

# Point operations for combining images

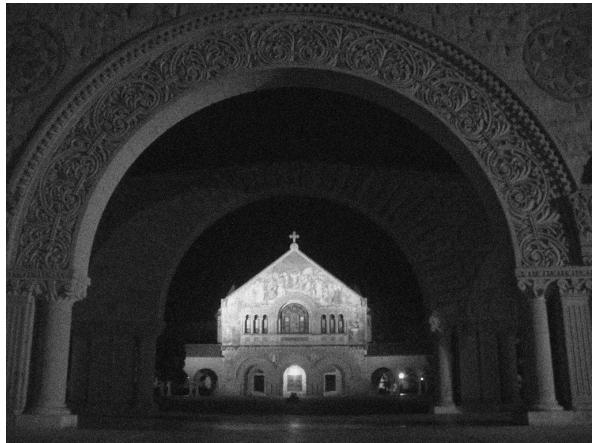
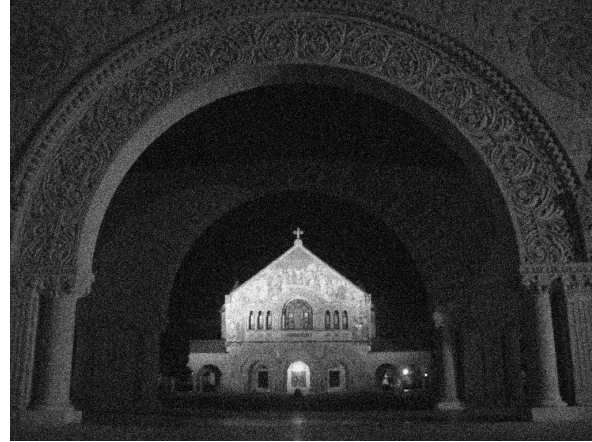
- Image averaging for noise reduction
- Combination of different exposures for high-dynamic range imaging
- Image subtraction for change detection
- Need for accurate alignment

# Image averaging for noise reduction

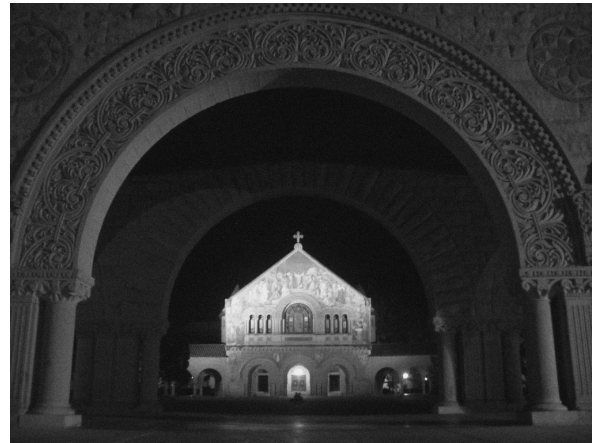
1 image



2 images



8 images



32 images



- Take N aligned images  $f_1(x, y), f_2(x, y), \dots, f_N(x, y)$

- Average image:  $\overline{f(x, y)} = \frac{1}{N} \sum_{i=1}^N f_i(x, y)$

- Mean squared error vs. noise-free image  $g$

$$\begin{aligned}
 E \left\{ \left( \overline{f} - g \right)^2 \right\} &= E \left\{ \left( \left( \frac{1}{N} \sum_i f_i \right) - g \right)^2 \right\} = E \left\{ \left( \left( \frac{1}{N} \sum_i (g + n_i) \right) - g \right)^2 \right\} \\
 &= E \left\{ \left( \frac{1}{N} \sum_i n_i \right)^2 \right\} = \frac{1}{N^2} \sum_i E \{ n_i^2 \} = \frac{1}{N} E \{ n^2 \}
 \end{aligned}$$

