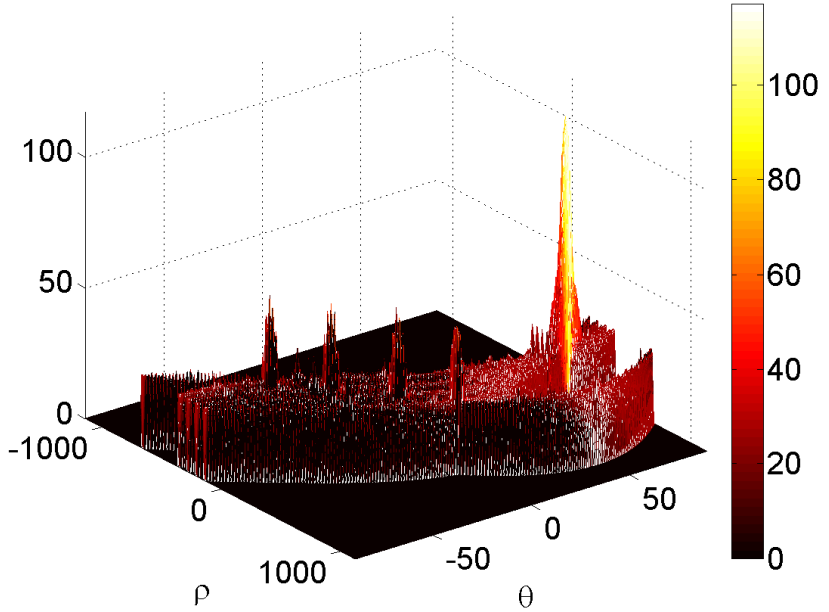
- Similar to Radon transform

Hough transform example

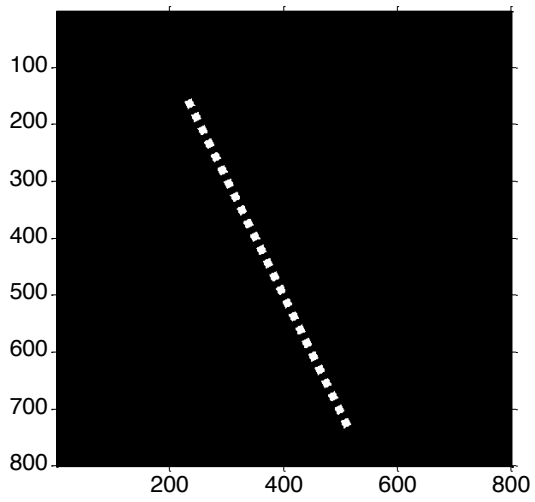


Original image

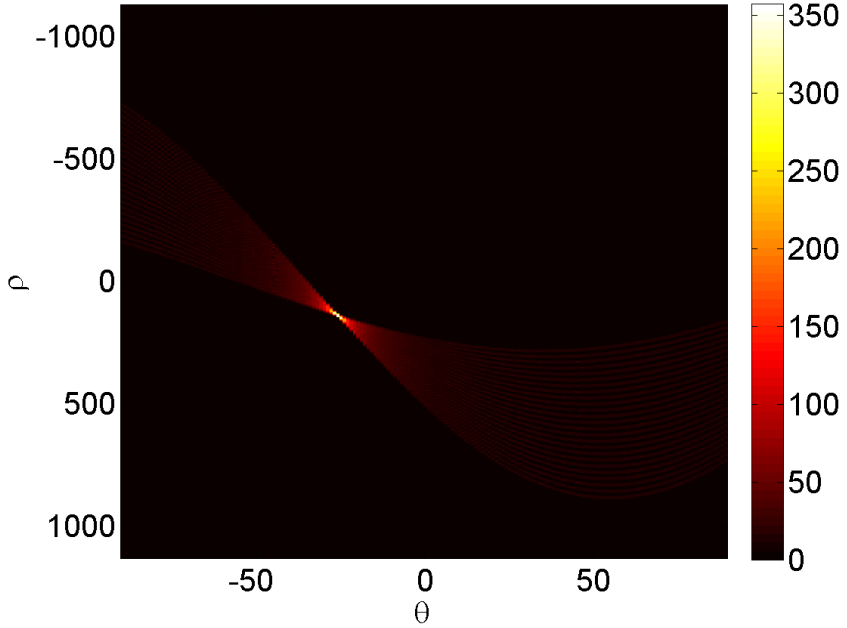


