



# PANORAMIC IMAGING – PART 1

# Take Home Message

Q: What is a Panorama?

A: Wider-angle image than a normal camera can capture

Compensate for shortcomings of traditional cameras by capturing and fusing many images

Technique:

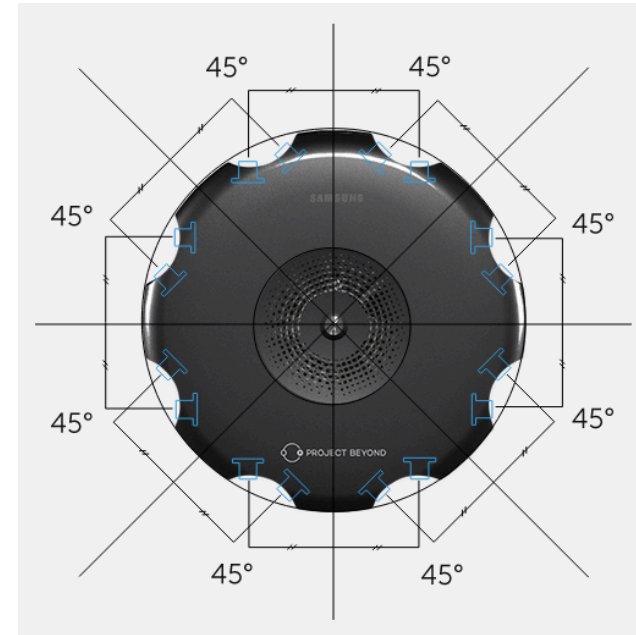
- Take  $N$  images at different directions
- Deduce the image that would have been taken by a wide angle lens

# Application: Virtual Reality



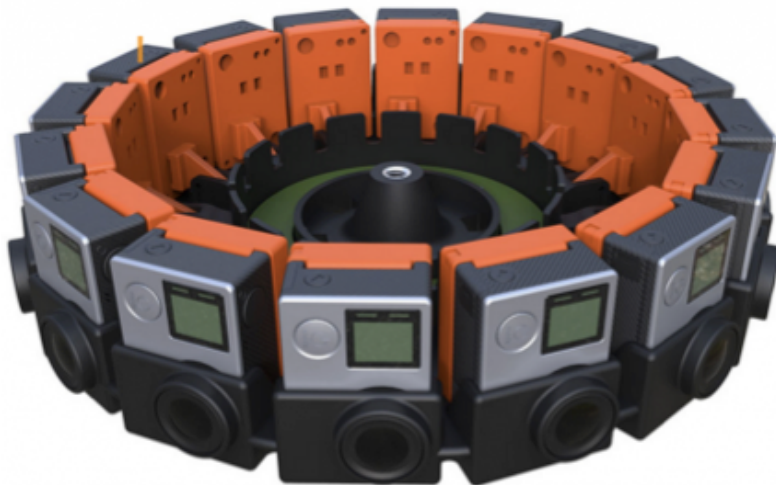
# Real World Capture Setups

Jaunt NEO



Samsung Beyond

Google Jump



Nokia OZO

# Panorama: Virtual Wide Angle



<http://people.csail.mit.edu/fredo/Panos/>

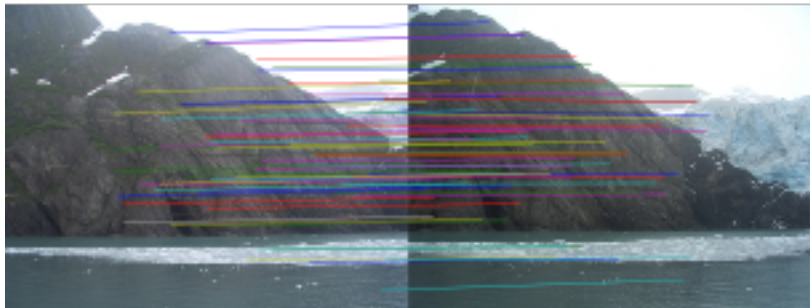
Two lectures:

Part 1: Monoscopic Panoramas  
(today's lecture)

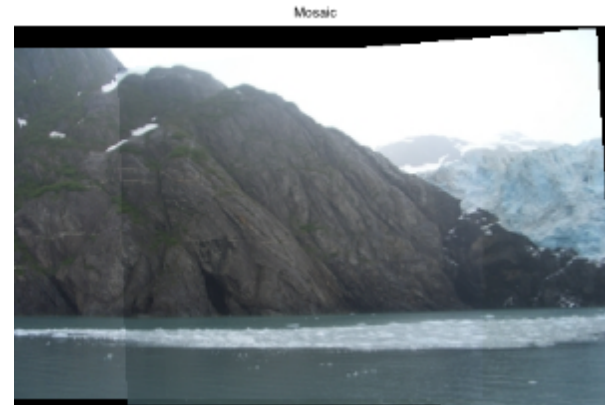
Part 2: Stereoscopic Panoramas  
(next lecture)

# Panoramic Imaging Pipeline

Estimating  
Correspondences



Warping

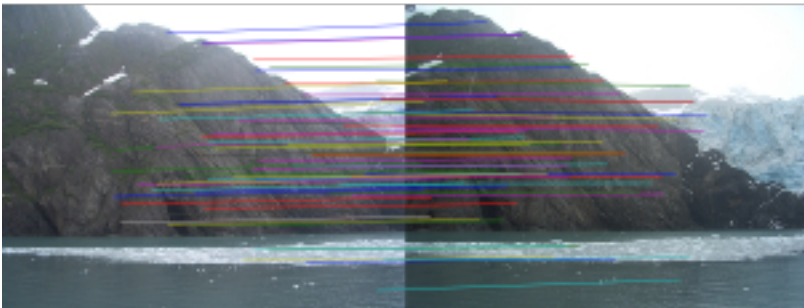


Blending



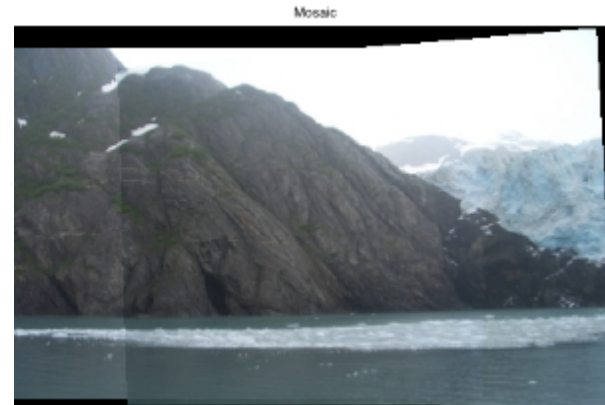
# Panoramic Imaging Pipeline

## Estimating Correspondences



1. Detect keypoints
2. Find out which keypoints in im1 match which keypoints in im2

## Warping



3. Compute transformation between im2 and im1
4. Transform im2 to overlap with im1

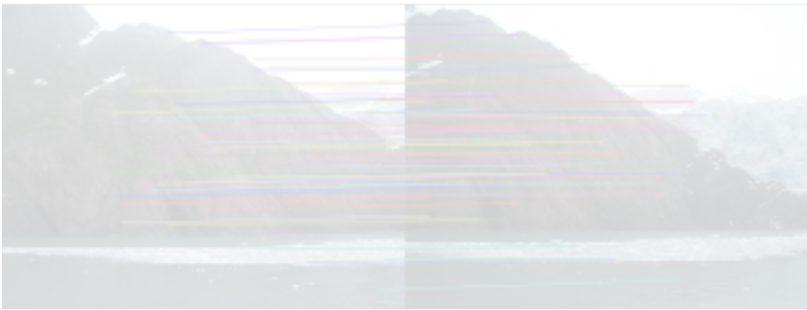
## Blending



5. Blend the images to create a smooth transition
6. Repeat for other images

# Panoramic Imaging Pipeline

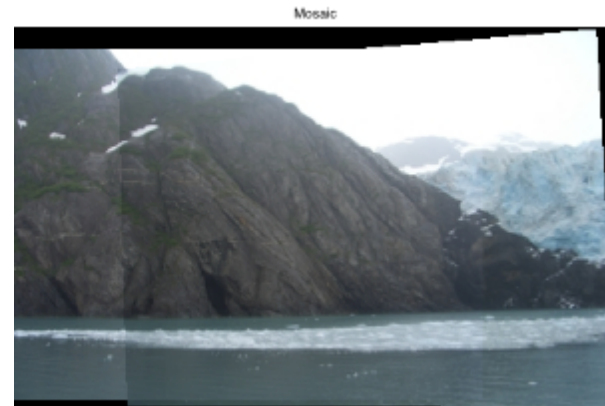
## Estimating Correspondences



Covered in upcoming lectures

For now: assume correspondences are known

## Warping



3. Compute transformation between  $im_2$  and  $im_1$
4. Transform  $im_2$  to overlap with  $im_1$

## Blending



5. Blend the images to create a smooth transition
6. Repeat for other images

# Warp to Align Images



Images: Levoy

# How to Warp?



translation?



rotation?



perspective!