provided  $E \{ n_i n_j \} = 0 \forall i, j$

$E \{ n_i \} = E \{ n \} \forall i$

# High-dynamic range imaging



-8 f-stops



-2 f-stops



+2 f-stops



+4 f-stops



Blended image from  
Exposure Fusion

*[Tom Mertens et al. 2007]*

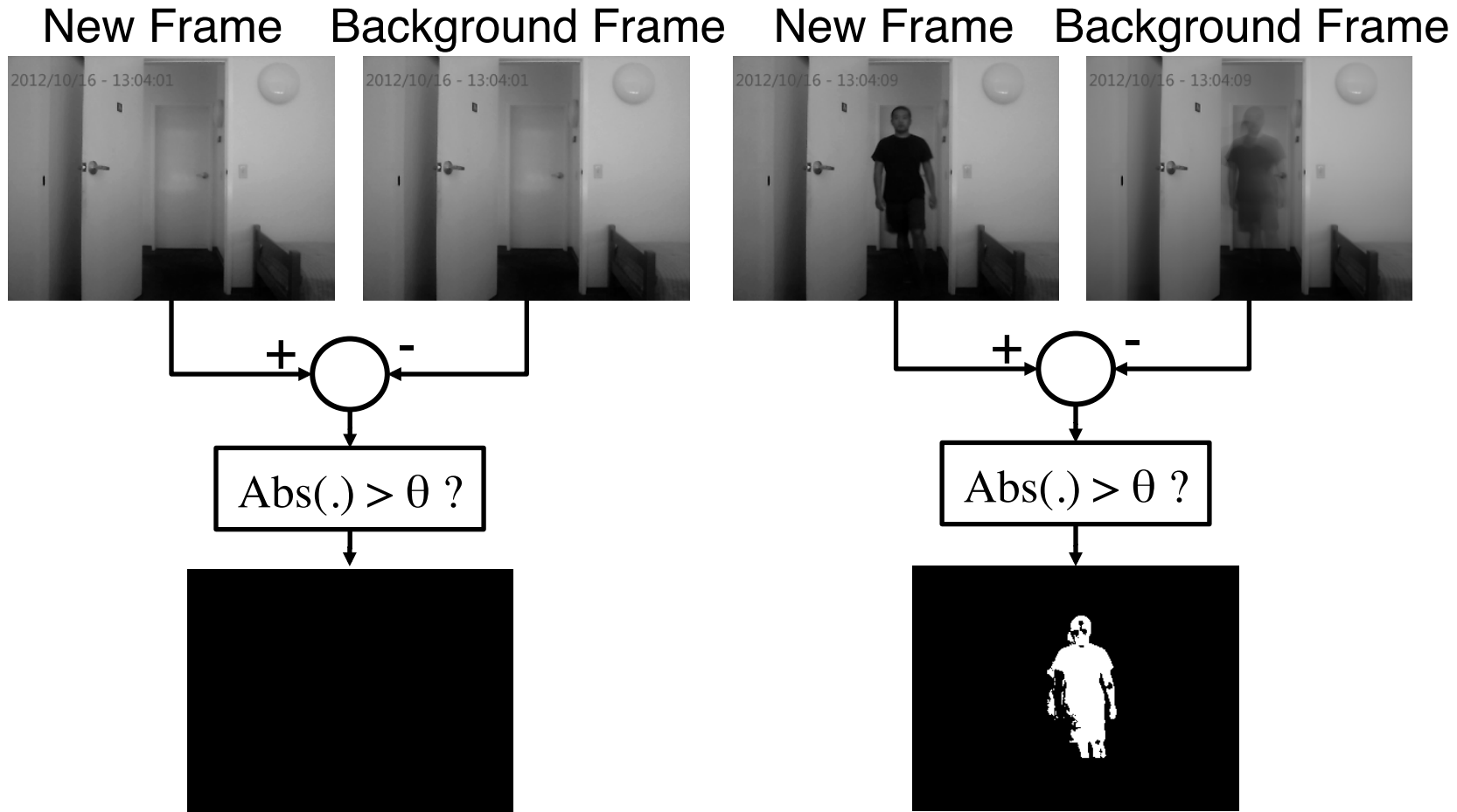


# Image subtraction

- Find differences/changes between 2 mostly identical images
- Example: digital subtraction angiography