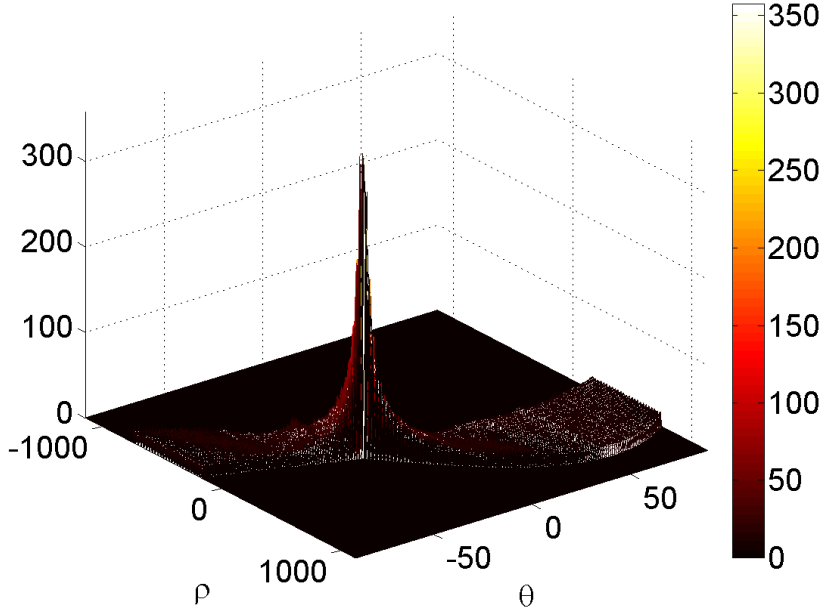


Hough transform example

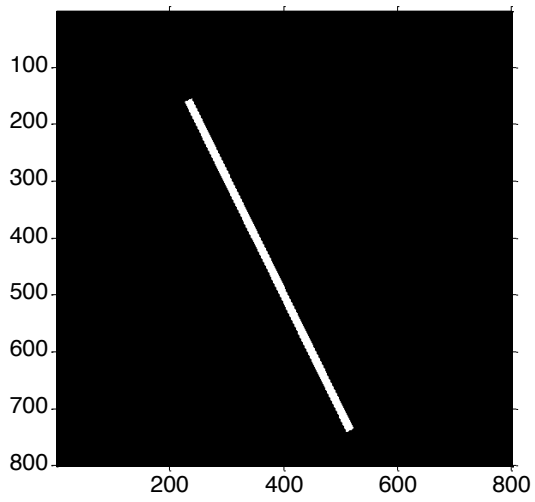


Original image

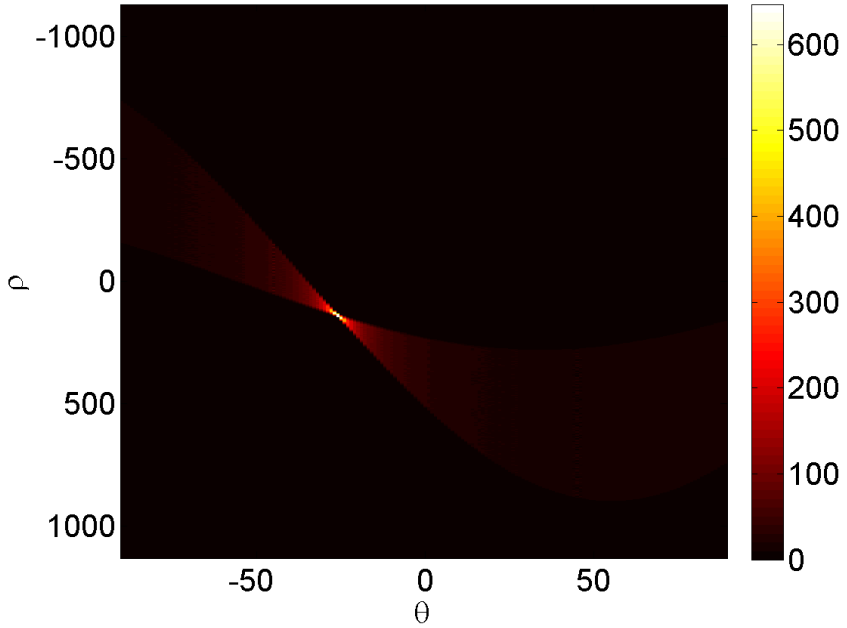


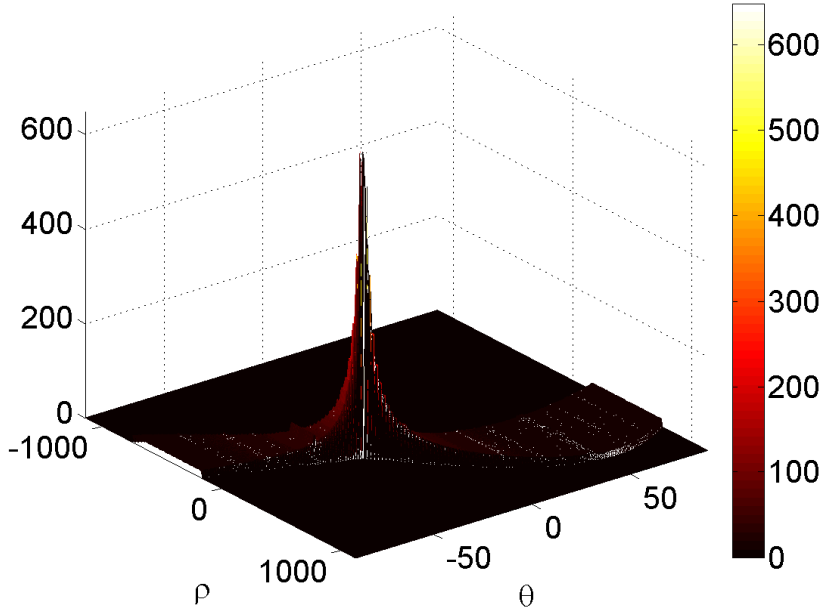


