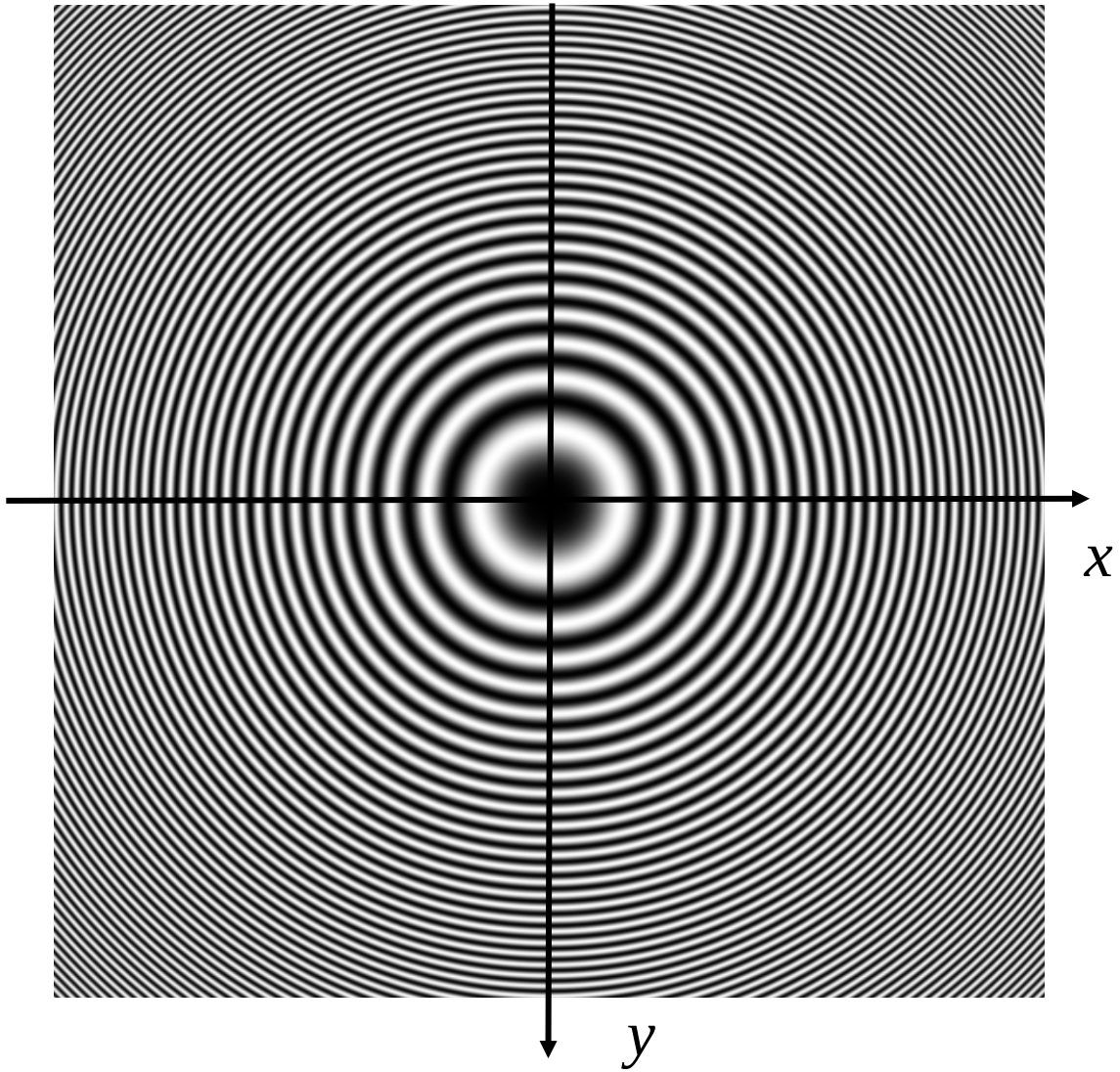
Zoneplate pattern to visualize frequency plane



Equation to generate pattern:

$$s(x,y) = \hat{s}\cos(a_x x^2 + a_y y^2) + s_0$$

Local frequency at (x,y)

$$\frac{\partial}{\partial x}\left(a_x x^2 + a_y y^2\right) = 2a_x x$$

$$\frac{\partial}{\partial y}\left(a_x x^2 + a_y y^2\right) = 2a_y y$$

$$\mathrm{s}[x,y] \underset{\text{sampling}}{\overset{\text{interpolation}}{\rightleftarrows}} s(x,y)$$

Sampling interpretation of 2-d discrete-space Fourier transform

- How is the Fourier transform of $s[m,n]$ related to the Fourier transform of the continuous signal

$$s(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} s[m,n]\, \delta_2(x-m, y-n)$$

