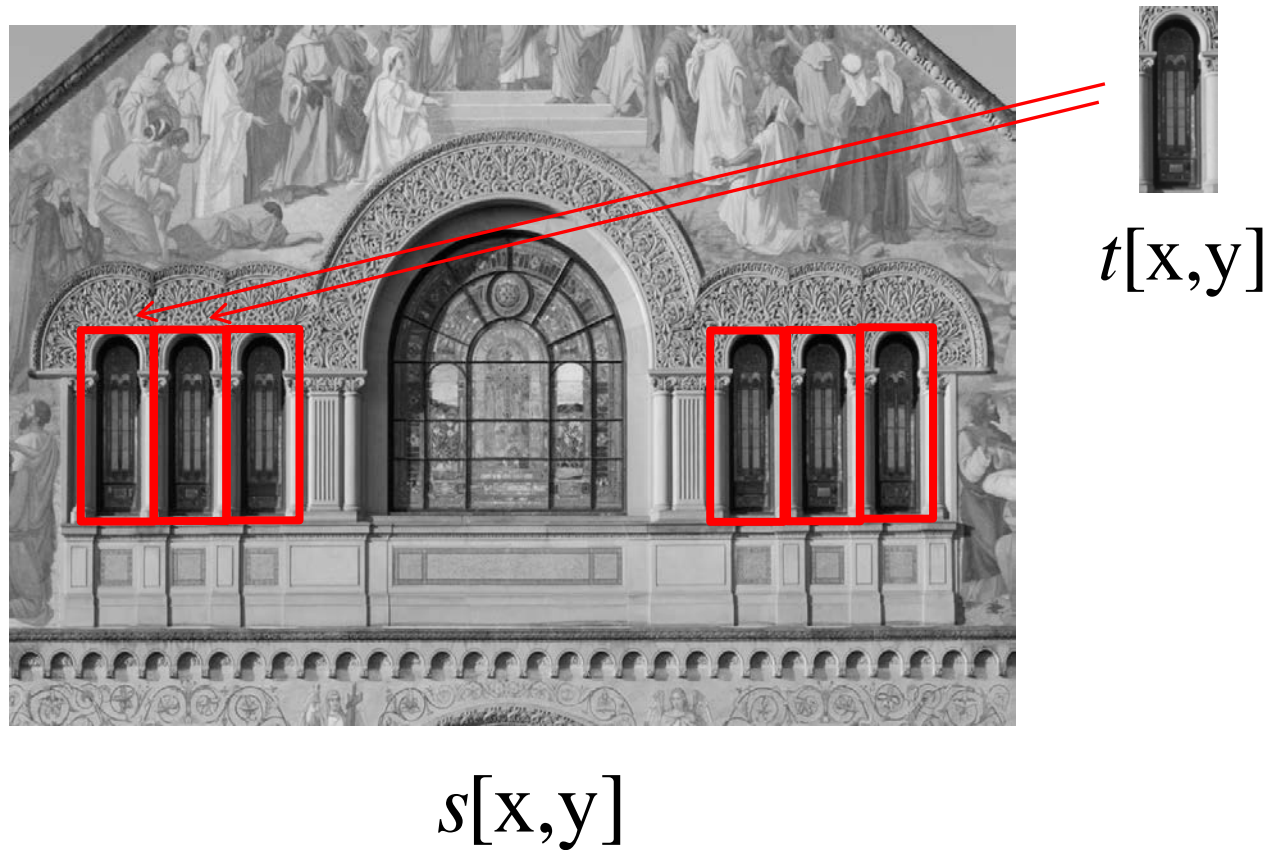


Template matching

- Problem: locate an object, described by a template $t[x,y]$, in the image $s[x,y]$
- Example



Template matching (cont.)

- Search for the best match by minimizing mean-squared error

$$\begin{aligned} E[p, q] &= \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} \left[s[x, y] - t[x - p, y - q] \right]^2 \\ &= \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} |s[x, y]|^2 + \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} |t[x, y]|^2 - 2 \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x, y] \cdot t[x - p, y - q] \end{aligned}$$