Slide: Levoy

# Image Warping: Mathematical Representation

image filtering: change *range* of image

$$g(x) = T(f(x))$$

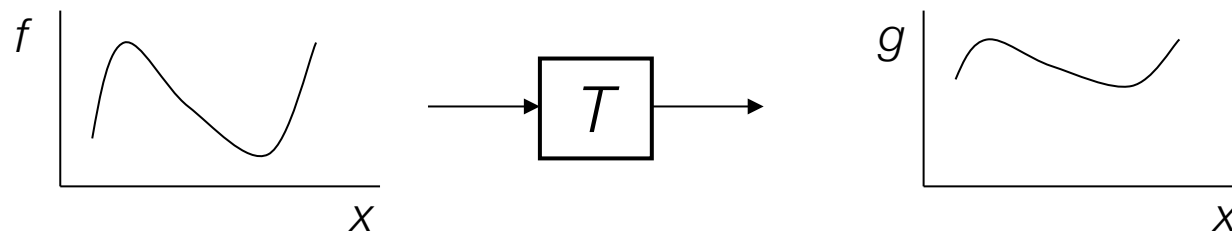
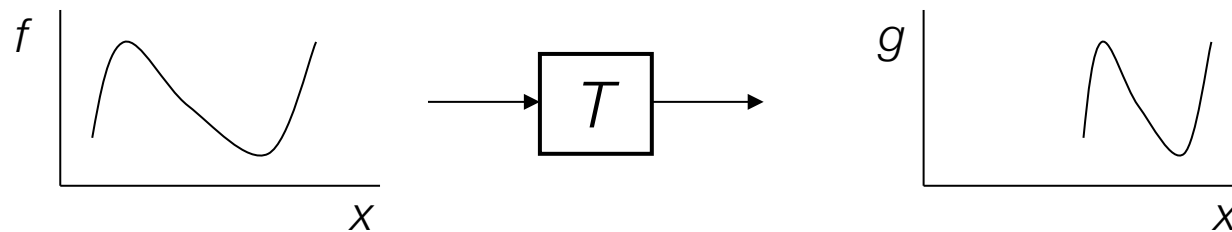


image warping: change *domain* of image

$$g(x) = f(T(x))$$



# Image Warping: Mathematical Representation

image filtering: change *range* of image

$$g(x) = T(f(x))$$

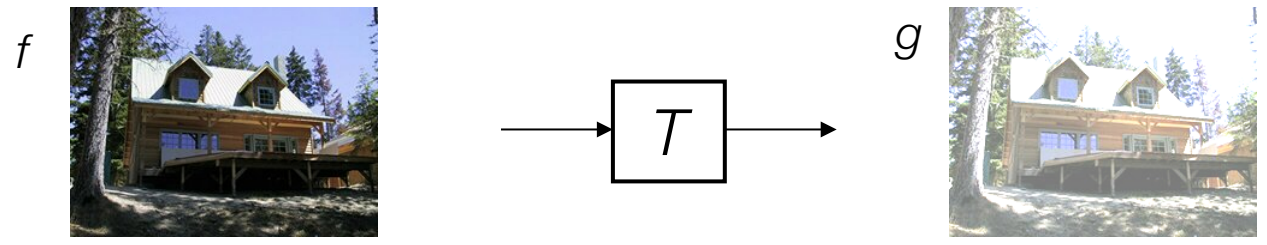
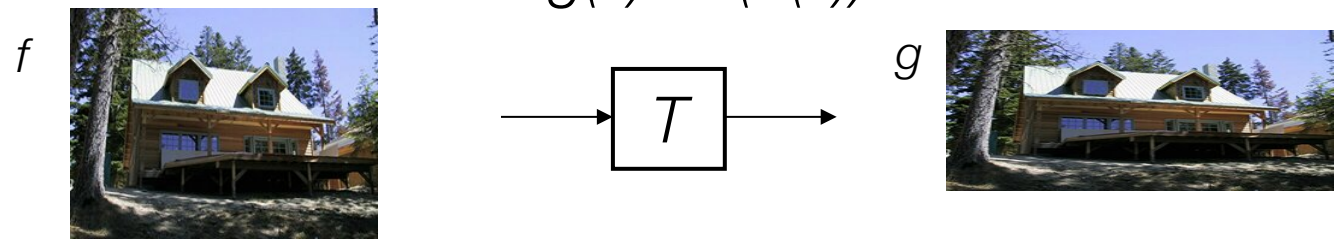


image warping: change *domain* of image

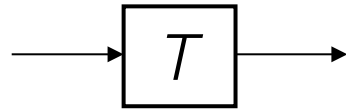
$$g(x) = f(T(x))$$



# Image Warping: Mathematical Representation



$$p = (u, v)$$



$$p' = (u', v')$$

Generic model

$$p' = T(p)$$

Can we model the transformation as a matrix?

# 2D Transformations

Scaling

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Rotation

$$\begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

What about translation?

$$u' = u + t_u$$

$$v' = v + t_v$$

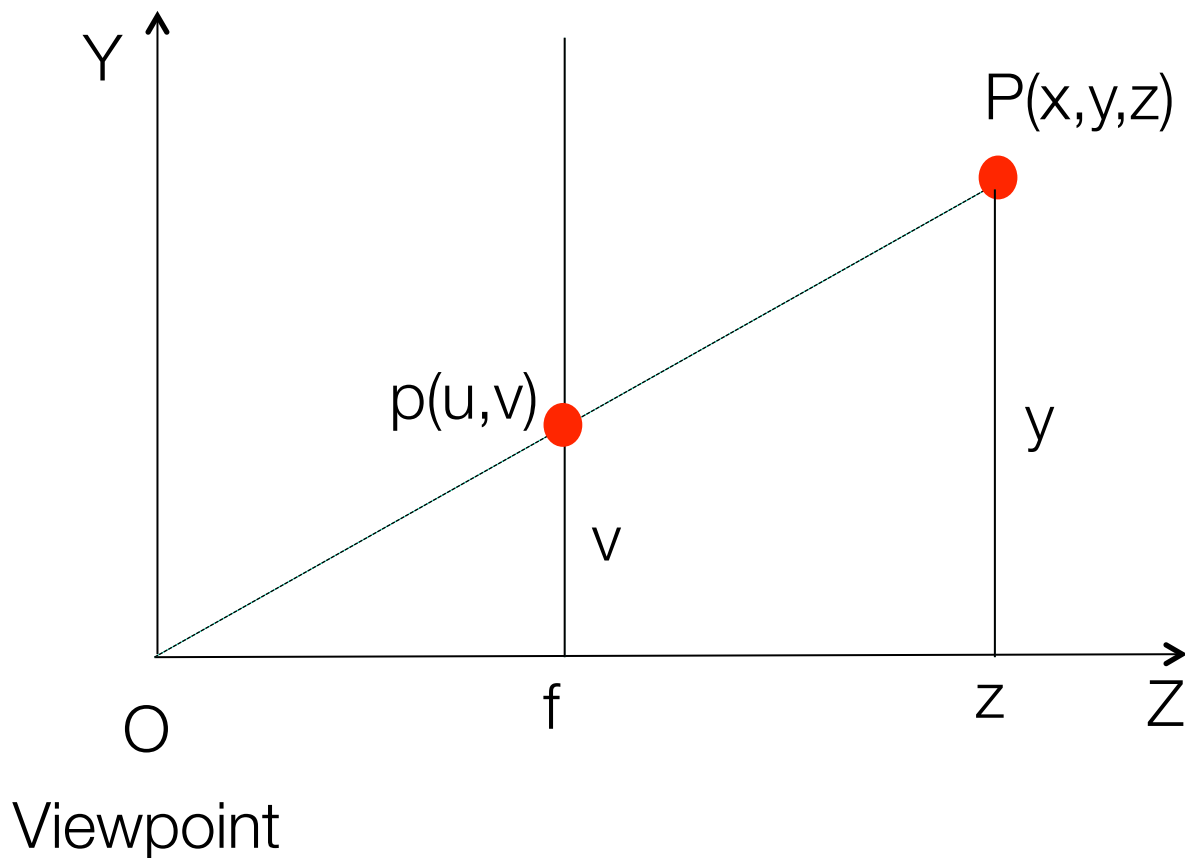
# Homogeneous Coordinates

- Notation trick
- Represent 2D points with 3 numbers
- Homogeneous coordinates  $(u, v, w) \rightarrow$  image coordinates  $(u/w, v/w)$

$$\begin{array}{l} u' = u + t_u \\ v' = v + t_v \end{array} \quad \longrightarrow \quad \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_u \\ 0 & 1 & t_v \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