continuous

discrete

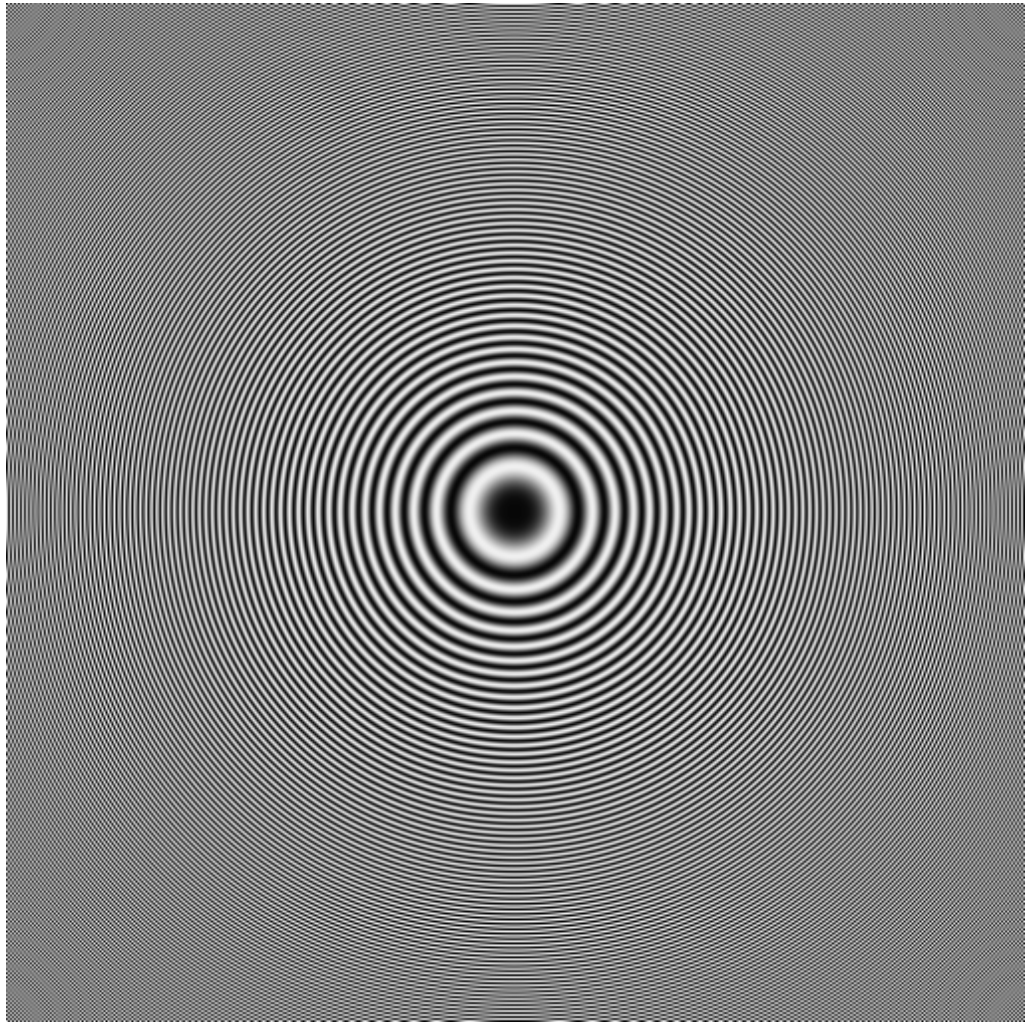
„continuous“ delta function

- Continuous-space 2-d Fourier transform

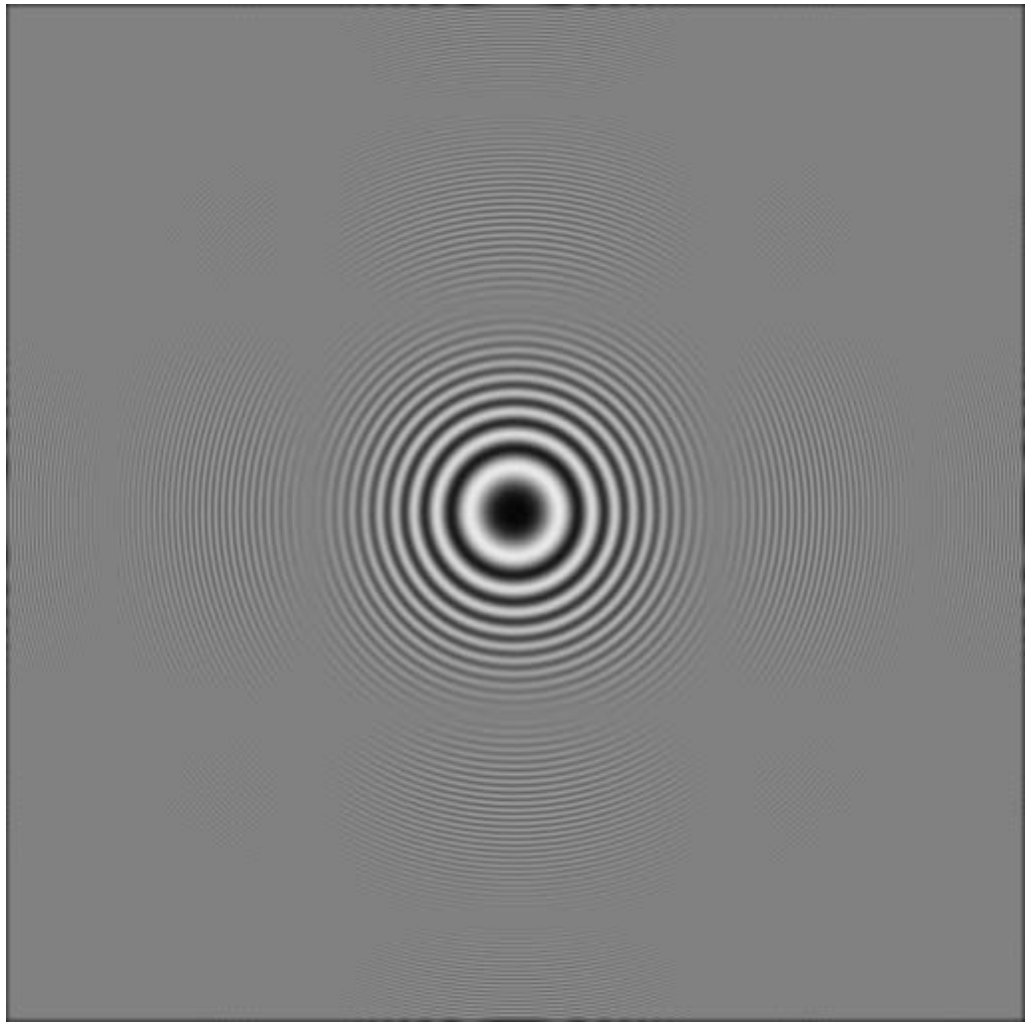
$$\hat{S}(\omega_x,\omega_y) = \int_x \int_y \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} s[m,n]\, \delta_2(x-m, y-n)\, e^{-j\omega_x x - j\omega_y y}\, dx\, dy$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} s[m,n] e^{-j\omega_x m - j\omega_y n} = S\left(e^{j\omega_x}, e^{j\omega_y}\right)$$