- Equivalently, maximize *area correlation*

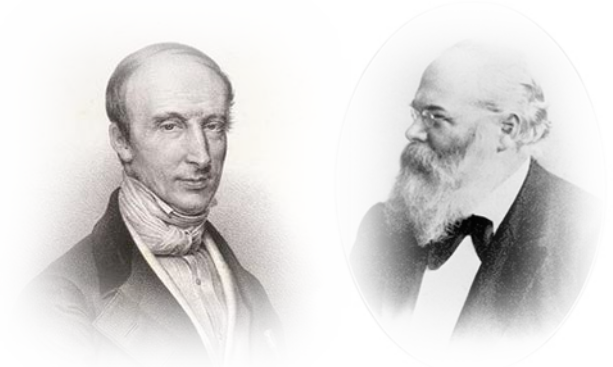
$$r[p, q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x, y] \cdot t[x - p, y - q] = s[p, q] * t[-p, -q]$$

- Area correlation is equivalent to convolution of image $s[x, y]$ with impulse response $t[-x, -y]$

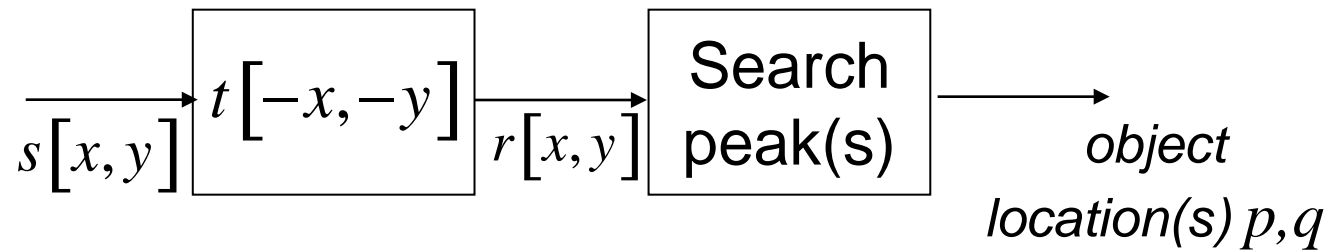
Template matching (cont.)

- From Cauchy-Schwarz inequality

$$r[p, q] = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} s[x, y] \cdot t[x-p, y-q] \leq \sqrt{\left(\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} |s[x, y]|^2 \right) \left(\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} |t[x, y]|^2 \right)}$$



- Equality, iff $s[x, y] = \alpha \cdot t[x-p, y-q]$ with $\alpha \geq 0$
- Block diagram of template matcher



- Remove mean before template matching to avoid bias towards bright image areas

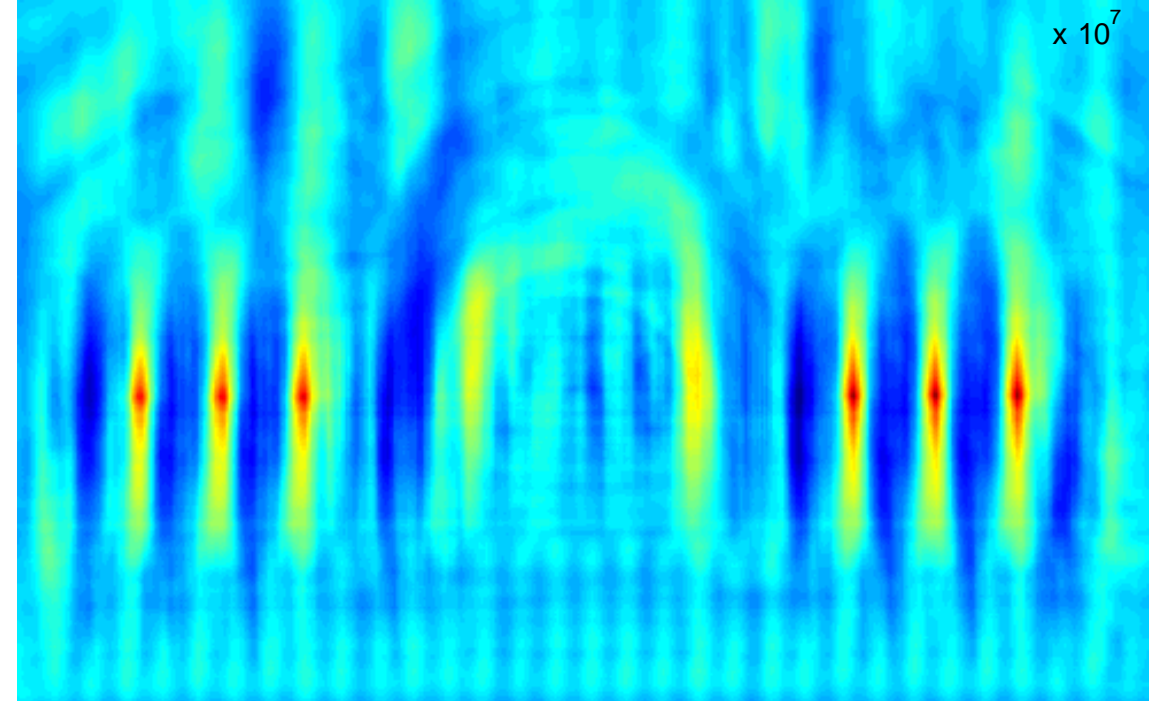
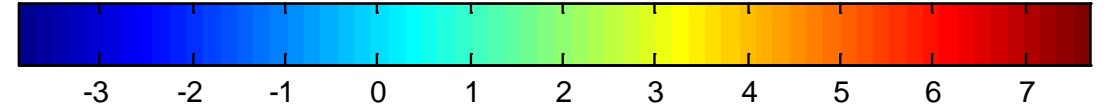
Template matching example