# Video background subtraction

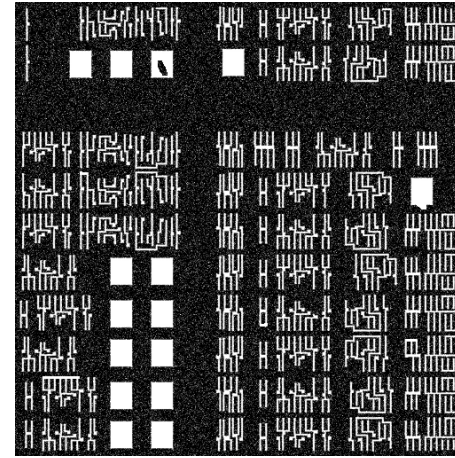
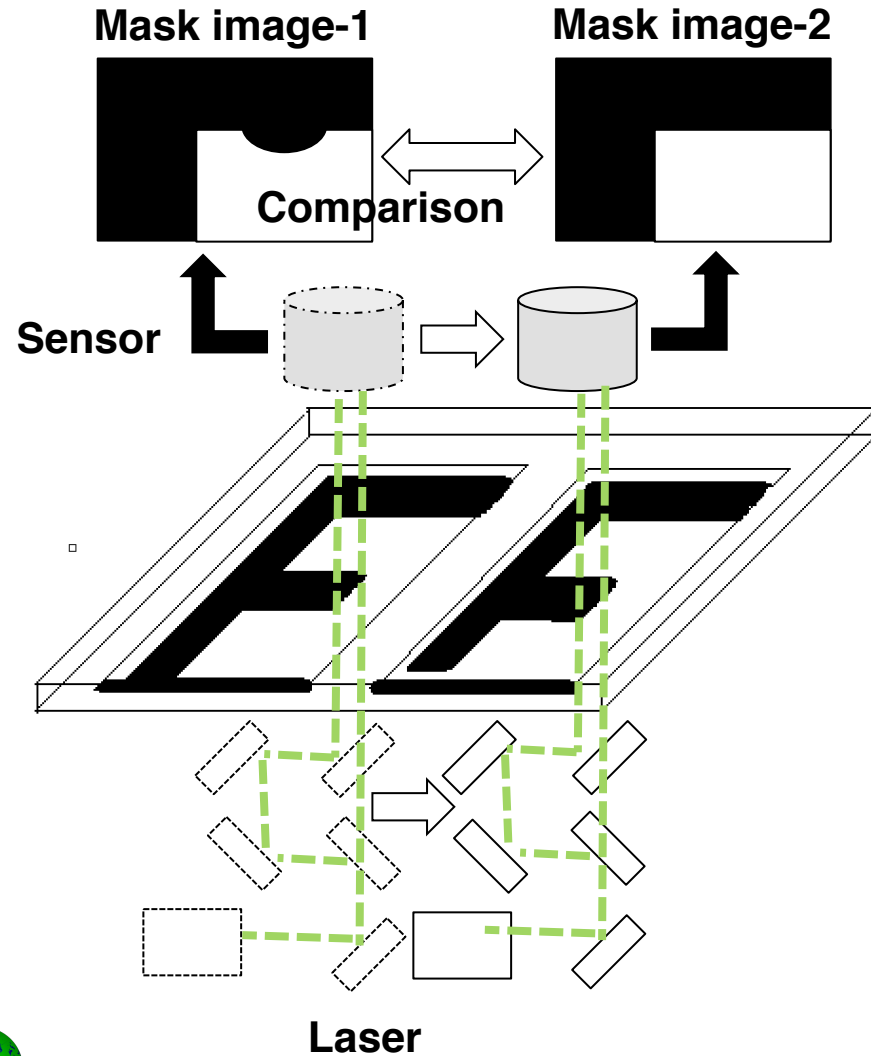