Lowpass filtering

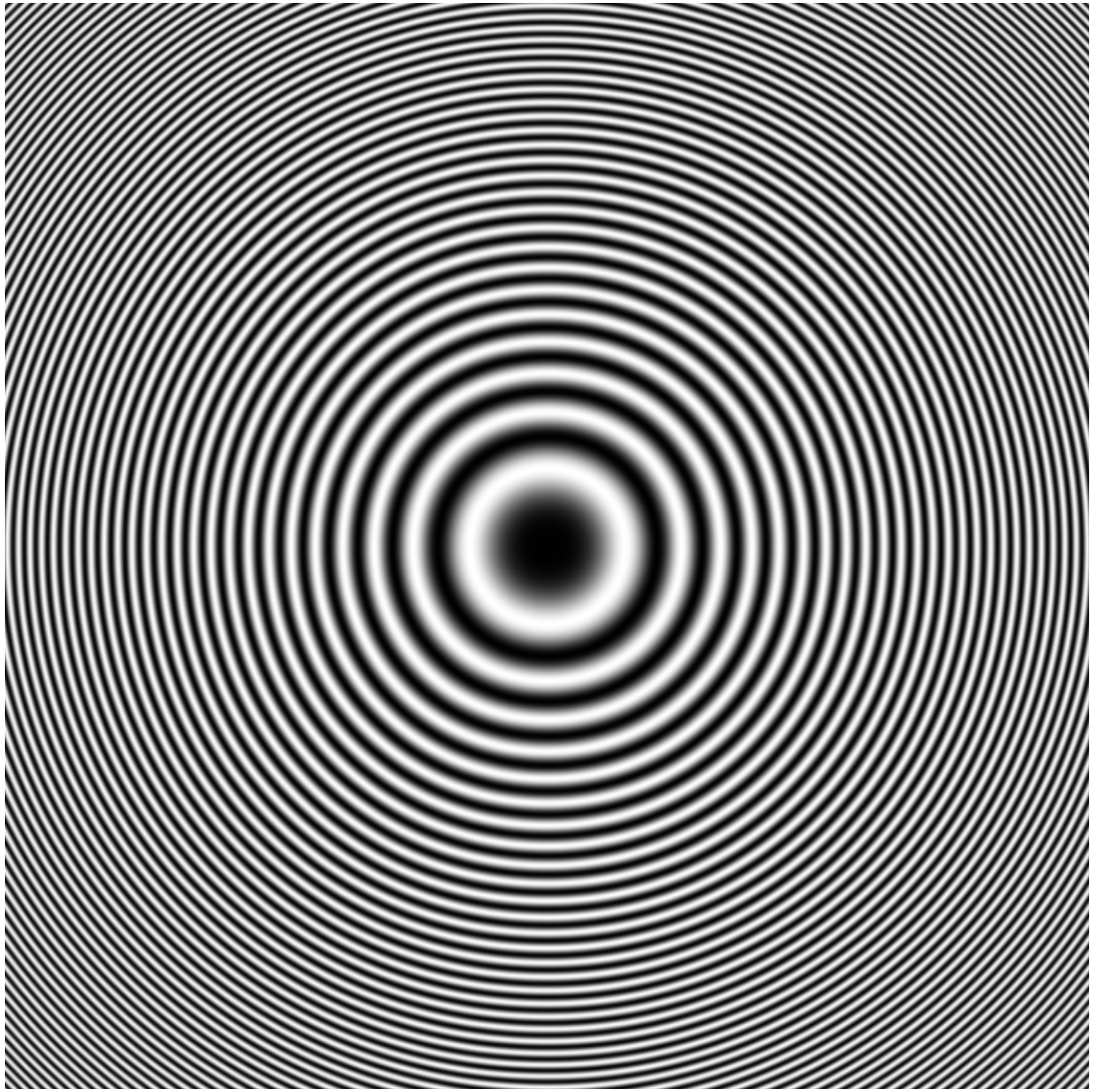


Original *Zoneplate* (512x512)