$t[x,y]$



$s[x,y]$

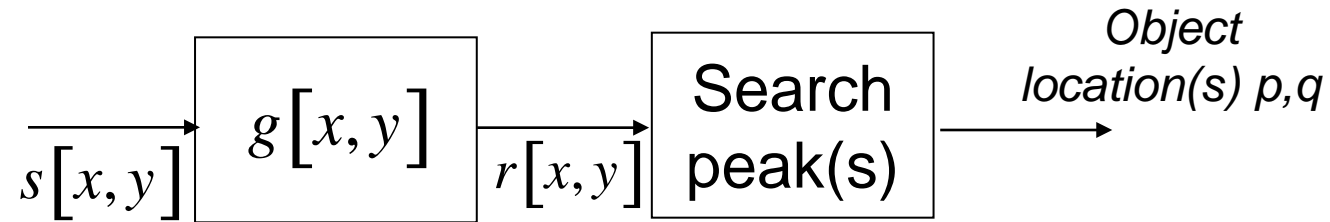


$r[p,q]$



Matched filtering

- Consider signal detection problem



- Signal model

$$s[x, y] = t[x - p, y - q] + n[x, y]$$

The equation is annotated with:

- A green box labeled "shifted template" pointing to $t[x - p, y - q]$.
- A blue box labeled "Other objects: 'noise' or 'clutter' psd $\Phi_{nn}(e^{j\omega_x}, e^{j\omega_y})$ " pointing to $n[x, y]$.

- Problem: design filter $g[x, y]$ to maximize

$$SNR = \frac{|r[p, q]|^2}{E \left\{ |n[x, y] * g[x, y]|^2 \right\}}$$

The equation is annotated with:

- A green box labeled "correct peak" pointing to the numerator $|r[p, q]|^2$.
- A blue box labeled "false readings" pointing to the denominator $E \left\{ |n[x, y] * g[x, y]|^2 \right\}$.

Matched filtering (cont.)

- Optimum filter has frequency response

$$G(e^{j\omega_x}, e^{j\omega_y}) = \frac{T^*(e^{j\omega_x}, e^{j\omega_y})}{\Phi_{nn}(e^{j\omega_x}, e^{j\omega_y})}$$

- Proof:

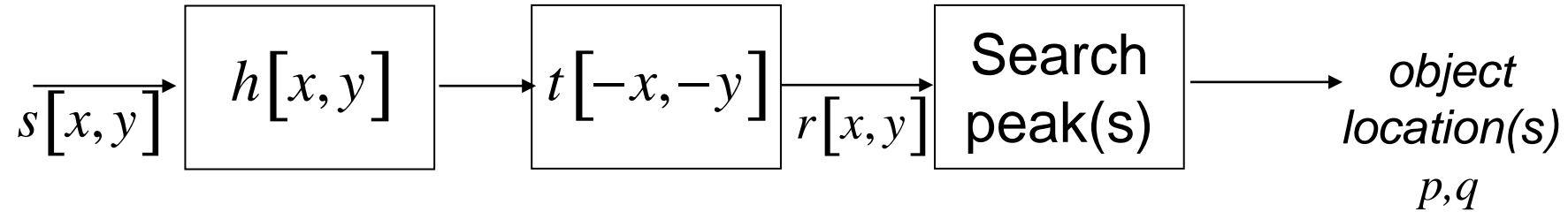
$$\begin{aligned}
 SNR &= \frac{|r[p, q]|^2}{E\{|n[x, y] * g[x, y]|^2\}} \approx \frac{\left| \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} G(e^{j\omega_x}, e^{j\omega_y}) T(e^{j\omega_x}, e^{j\omega_y}) d\omega_x d\omega_y \right|^2}{\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |G(e^{j\omega_x}, e^{j\omega_y})|^2 \Phi_{nn}(e^{j\omega_x}, e^{j\omega_y}) d\omega_x d\omega_y} \\
 &= \frac{\left| \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} [G \Phi_{nn}^{1/2}] [\Phi_{nn}^{-1/2} T] d\omega_x d\omega_y \right|^2}{\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |G|^2 \Phi_{nn} d\omega_x d\omega_y} \leq \frac{\left[\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |G|^2 \Phi_{nn} d\omega_x d\omega_y \right] \left[\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |T|^2 \Phi_{nn}^{-1} d\omega_x d\omega_y \right]}{\int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |G|^2 \Phi_{nn} d\omega_x d\omega_y} \\
 &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} |T|^2 \Phi_{nn}^{-1} d\omega_x d\omega_y
 \end{aligned}$$

