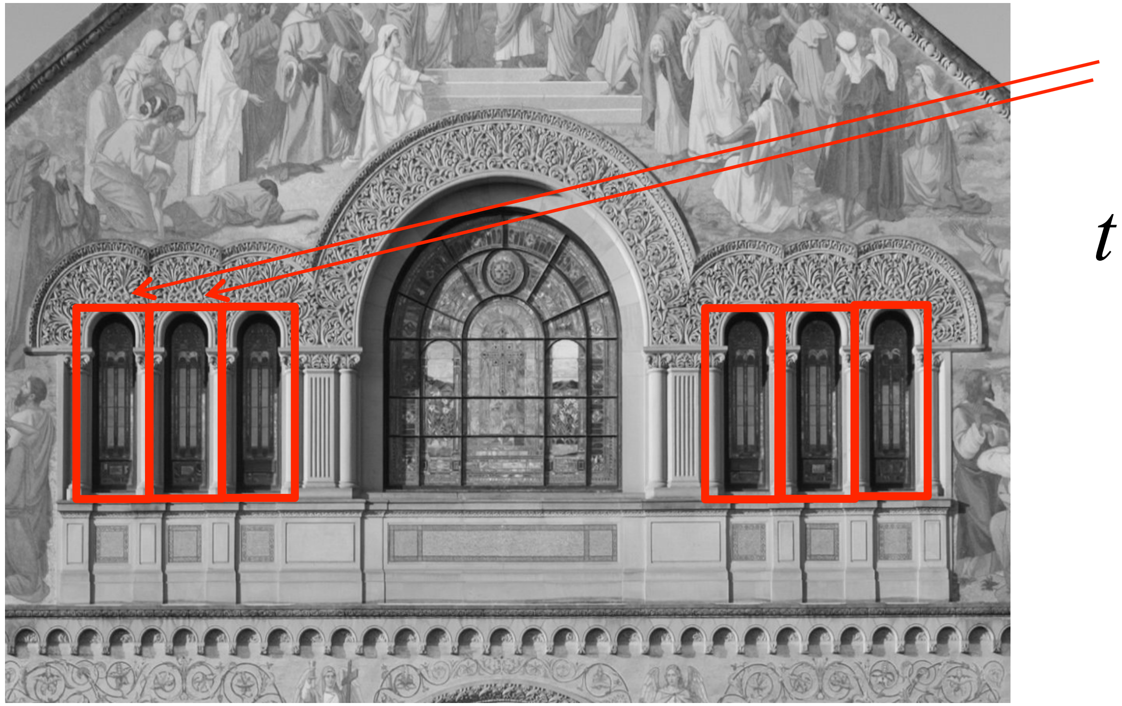
Template matching

- Problem: locate an object, described by a template $t[x,y]$, in the image $s[x,y]$
- Example

$t[x,y]$



$s[x,y]$

Template matching (cont.)

- Search for the best match by minimizing mean-squared error

$$E[p,q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left[s[x,y] - t[x-p, y-q] \right]^2$$

$$= \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| s[x,y] \right|^2 + \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| t[x,y] \right|^2 - 2 \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x,y] \cdot t[x-p, y-q]$$

- Equivalently, maximize *area correlation*

$$r[p,q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x,y] \cdot t[x-p, y-q] = s[p,q] * t[-p,-q]$$

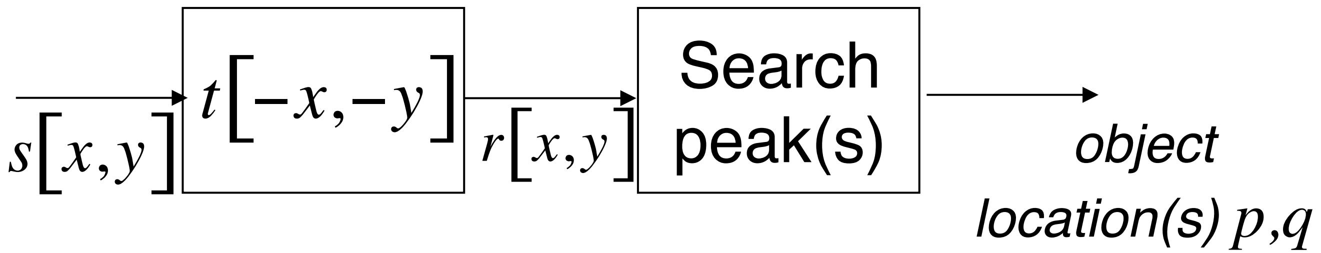
- Area correlation is equivalent to convolution of image $s[x,y]$ with impulse response $t[-x,-y]$

Template matching (cont.)