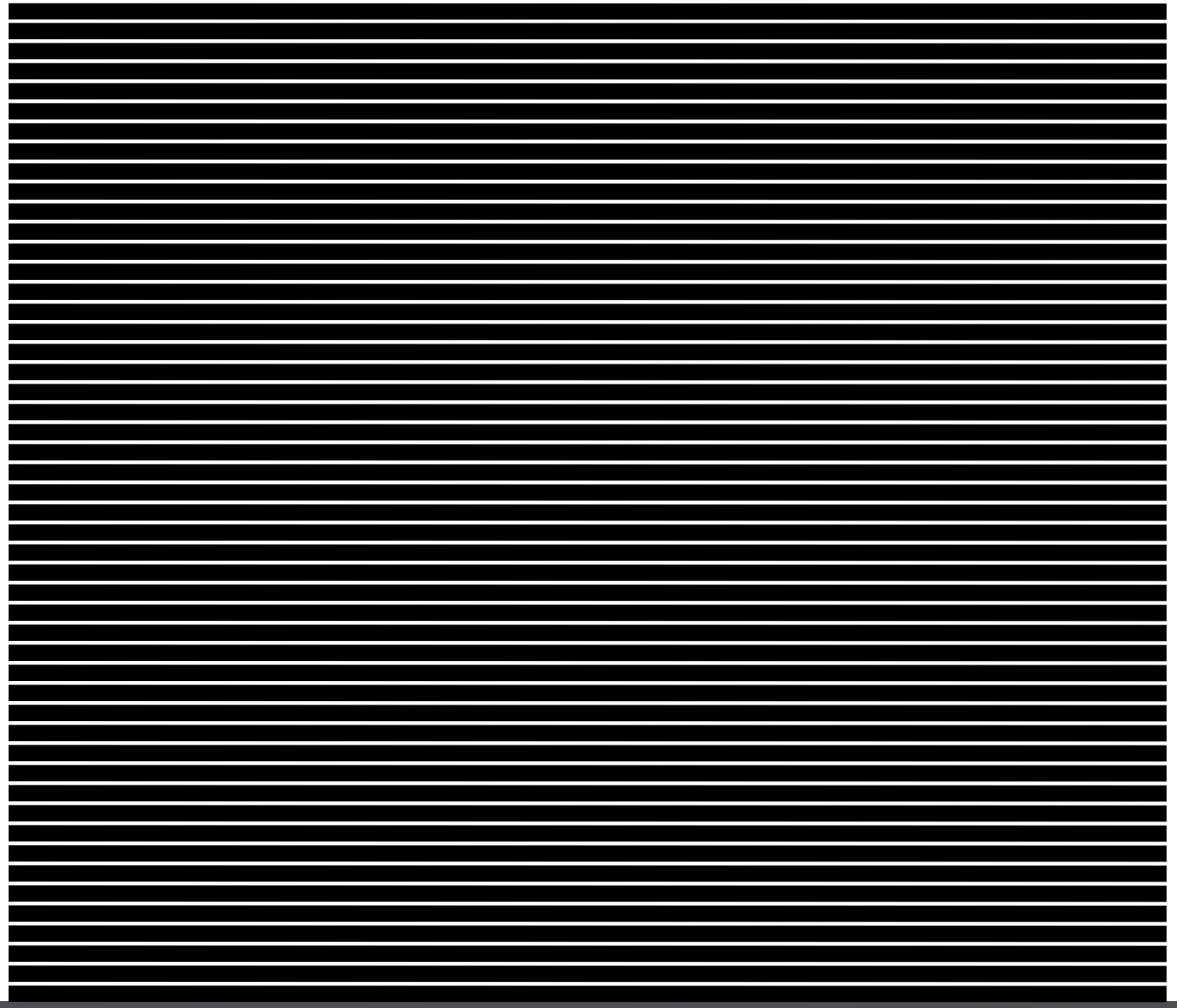


Lowpass filtered with 5x5 box filter

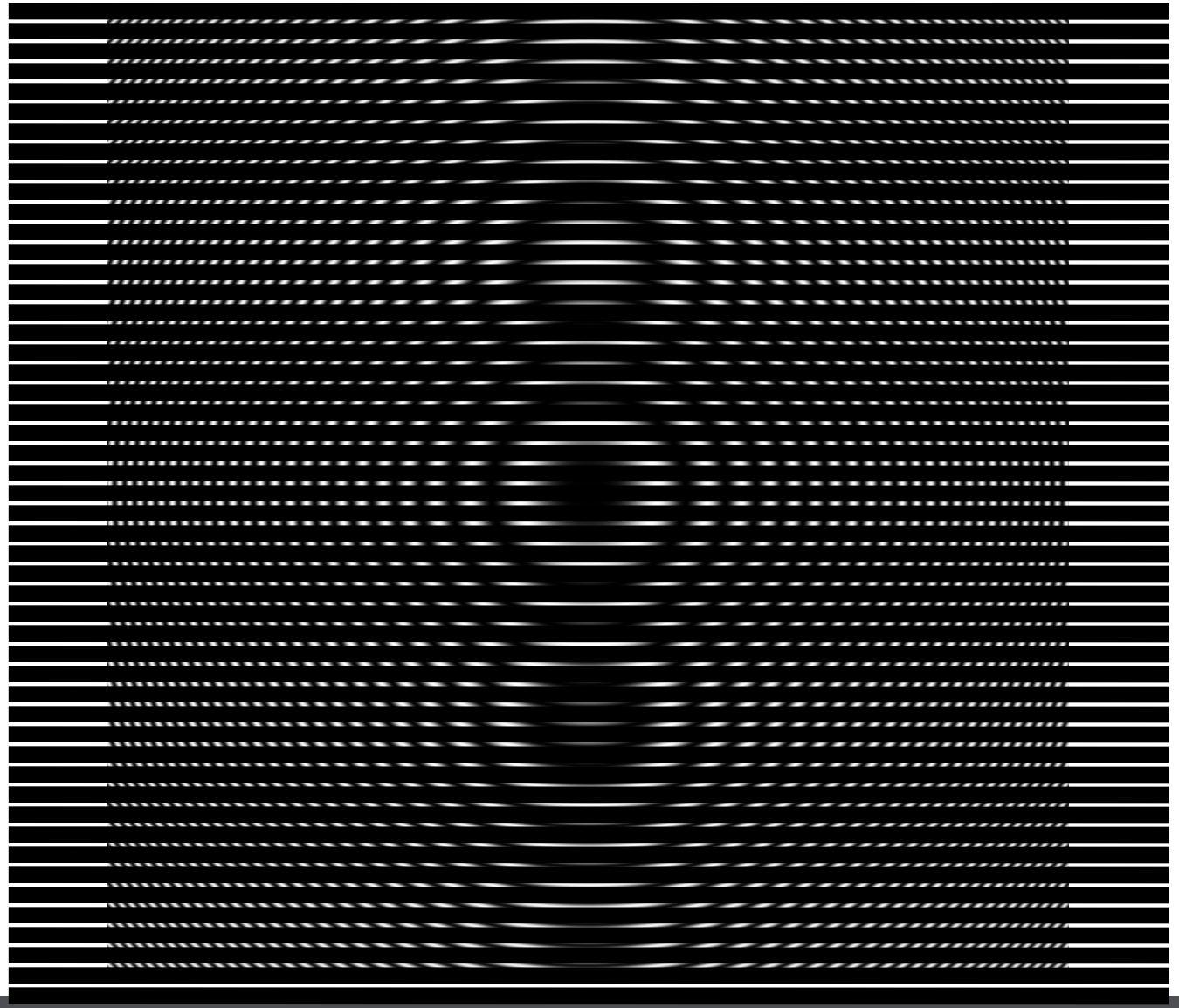