# Simple Perspective Projection

Project all points to  $z = f$  plane



$$u = f \cdot x / z$$

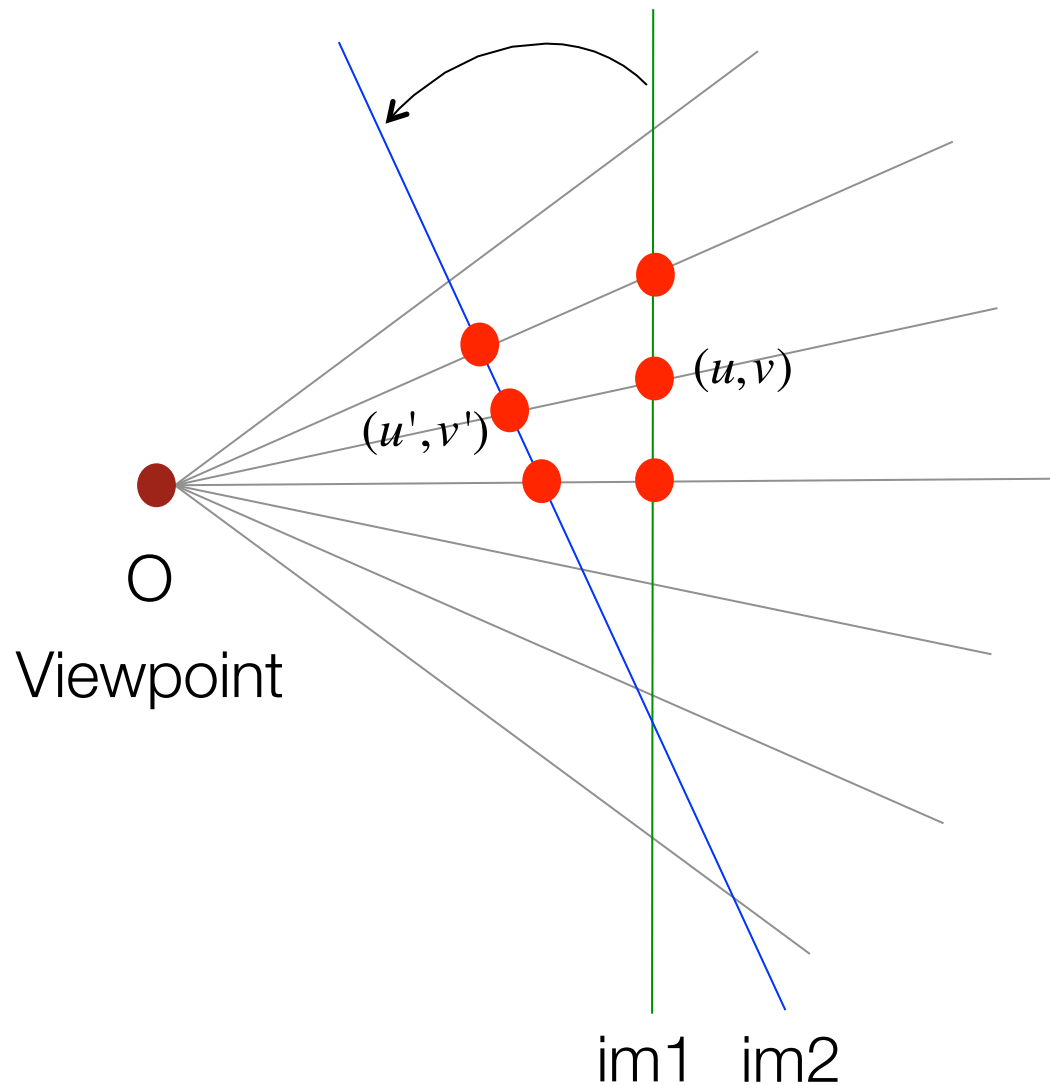
$$v = f \cdot y / z$$



$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \sim \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Image coordinates      Homogeneous coordinates

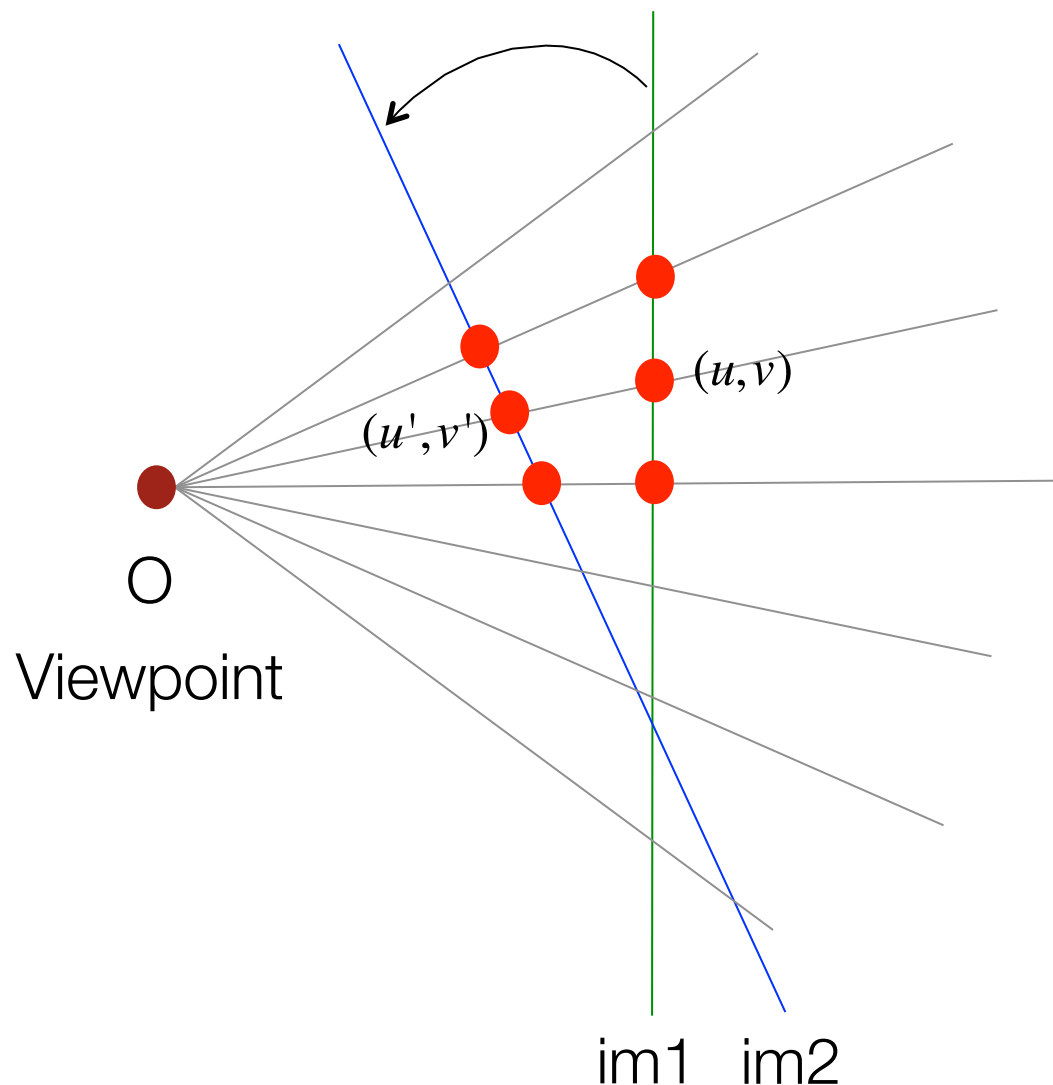
# Central Projection with Rotating Camera



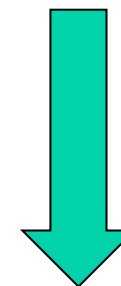
Transform between two images  
with the same camera center

Which transform links  $(u, v)$  to  $(u', v')$  ?