- From Cauchy-Schwarz inequality

$$r[p,q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x,y] \cdot t[x-p, y-q] \leq \sqrt{\left(\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| s[x,y] \right|^2 \right) \left(\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left| t[x,y] \right|^2 \right)}$$

- Equality, iff $s[x,y] = \alpha \cdot t[x-p, y-q] \quad \text{with } \alpha \geq 0$
- Block diagram of template matcher

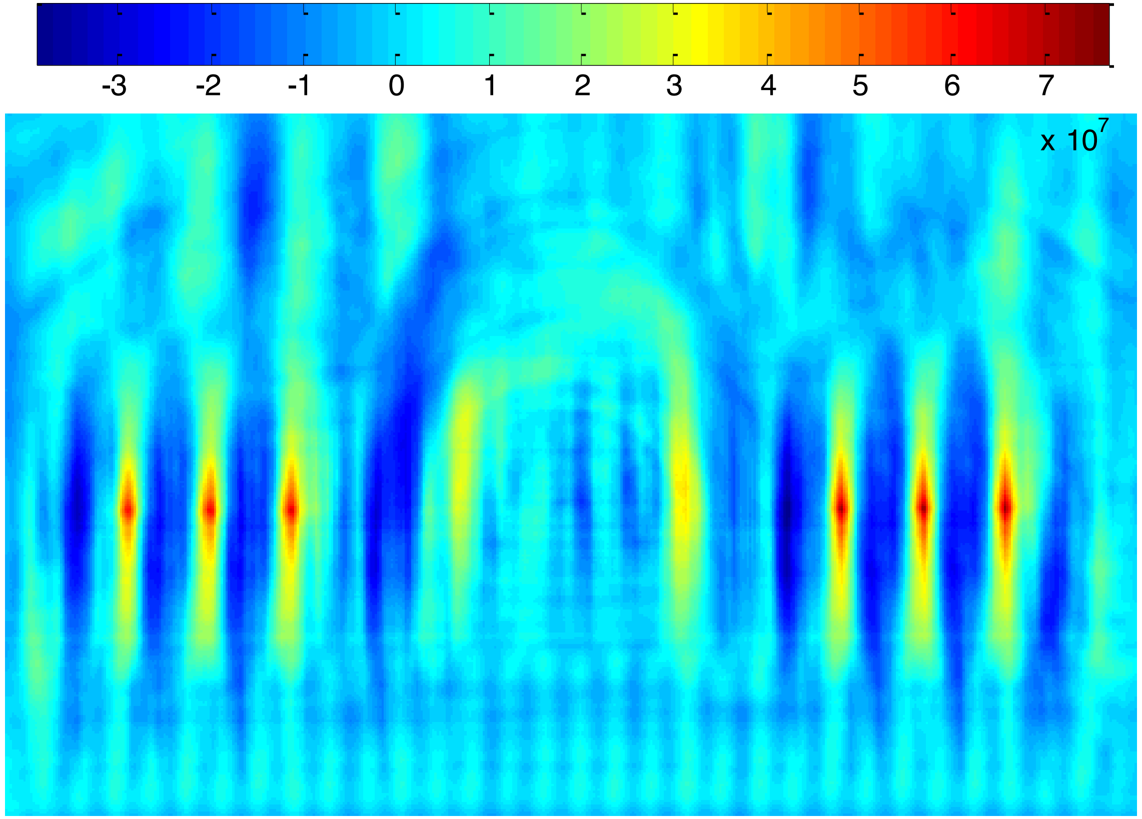


- Remove mean before template matching to avoid bias towards bright image areas

Template matching example

$t[x,y]$

$s[x,y]$



$r[p,q]$

Matched filtering

- Consider signal detection problem

$$s[x,y] \longrightarrow \boxed{g[x,y]} \xrightarrow{\;r[x,y]\;} \boxed{\text{Search peak(s)}} \longrightarrow \textit{Object location(s) p,q}$$

- Signal model

shifted template

Other objects: "noise" or "clutter" psd $\Phi_{nn}\left(e^{j\omega_x}, e^{j\omega_y}\right)$

$$s[x,y] = t[x-p, y-q] + n[x,y]$$

- Problem: design filter $g[x,y]$ to maximize

correct peak

$$SNR = \frac{\left| r[p,q] \right|^2}{E\left\{ \left| n[x,y] * g[x,y] \right|^2 \right\}}$$

false readings

Vector-matrix formulation

$$r[p,q] = \vec{g}^H \vec{s}$$

covariance $R_{nn} = E\left\{ \vec{n}\vec{n}^H \right\}$

$$\vec{s} = \vec{t} + \vec{n}$$

$$SNR = \frac{\left| r[p,q] \right|^2}{E\left\{ \left| \vec{g}^H \vec{n} \right|^2 \right\}}$$

Matched filtering (cont.)