Zoneplate demonstration: 1-d sampling of 2-d signals



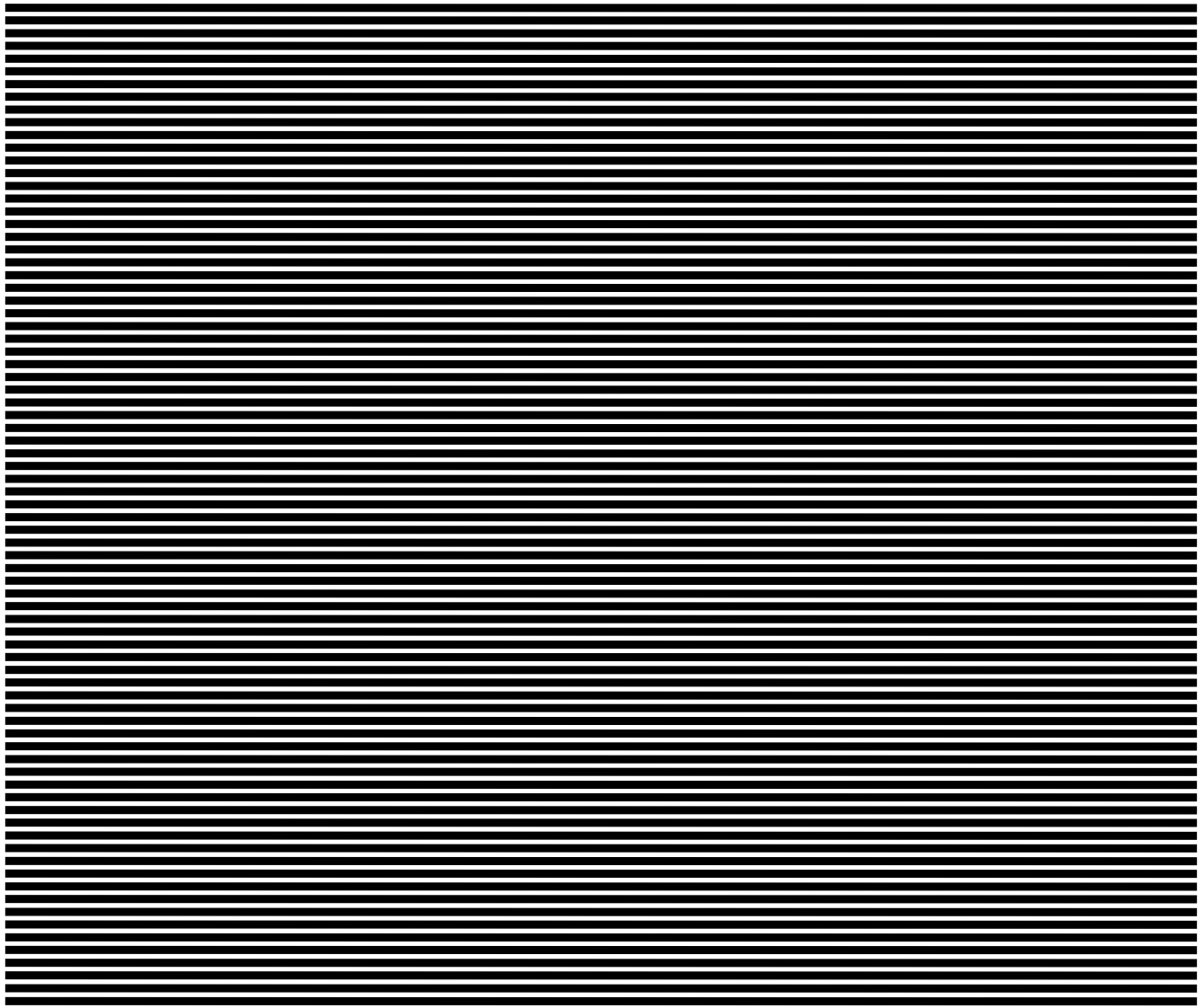
Zoneplate demonstration: 1-d sampling of 2-d signals