# Central Projection and Homography



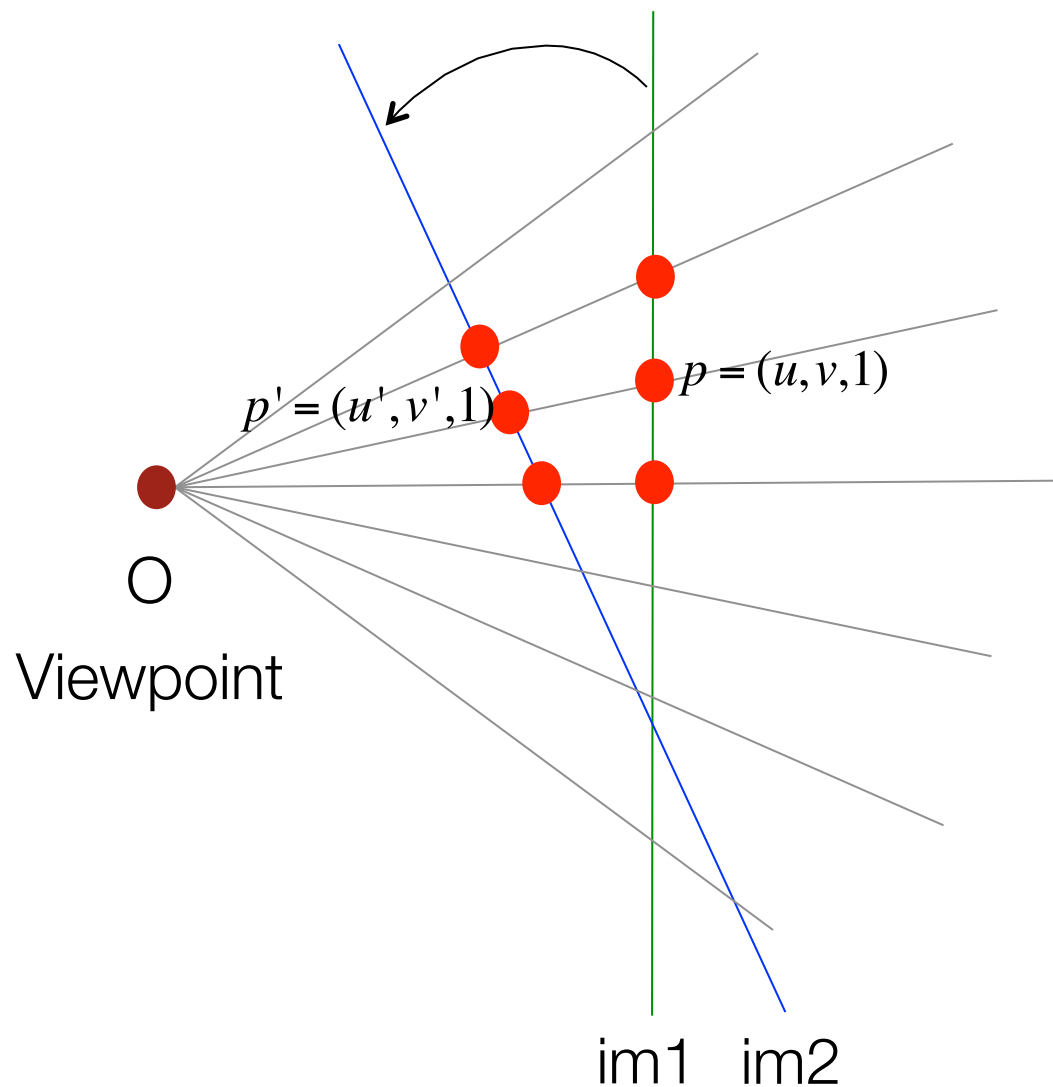
Observation:  
Lines in  $im1$  map onto lines in  $im2$



Any transformation of a plane onto a plane that preserves straight lines must be a homography

Hartley & Zisserman, Theorem 2.10

# Central Projection and Homography



$H$ : non-singular 3x3 matrix

$$p' = H.p$$

$$\begin{bmatrix} wu' \\ wv' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

Valid up to scale

So can set  $i = 1$

# Estimating Homography

$$u'_k = \frac{a.u_k + b.v_k + c}{g.u_k + h.v_k + 1} \Rightarrow u_k a + v_k b + c - u_k u'_k g - v_k v'_k h = u'_k$$

$$v'_k = \frac{d.u_k + e.v_k + f}{g.u_k + h.v_k + 1} \Rightarrow u_k d + v_k e + f - u_k u'_k g - v_k v'_k h = v'_k$$

•  
•  
•

Set up a system of linear equations

$$Ax = b$$

$$x = [a, b, c, d, e, f, g, h]^T$$

# Solve for Homography

Each correspondence gives 2 equations

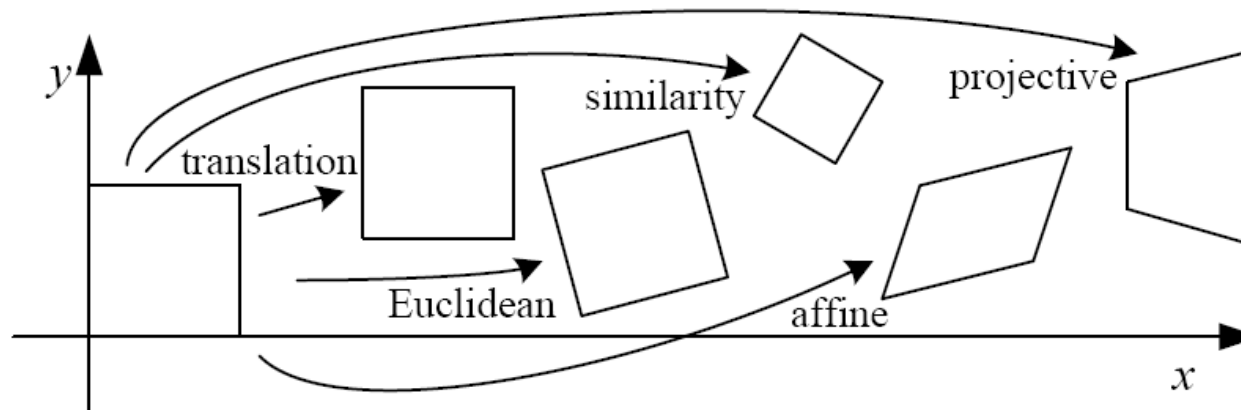
Need at least 8 equations, but the more the better...(why?)






If overdetermined, solve using least squares

Minimize  $\|Ax - b\|^2$

Can be done using “\” in Matlab

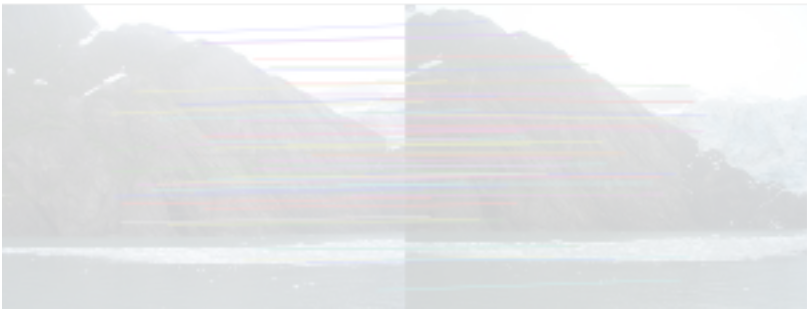
# 2D Image Transformations



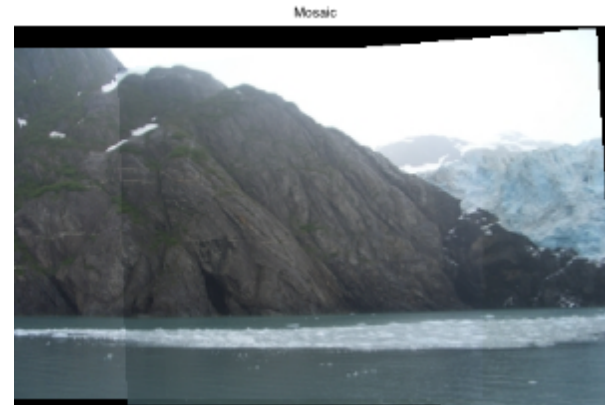
Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	2	orientation + ...	
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	3	lengths + ...	
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$	4	angles + ...	
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$	6	parallelism + ...	
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$	8	straight lines	