- Optimum filter has frequency response

$$\boxed{G\left(e^{j\omega_x},e^{j\omega_y}\right)=\frac{T^*\left(e^{j\omega_x},e^{j\omega_y}\right)}{\Phi_{nn}\left(e^{j\omega_x},e^{j\omega_y}\right)}}$$

- Proof:

$$SNR=\frac{\left|r[p,q]\right|^2}{E\left\{\left|n[x,y]*g[x,y]\right|^2\right\}}\approx\frac{\left|\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}G\left(e^{j\omega_x},e^{j\omega_y}\right)T\left(e^{j\omega_x},e^{j\omega_y}\right)d\omega_x d\omega_y\right|^2}{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|G\left(e^{j\omega_x},e^{j\omega_y}\right)\right|^2\Phi_{nn}\left(e^{j\omega_x},e^{j\omega_y}\right)d\omega_x d\omega_y}$$

$$=\frac{\left|\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left[G\Phi_{nn}^{1/2}\right]\left[\Phi_{nn}^{-1/2}T\right]d\omega_x d\omega_y\right|^2}{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|G\right|^2\Phi_{nn}\,d\omega_x d\omega_y}\le\frac{\left[\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|G\right|^2\Phi_{nn}\,d\omega_x d\omega_y\right]\left[\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|T\right|^2\Phi_{nn}^{-1}\,d\omega_x d\omega_y\right]}{\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|G\right|^2\Phi_{nn}\,d\omega_x d\omega_y}$$

$$=\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\left|T\right|^2\Phi_{nn}^{-1}\,d\omega_x d\omega_y$$

max. SNR

Cauchy-Schwarz inequality,

with equality, iff $G\Phi_{nn}^{1/2}=\alpha\cdot\left[\Phi_{nn}^{-1/2}T\right]^*$

Vector-matrix formulation

$$\boxed{\vec{g}=R_{nn}^{-1}\vec{t}}$$

Matched filtering (cont.)

- Optimum filter corresponds to projection on

$$\boxed{\vec{g} = R_{nn}^{-1}\vec{t}}$$

- Proof:

$$SNR = \frac{\left|r\left[p,q\right]\right|^2}{E\left\{\left|\vec{g}^H\vec{n}\right|^2\right\}} \approx \frac{\left|\vec{g}^H\vec{t}\right|^2}{\vec{g}^H R_{nn}\vec{g}}$$

$$= \frac{\left|\left(R_{nn}^{1/2}\vec{g}\right)^H\left(R_{nn}^{-1/2}\vec{t}\right)\right|^2}{\left(R_{nn}^{1/2}\vec{g}\right)^H\left(R_{nn}^{1/2}\vec{g}\right)} \leq \frac{\left|\left[\left(R_{nn}^{1/2}\vec{g}\right)^H\left(R_{nn}^{1/2}\vec{g}\right)\right]\left[\left(R_{nn}^{-1/2}\vec{t}\right)^H\left(R_{nn}^{-1/2}\vec{t}\right)\right]\right|}{\left(R_{nn}^{1/2}\vec{g}\right)^H\left(R_{nn}^{1/2}\vec{g}\right)}$$

$$= \vec{t}^H R_{nn}^{-1}\vec{t}$$

max. SNR

Cauchy-Schwarz inequality,

with equality, iff $R_{nn}^{1/2}\vec{g} = \alpha \cdot R_{nn}^{-1/2}\vec{t}$

Matched filtering (cont.)

- Optimum detection: prefiltering & template matching

$$s[x,y] \rightarrow \boxed{h[x,y]} \rightarrow \boxed{t[-x,-y]} \xrightarrow{r[x,y]} \boxed{\text{Search peak(s)}} \rightarrow \textit{object location(s)}\ p,q$$

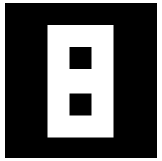
$$h[x,y] = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{1}{\Phi_{nn}\left(e^{j\omega_x}, e^{j\omega_y}\right)} e^{j\omega_x x + j\omega_y y}\, d\omega_x d\omega_y$$

- For white noise $n[x,y]$, no prefiltering $h[x,y]$ required
- Low frequency clutter: highpass prefilter