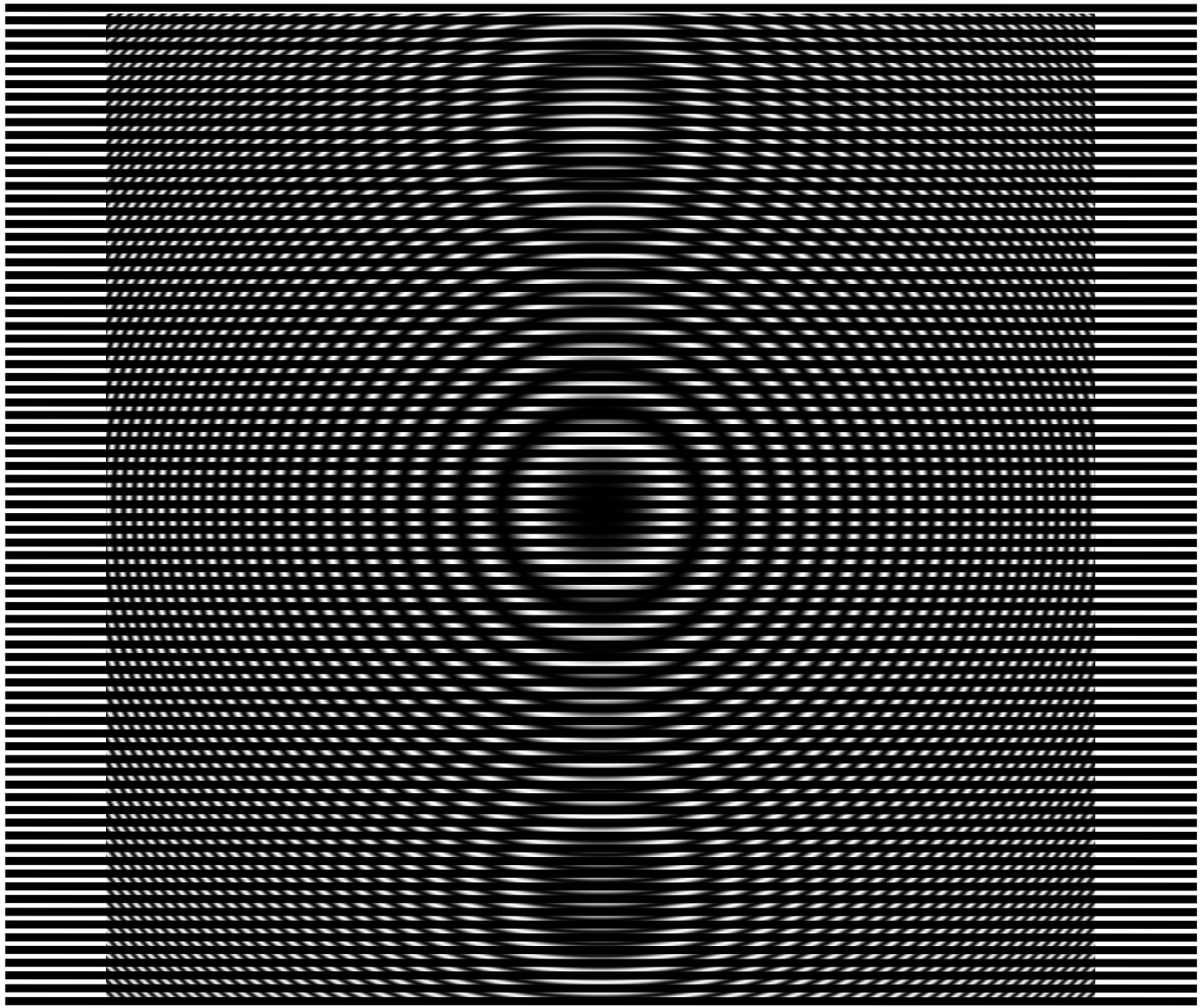
Zoneplate demonstration: 1-d sampling of 2-d signals