# Panoramic Imaging Pipeline

## Estimating Correspondences



## Warping



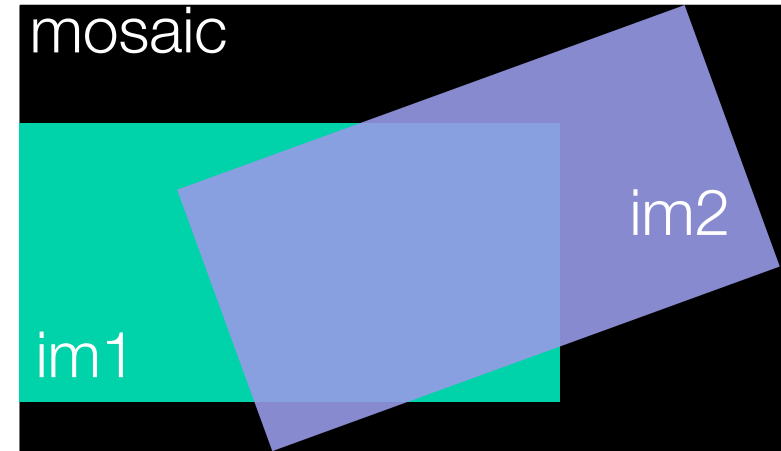
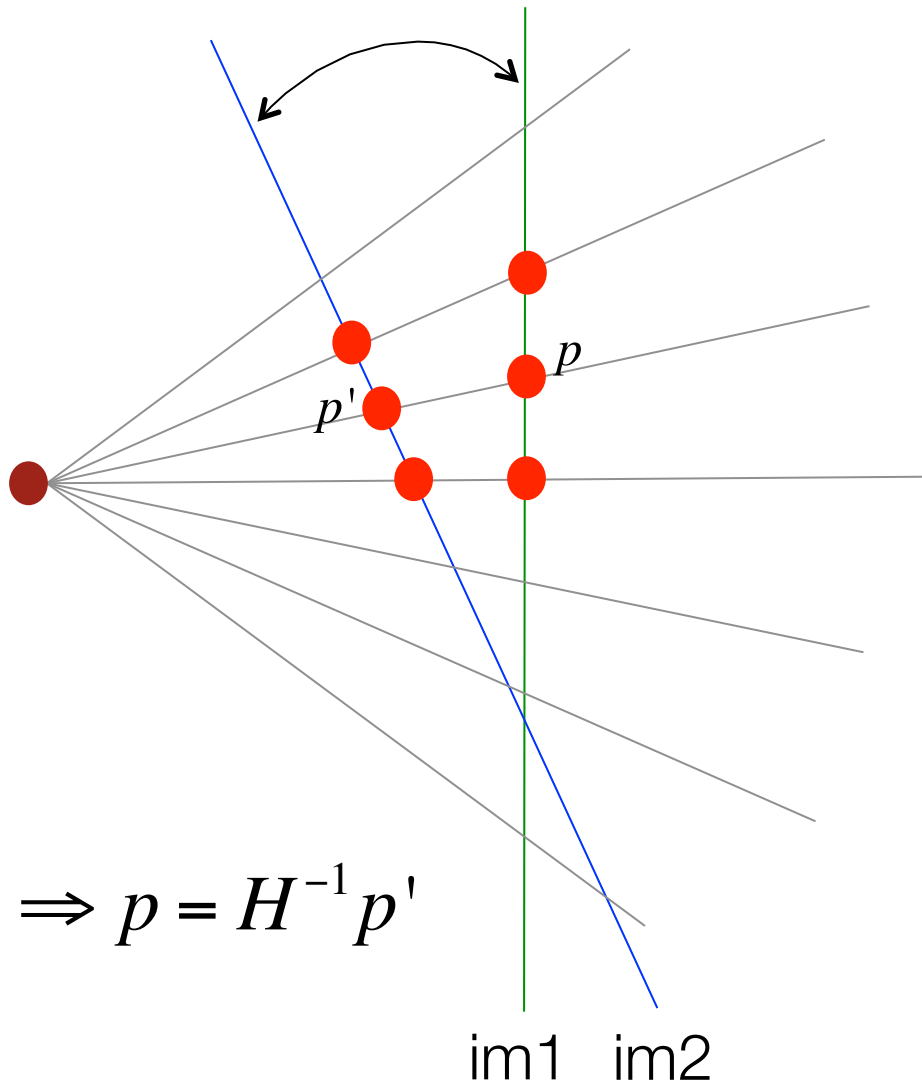
3. Compute transformation between  $im_2$  and  $im_1$
4. Transform  $im_2$  to overlap with  $im_1$

## Blending

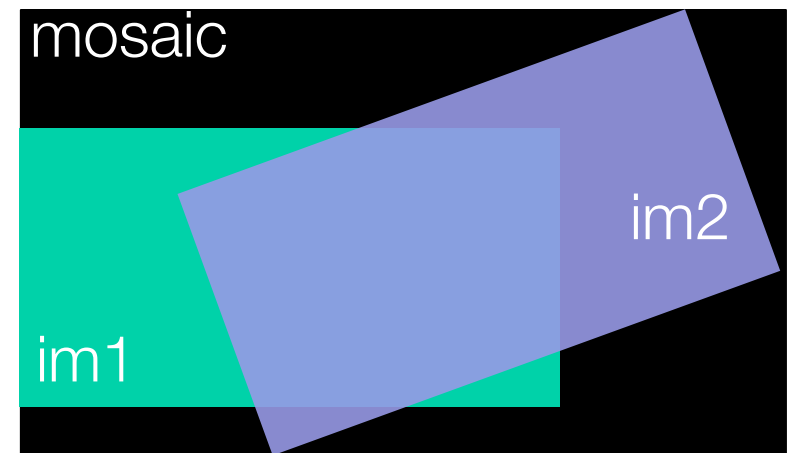
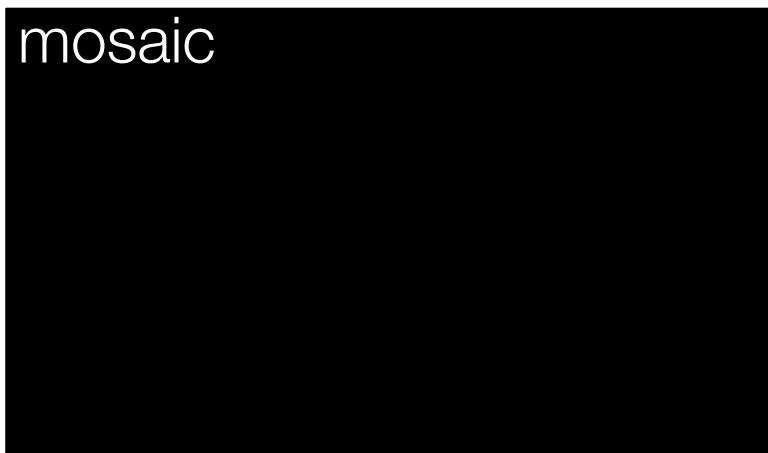
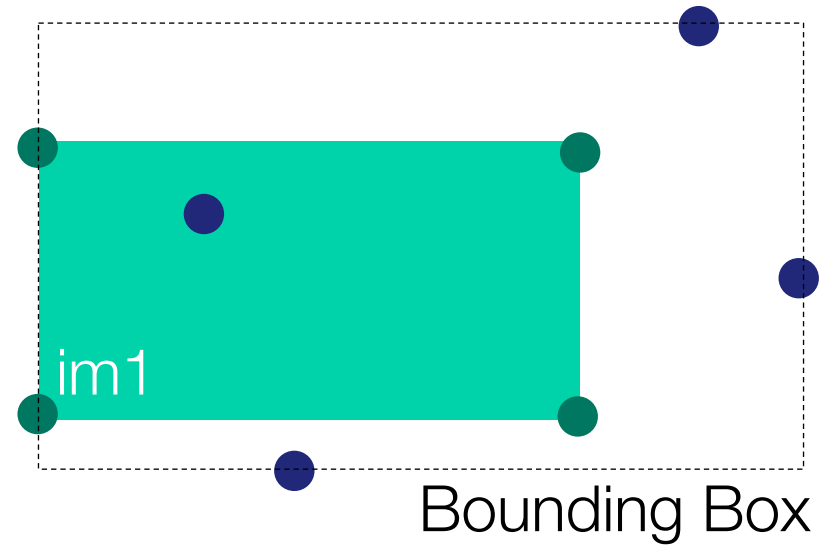