Hough transform example

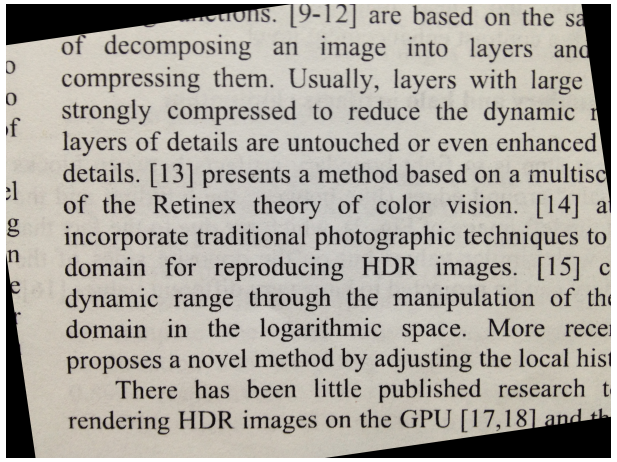


Original image

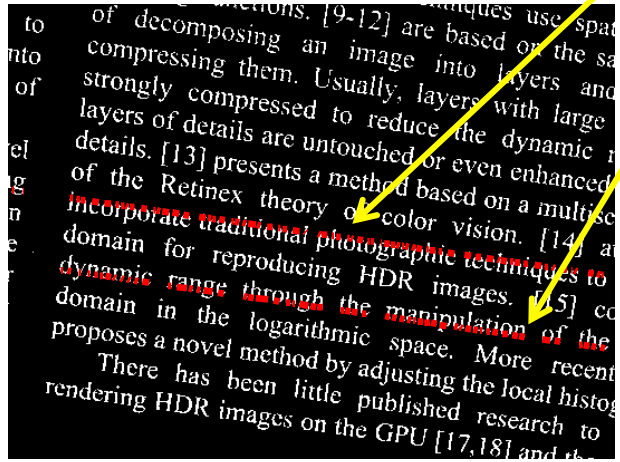




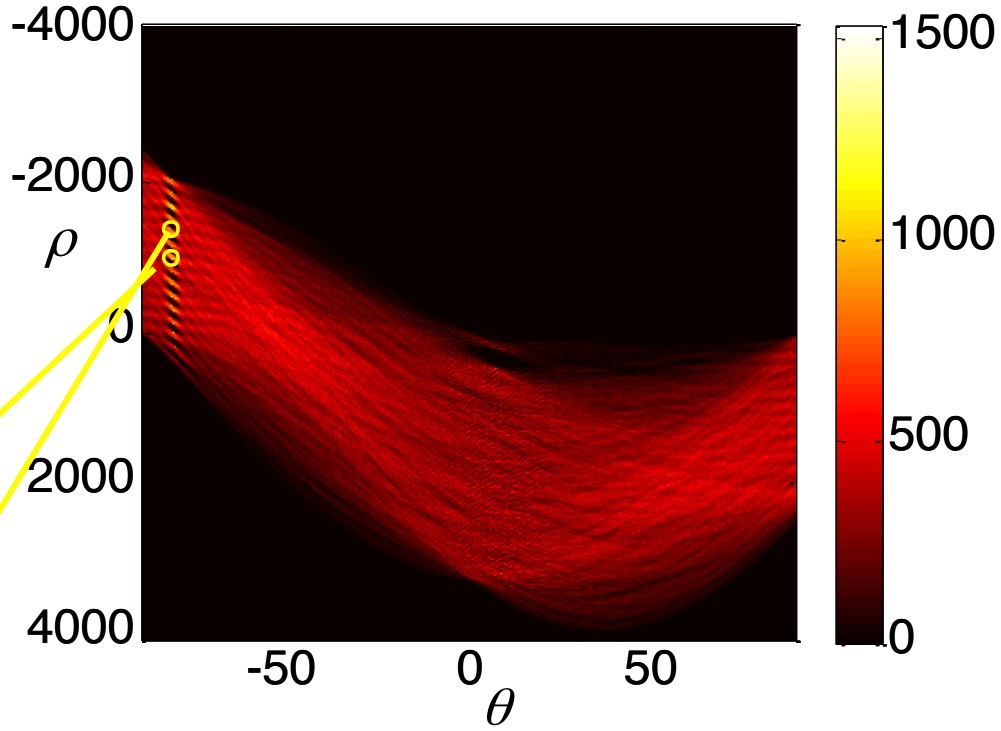