**Update:**

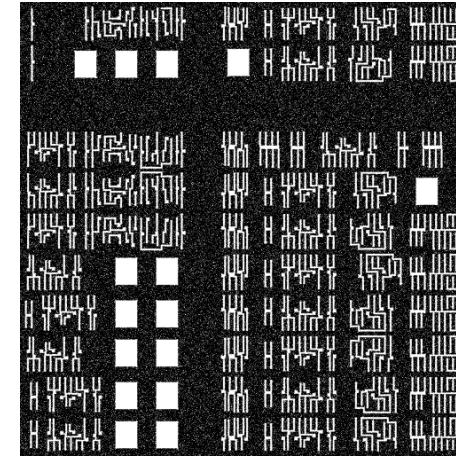
$$\text{Background}[t] := \alpha \text{Background}[t-1] + (1 - \alpha) \text{New}[t]$$



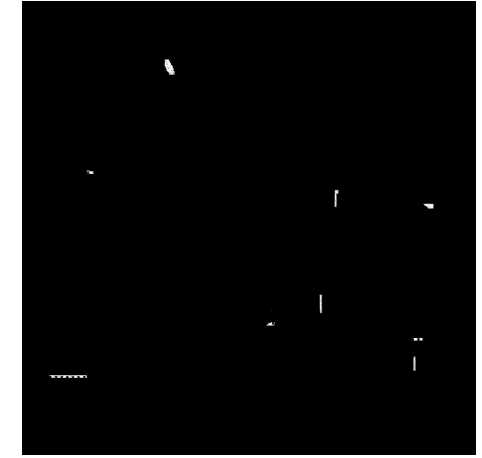
# Image subtraction in IC manufacturing: inspection of photomasks



Mask image-1



Mask image-2



Difference image



# Where is the defect?

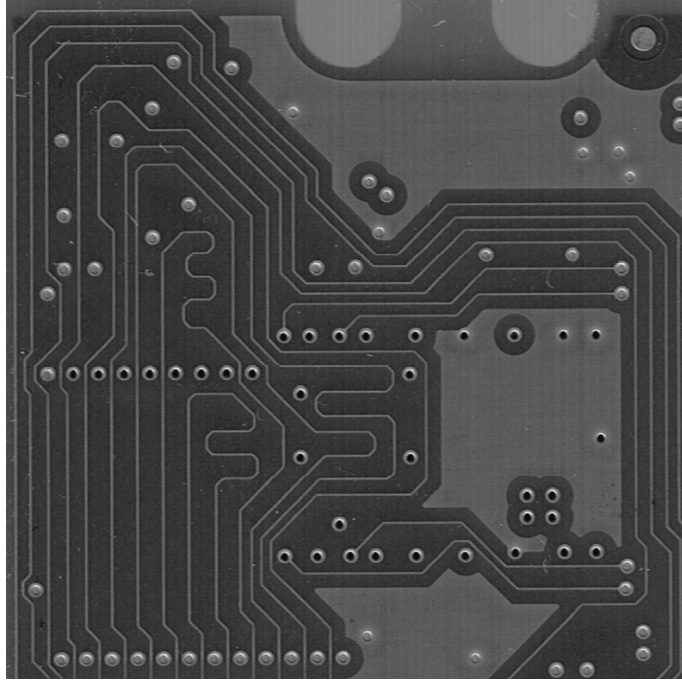


Image  $g[x,y]$  (no defect)