5. Blend the images to create a smooth transition
6. Repeat for other images

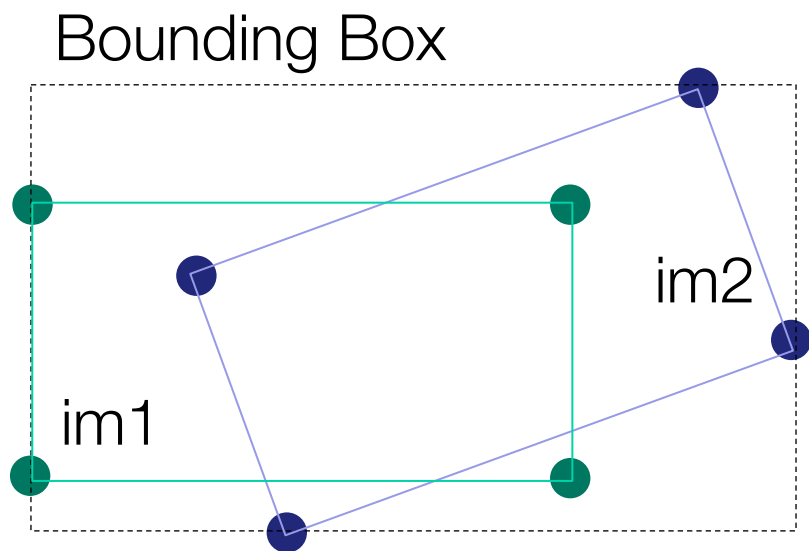
# Transform between images



# Building a Mosaic



# Filling the Bounding Box



Determine coordinates of bounding box in im1 frame

Get samples from im1, possibly with interpolation

Apply homography to mosaic coordinates to go to im2 frame

Get samples from im2, possibly with interpolation

Overlay samples from im1 and im2

Can use `meshgrid` Matlab function to create mosaic coordinates

Can use `interp2` Matlab function to get sub-pixel values

Watch out for NaN

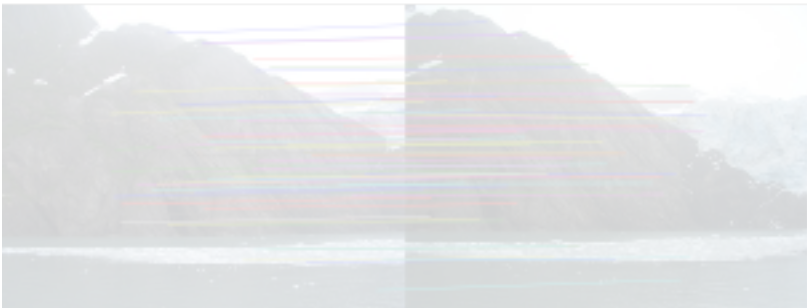
# Homography Example: 3D Pavement Drawings



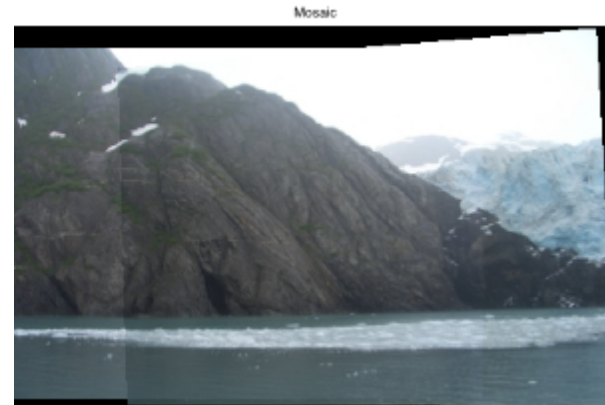
<http://www.julianbeever.net/>

# Panoramic Imaging Pipeline

## Estimating Correspondences



## Warping



3. Compute transformation between  $im_2$  and  $im_1$
4. Transform  $im_2$  to overlap with  $im_1$

## Blending



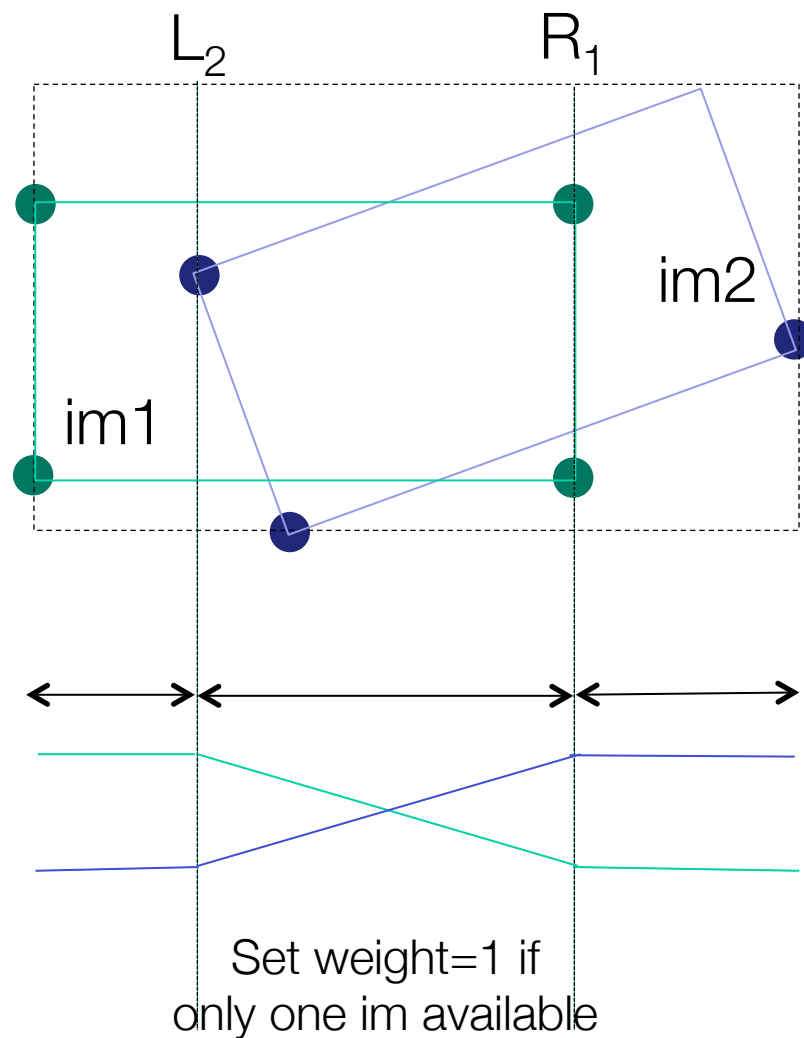
5. Blend the images to create a smooth transition
6. Repeat for other images

# Seams during Stitching



Image: Durand

# Simple Linear Blending



# Potential Artifacts



Smooth blending



Ghosting  
Blurring

Basic reprojection



Image: Durand

# Two-scale Blending

*Idea: Combine smooth and abrupt transition*

Split image into low and high frequency

Smooth blending for low frequencies: corrects exposure differences

Abrupt blending for high frequencies: avoids ghosting, preserves sharpness

# Two-scale Blending Result



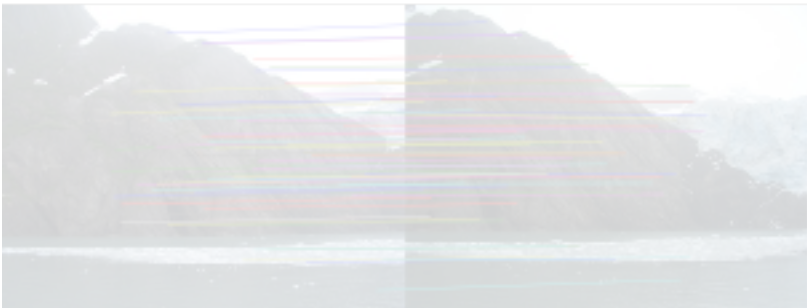
Other methods:

- Pyramid based
- Gradient domain

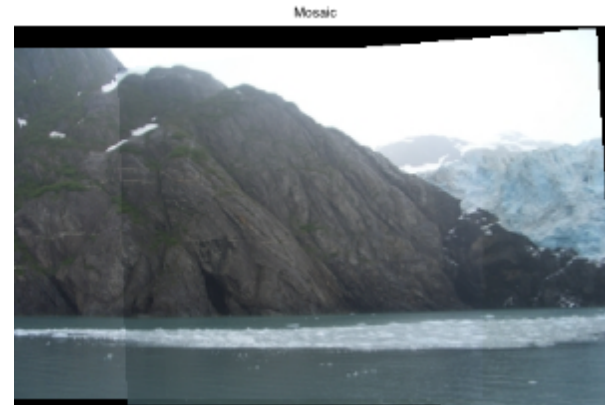
Image: Durand

# Panoramic Imaging Pipeline

## Estimating Correspondences



## Warping



3. Compute transformation between  $im_2$  and  $im_1$
4. Transform  $im_2$  to overlap with  $im_1$

## Blending



5. Blend the images to create a smooth transition
6. Repeat for other images

# Example: Stitching 3 Images



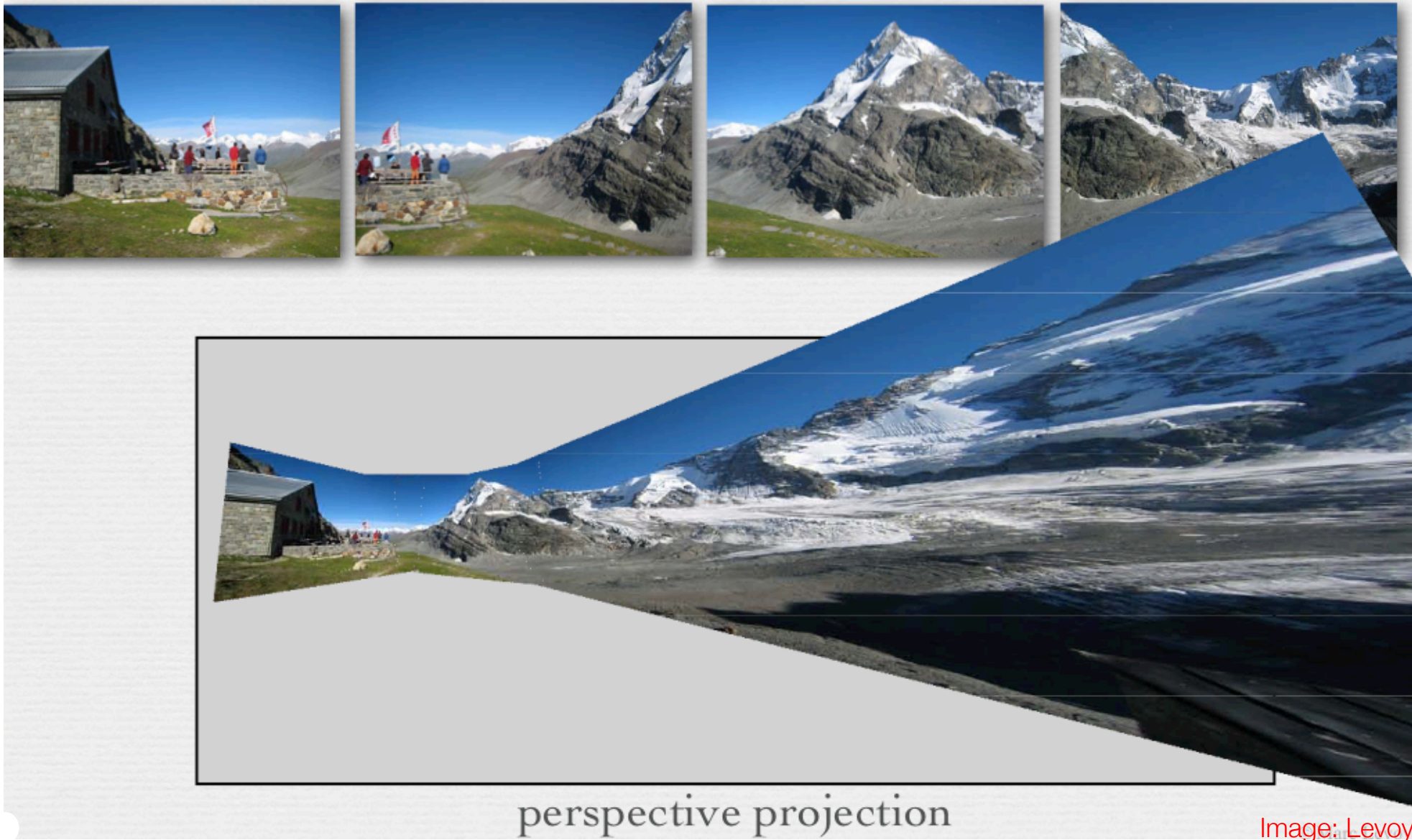
common  
picture  
plane of  
mosaic  
image



perspective projection

Image: Levoy

# Example: Stitching 4 Images



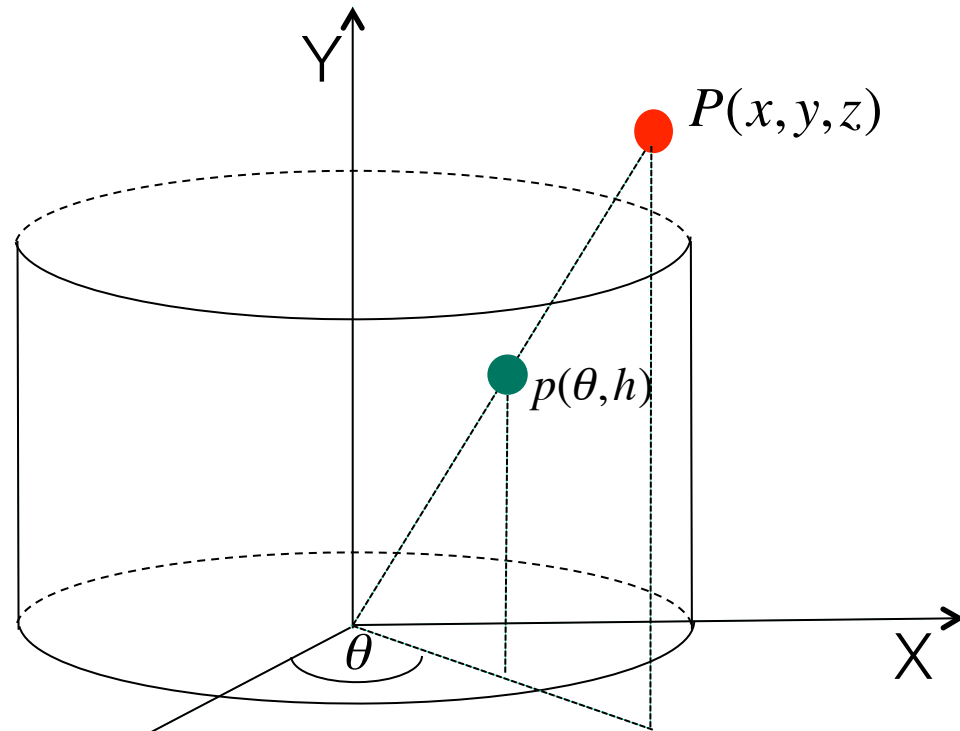
How to avoid this stretching?