↙
 Cauchy-Schwarz inequality,
 with equality, iff $G \Phi_{nn}^{1/2} = \alpha \cdot [\Phi_{nn}^{-1/2} T]^*$

↑
 max. SNR

Matched filtering (cont.)

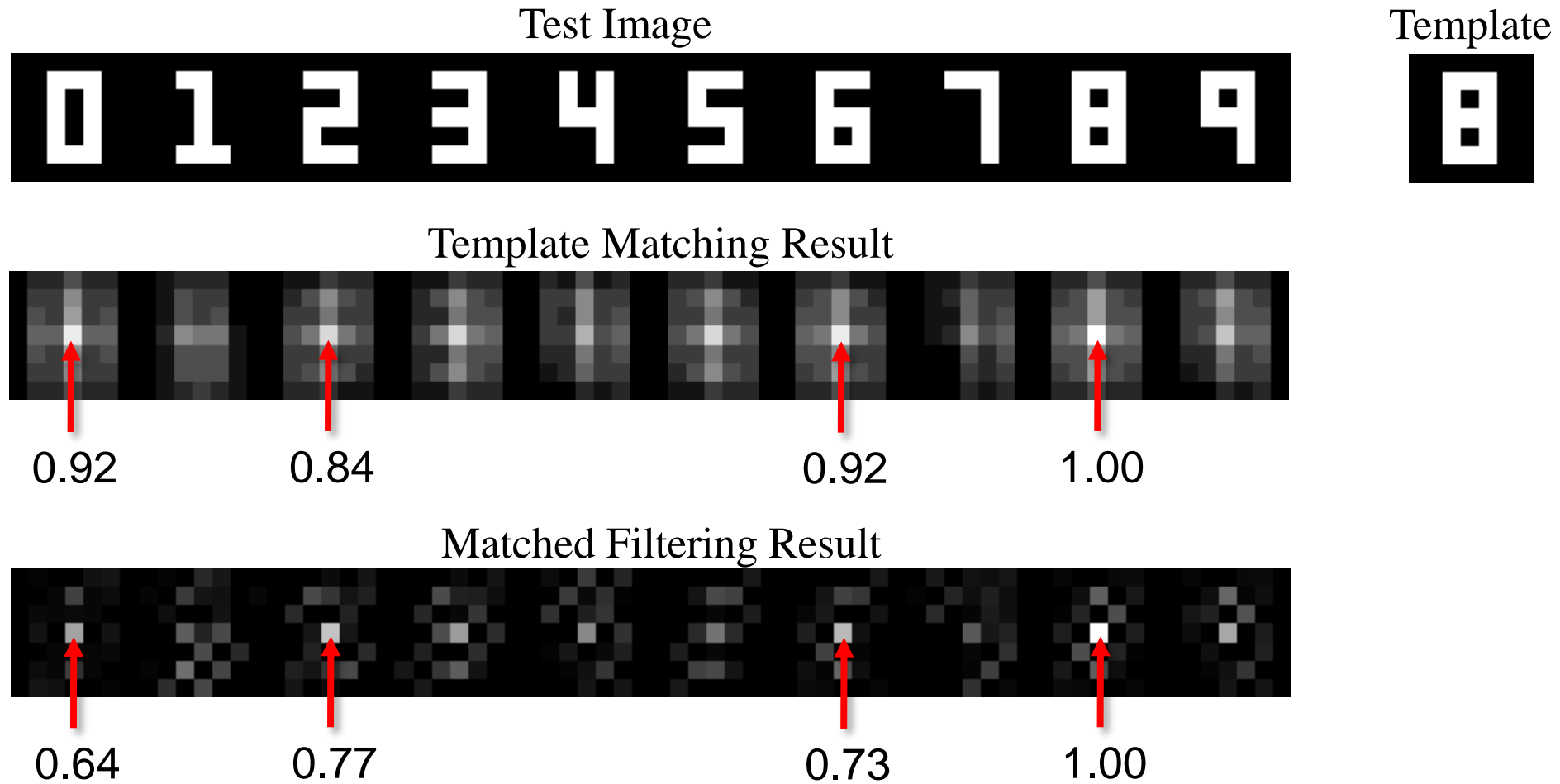
- Optimum detection: prefiltering & template matching