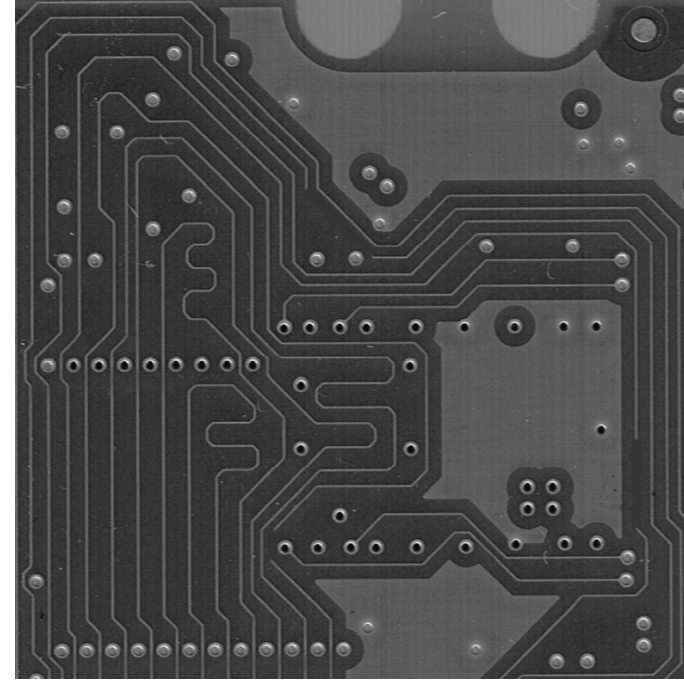
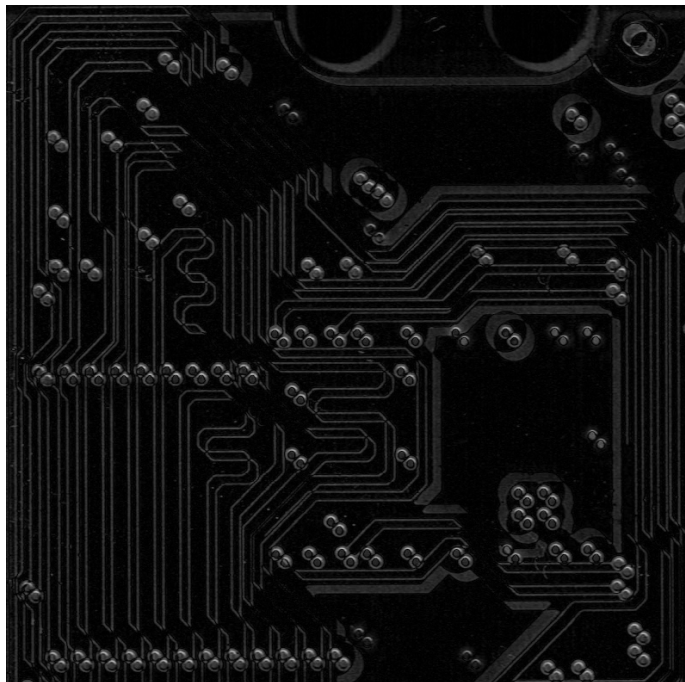


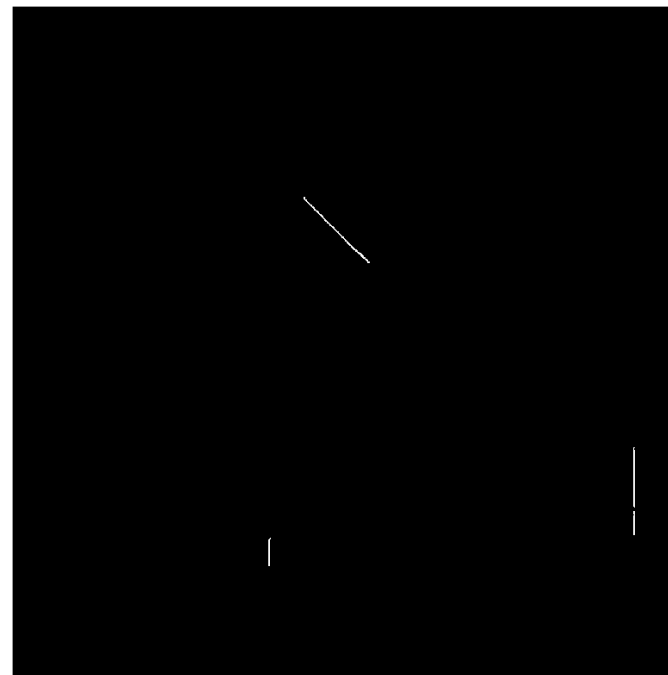
Image  $f[x,y]$  (w/ defect)



# Absolute difference between two images



$|f-g|$  w/o alignment



$|f-g|$  w/ alignment