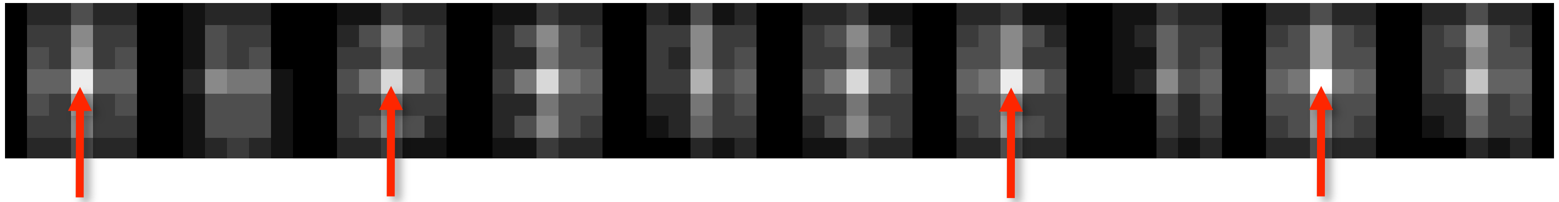
Matched filtering example

Test Image

Template



Template Matching Result

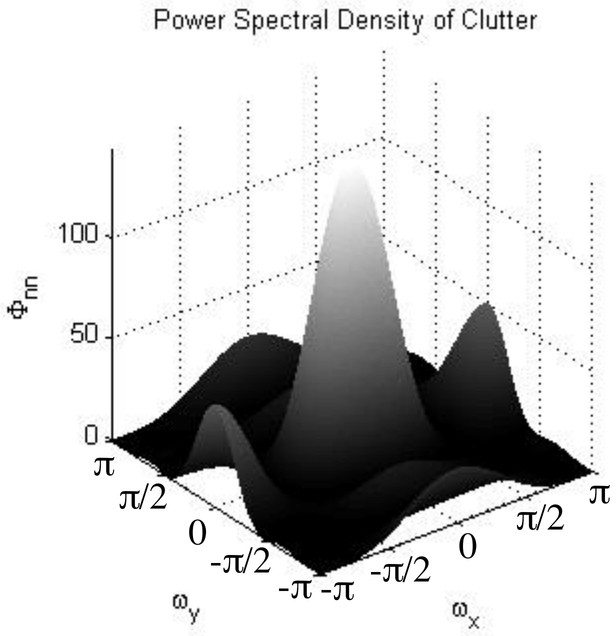


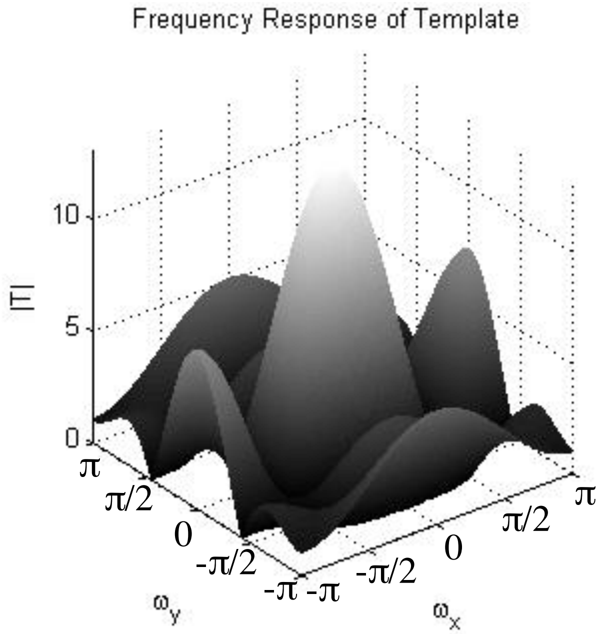
0.92 0.84 0.92 1.00

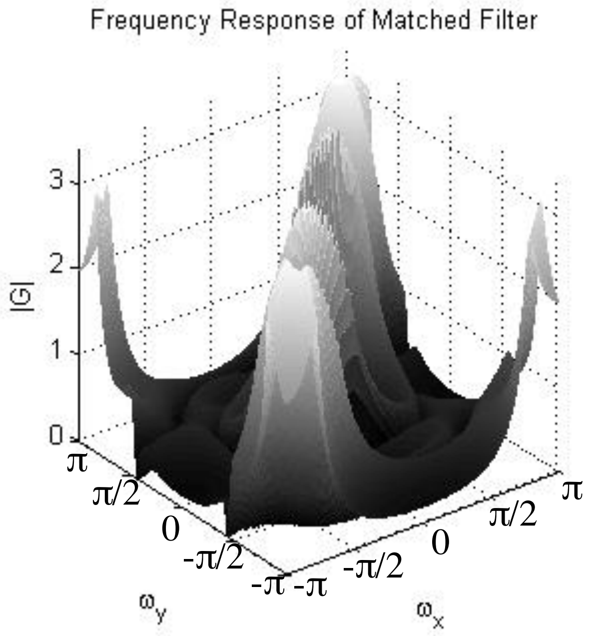
Matched Filtering Result

0.64 0.77 0.73 1.00

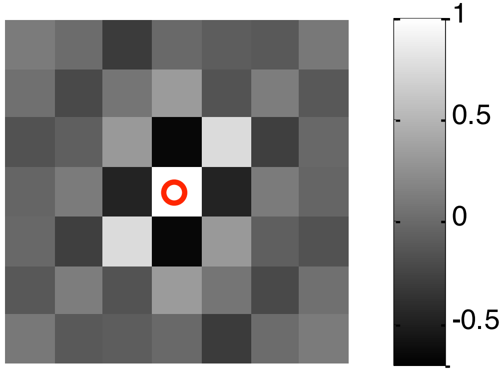
Matched filtering example (cont.)



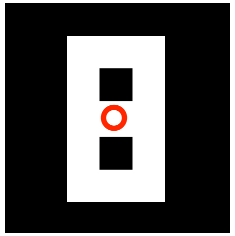




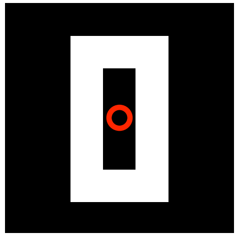