$$h[x, y] = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{1}{\Phi_{nn}(e^{j\omega_x}, e^{j\omega_y})} e^{j\omega_x x + j\omega_y y} d\omega_x d\omega_y$$

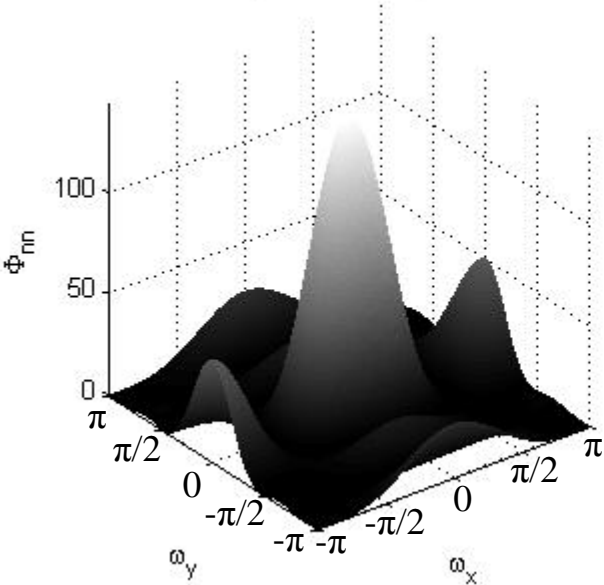
- For white noise $n[x, y]$, no prefiltering $h[x, y]$ required
- Low frequency clutter: highpass prefilter

Matched filtering example

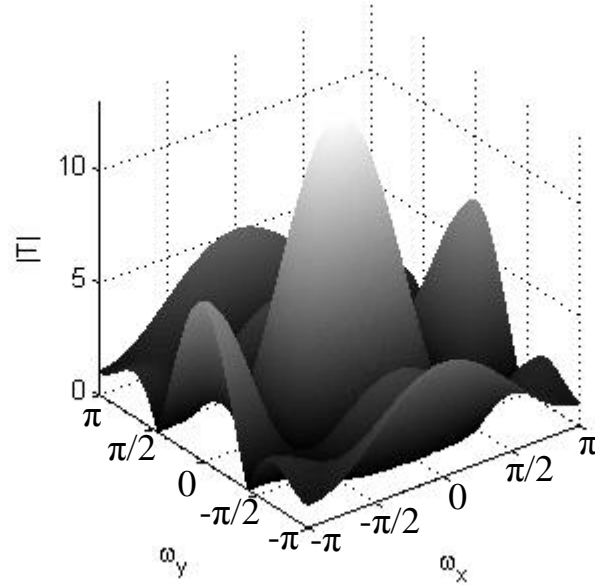


Matched filtering example (cont.)