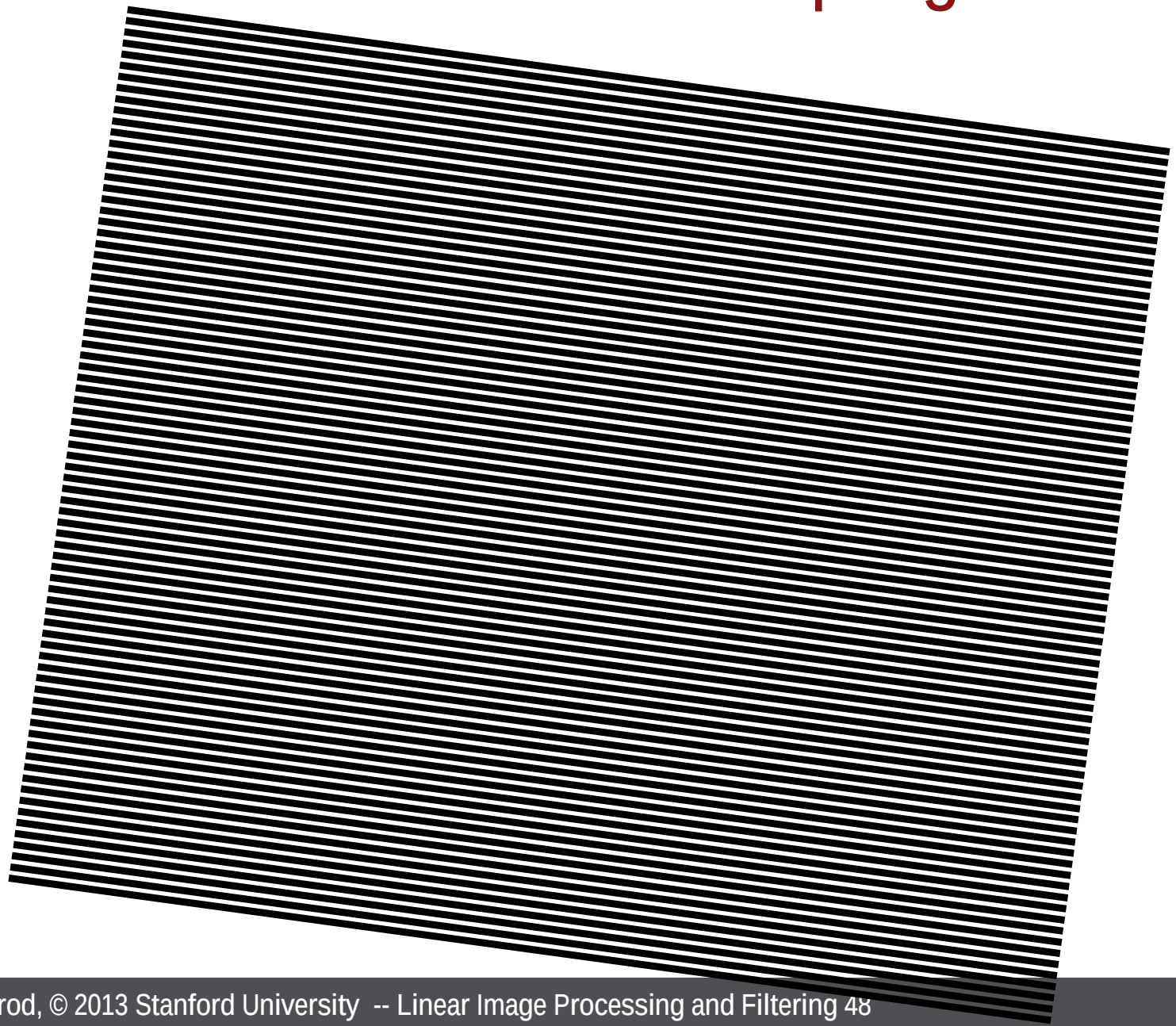
Zoneplate demonstration: 1-d sampling of 2-d signals



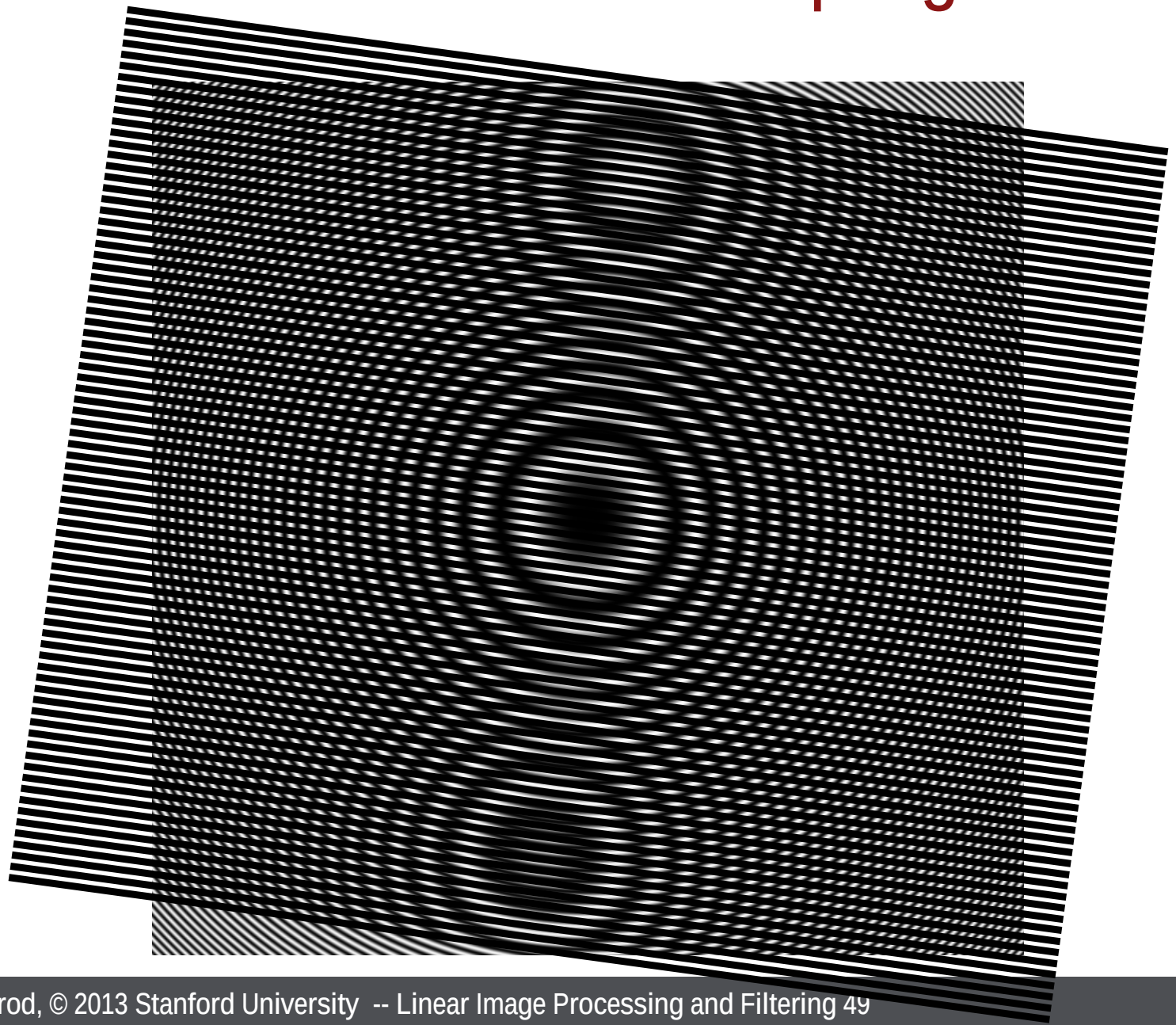
Zoneplate demonstration: 1-d sampling of 2-d signals