Matched Filter Impulse Response (180^o rotated)

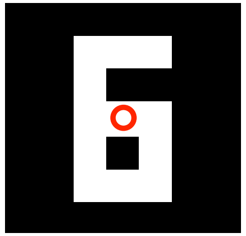


Template



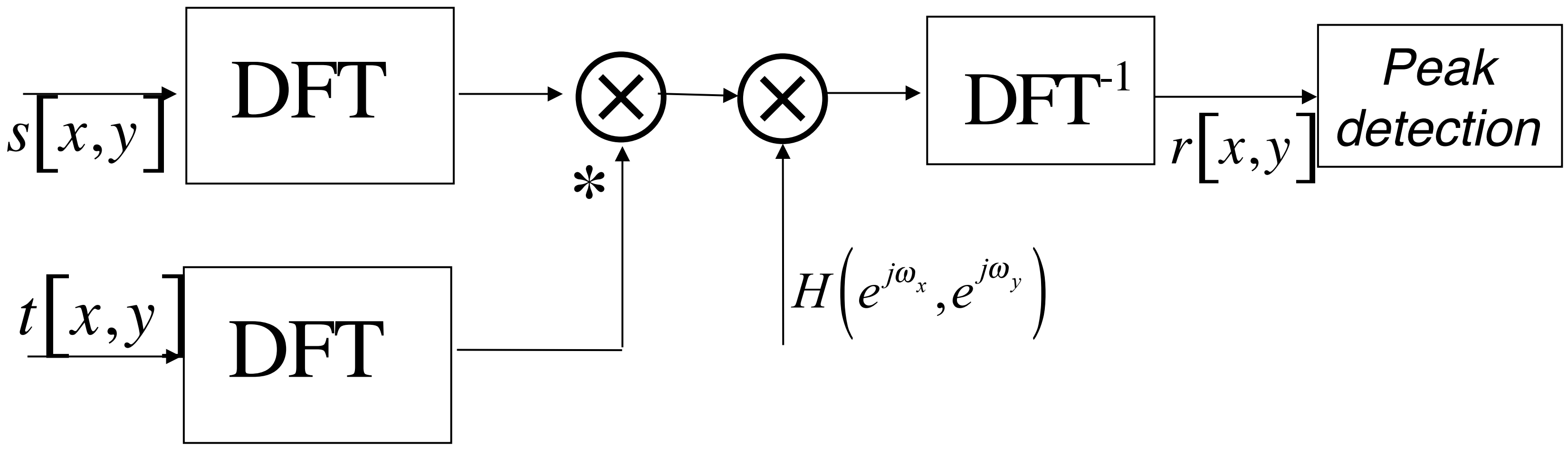
Clutter





Phase correlation

- Efficient implementation employing the Discrete Fourier Transform



- Phase correlation

$$H\left(e^{j\omega_x}, e^{j\omega_y}\right) = \frac{1}{\left|S\left(e^{j\omega_x}, e^{j\omega_y}\right)\right| \left|T\left(e^{j\omega_x}, e^{j\omega_y}\right)\right|}$$