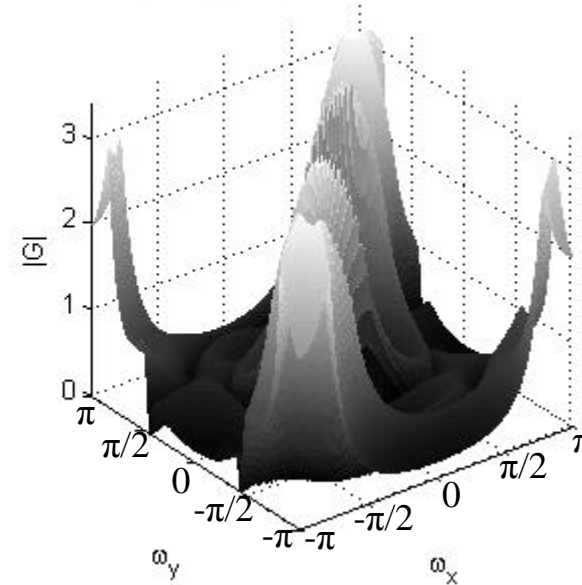
Power Spectral Density of Clutter



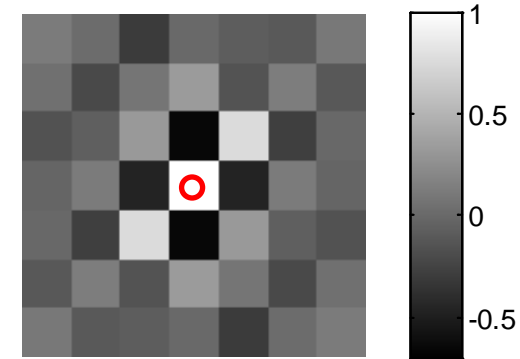
Frequency Response of Template



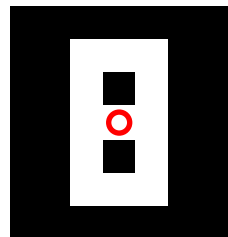
Frequency Response of Matched Filter



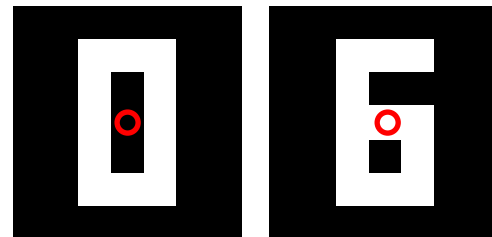
Matched Filter
Impulse Response
(180° rotated)



Template

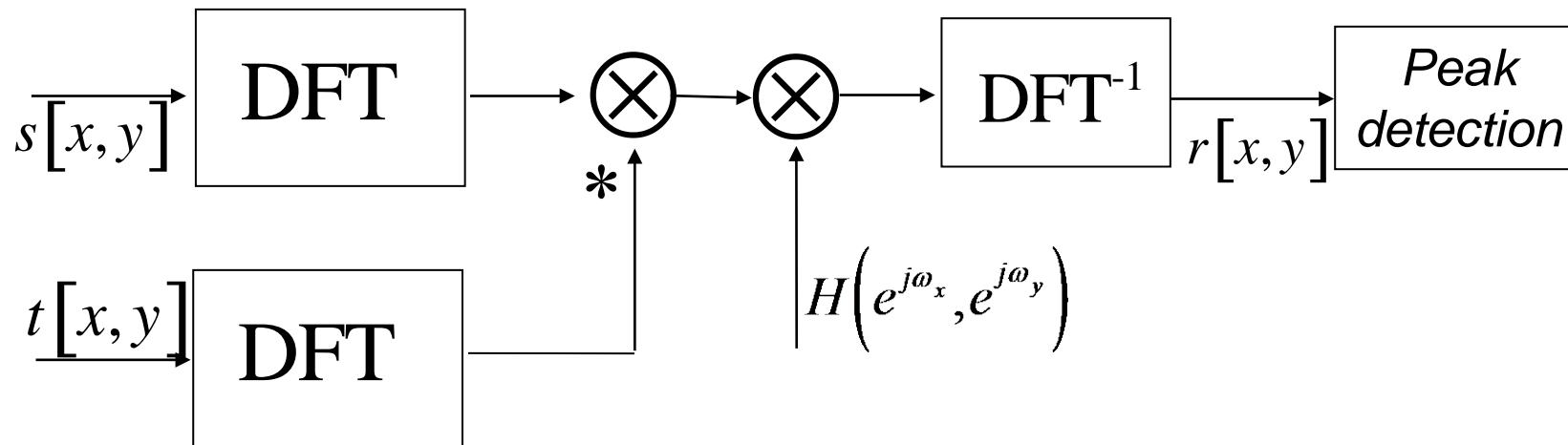


Clutter



Phase correlation

- Efficient implementation employing the Discrete Fourier Transform



- Phase correlation

$$H(e^{j\omega_x}, e^{j\omega_y}) = \frac{1}{\left| S(e^{j\omega_x}, e^{j\omega_y}) \right| \left| T(e^{j\omega_x}, e^{j\omega_y}) \right|}$$