Don't restrict image surface to a plane

Image: Levoy

# Cylindrical Panorama

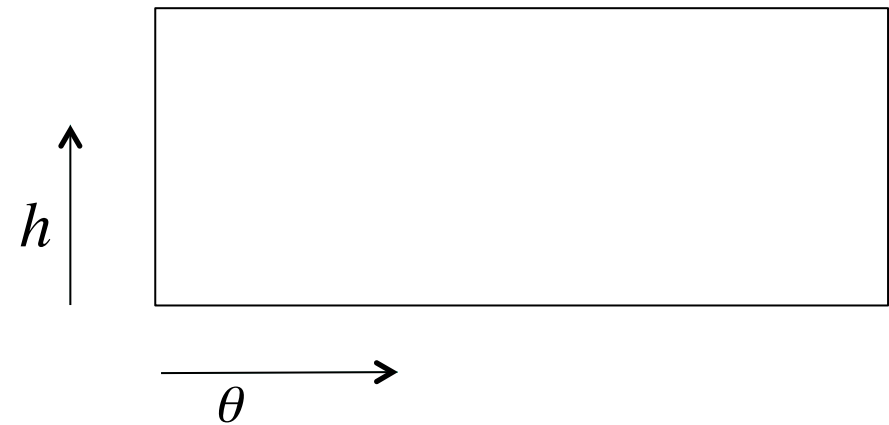
Assume cylinder of unit radius



$$\theta = \tan^{-1}(x/z)$$

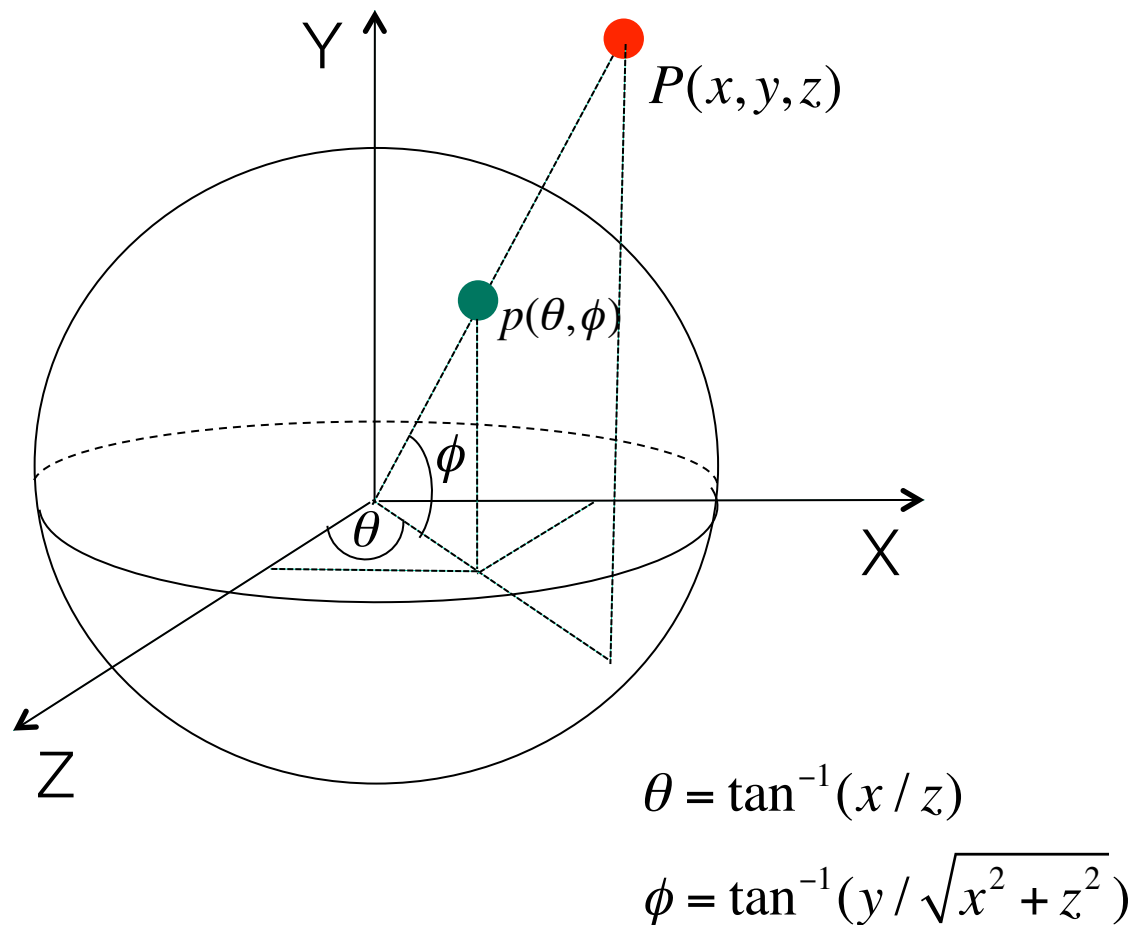
$$h = y / \sqrt{x^2 + z^2}$$

Unwrapped Cylinder

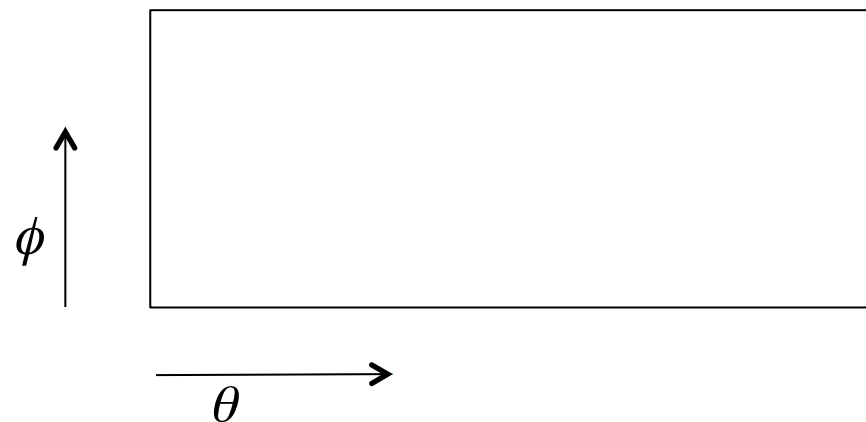


# Spherical Panorama

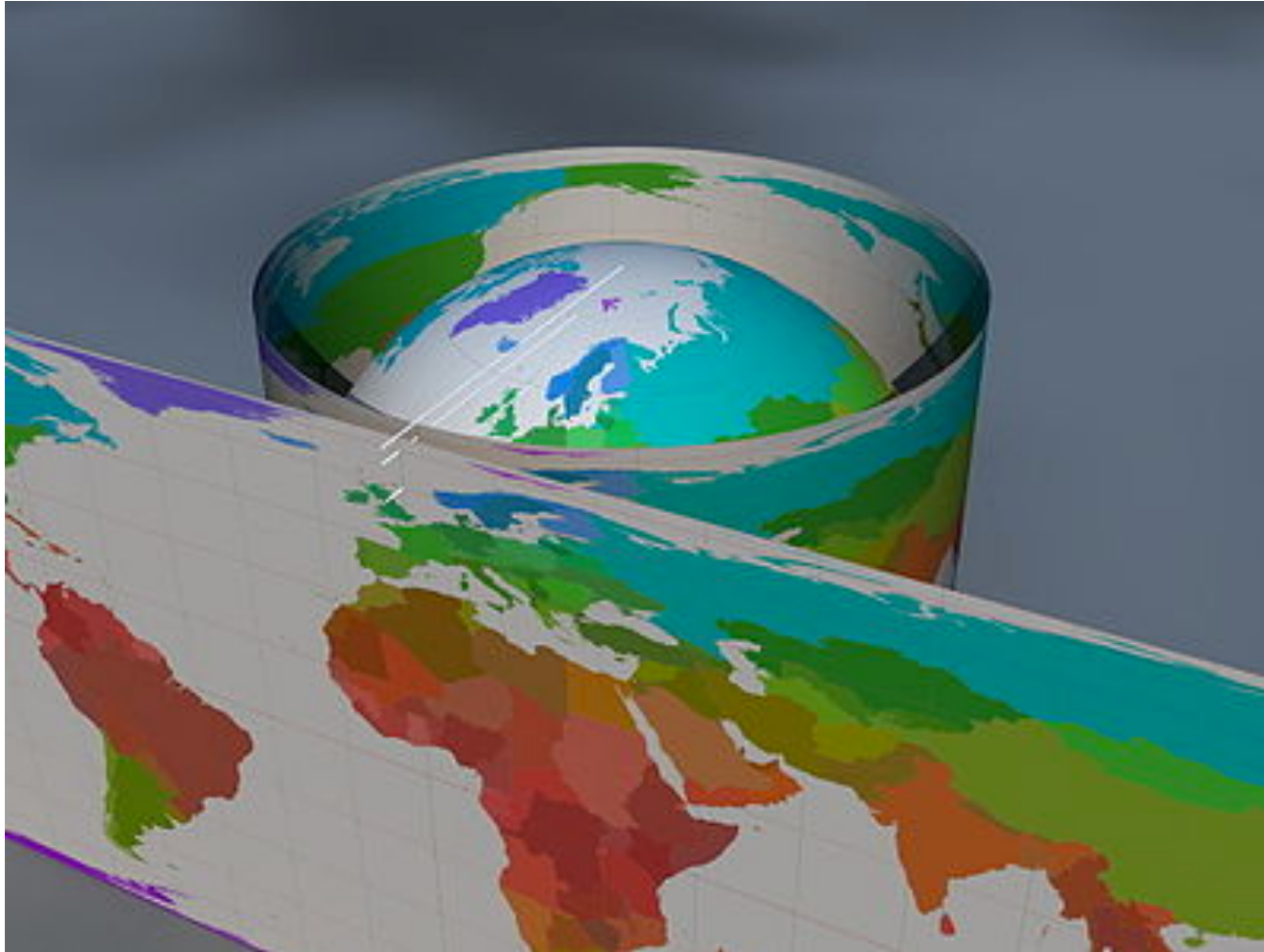
Assume sphere of unit radius



Unwrapped Sphere



# Sphere-to-plane mapping



Popular Mappings:

- Equirectangular
- Equal-area projection
- Mercator
- Cube maps
- ...

# Example of Spherical Panorama



Constructed from 54 photographs  
[Image Alignment and Stitching: A Tutorial, by Szeliski, 2006]

# Demo

`https://graphics.stanford.edu/courses/cs178/applets/projection.html`

`youtube.com/360`