Hough transform example



De-skewed Paper

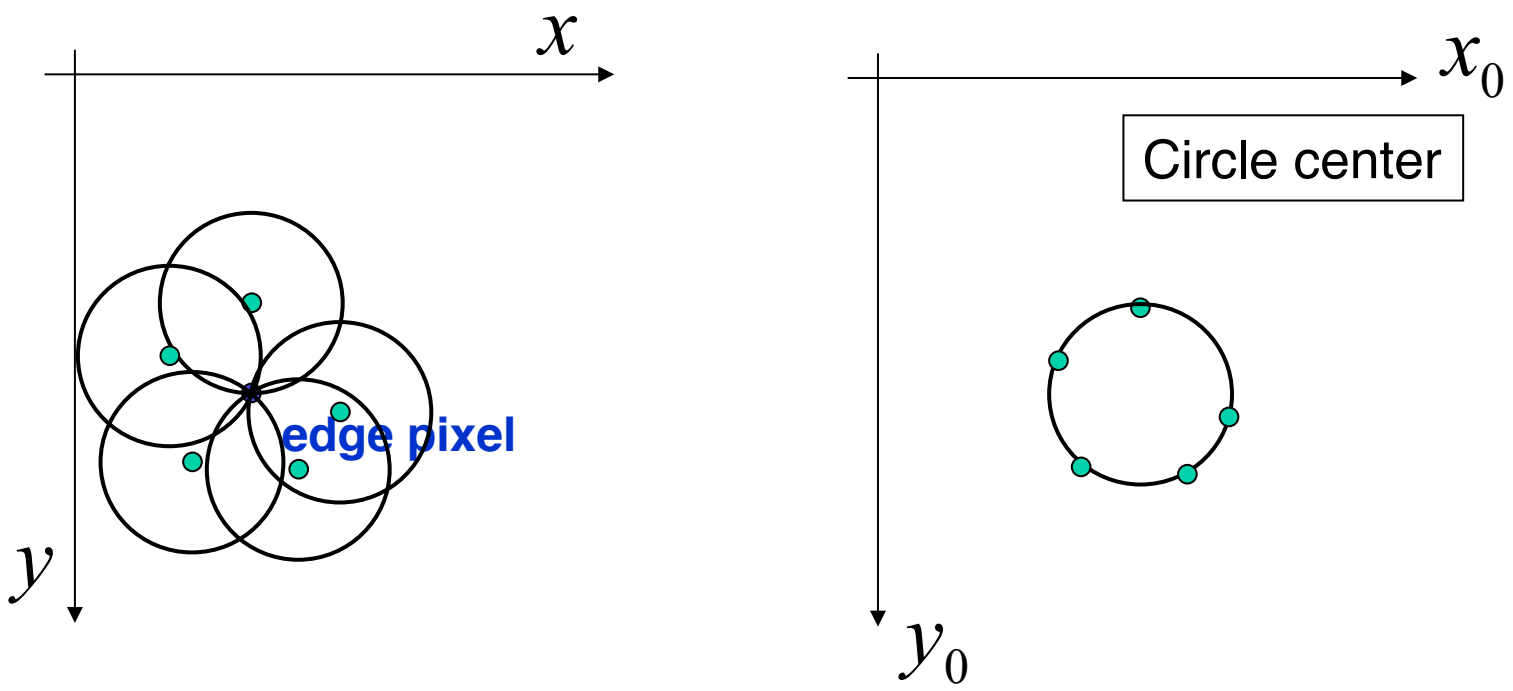


Global thresholding



Circle Hough Transform

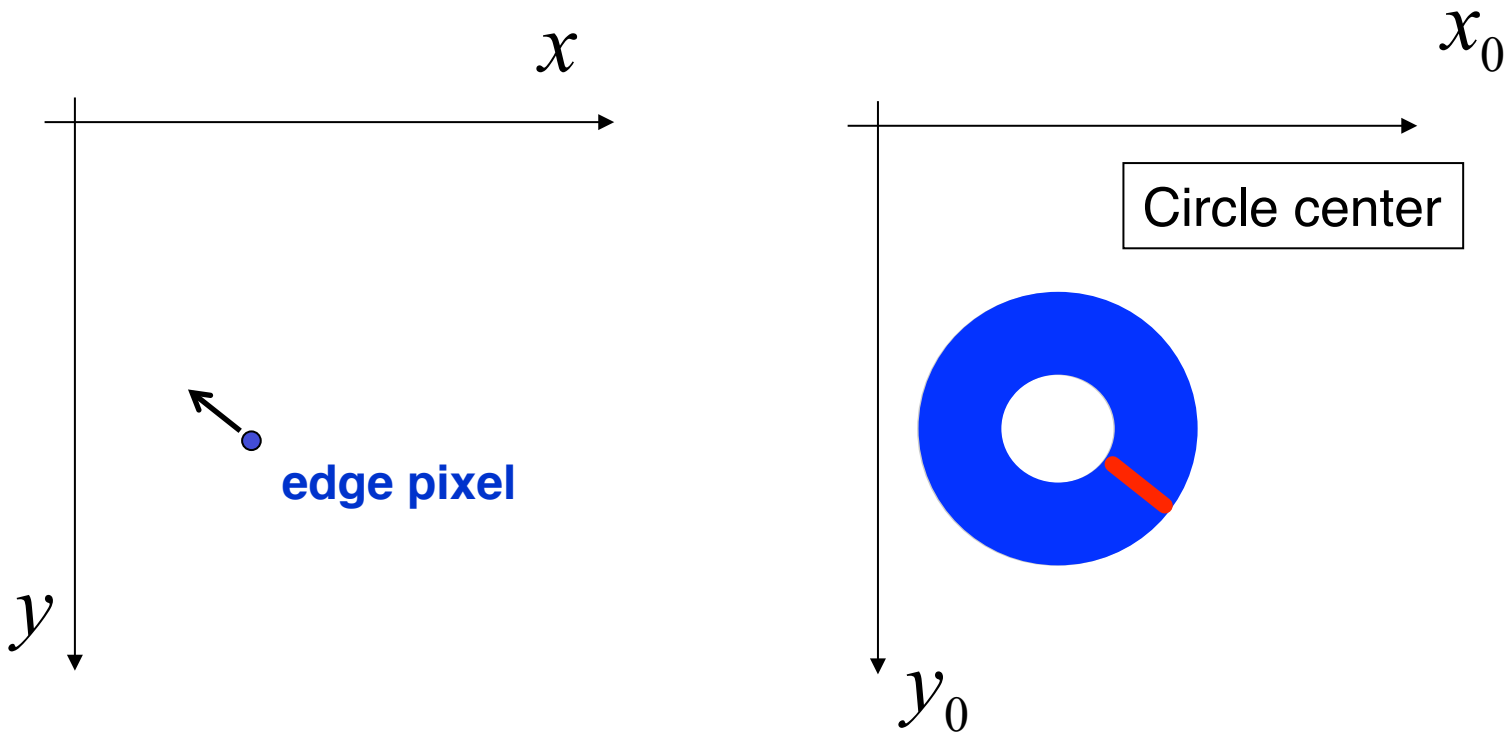
- Find circles of fixed radius r



- Equivalent to convolution (template matching) with a circle

Circle Hough Transform for unknown radius

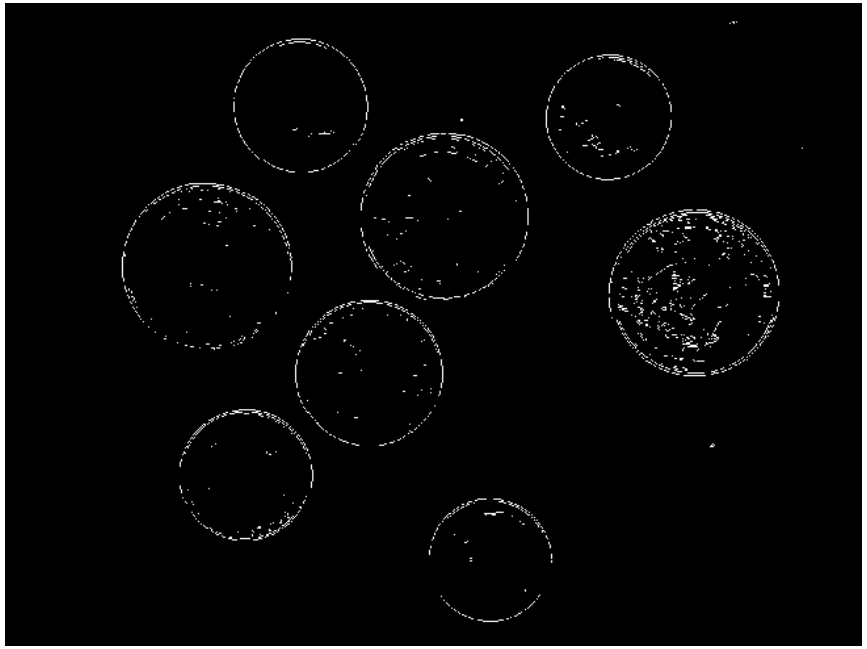
- 3-d Hough transform for parameters (x_0, y_0, r)
- 2-d Hough transform aided by edge orientation ➔ "spokes" in parameter space



Example: circle detection by Hough transform

Original
coins image

Prewitt edge detection



Detected circles