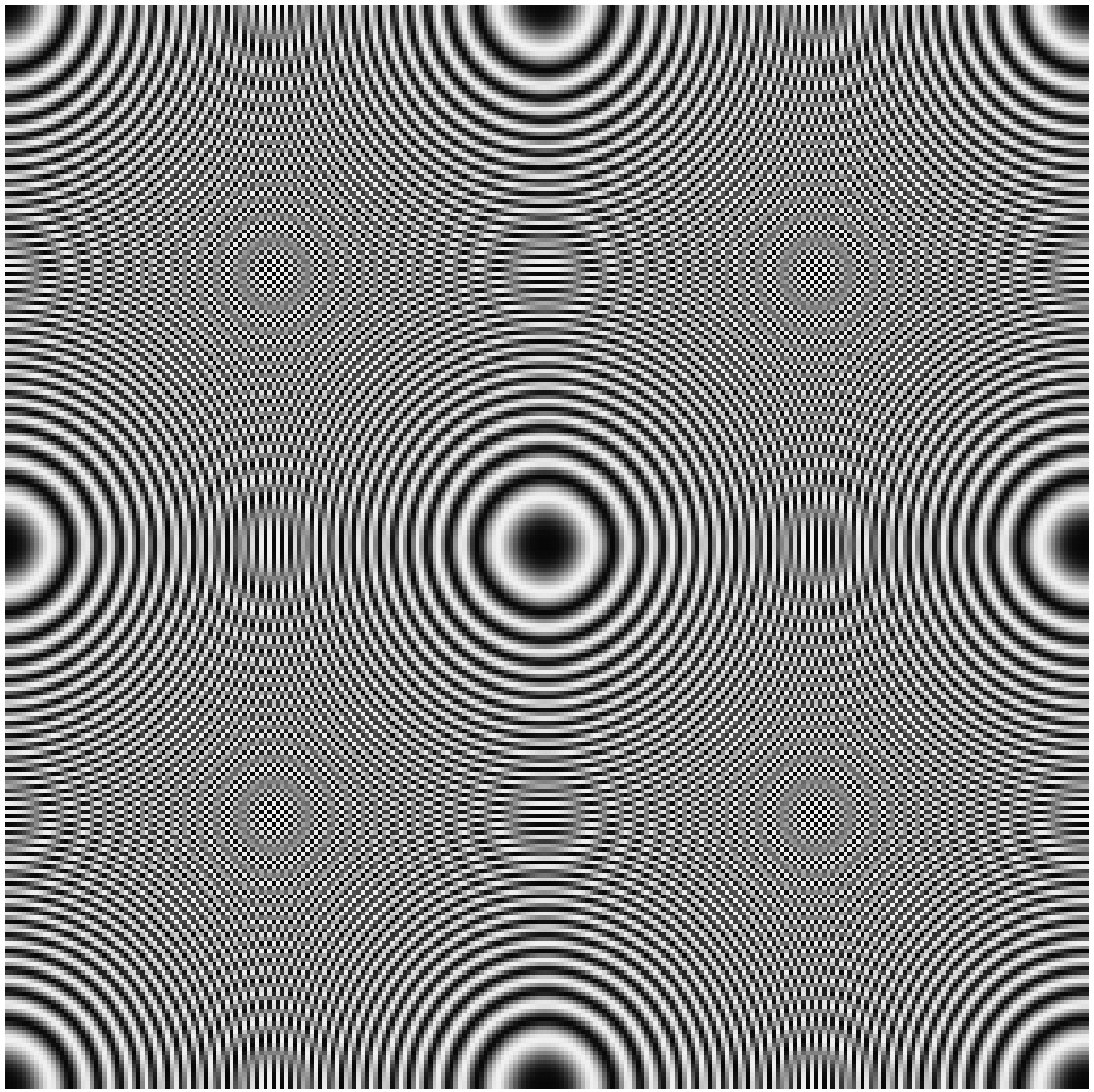
Zoneplate demonstration: 1-d sampling of 2-d signals



Zoneplate demonstration: 1-d sampling of 2-d signals



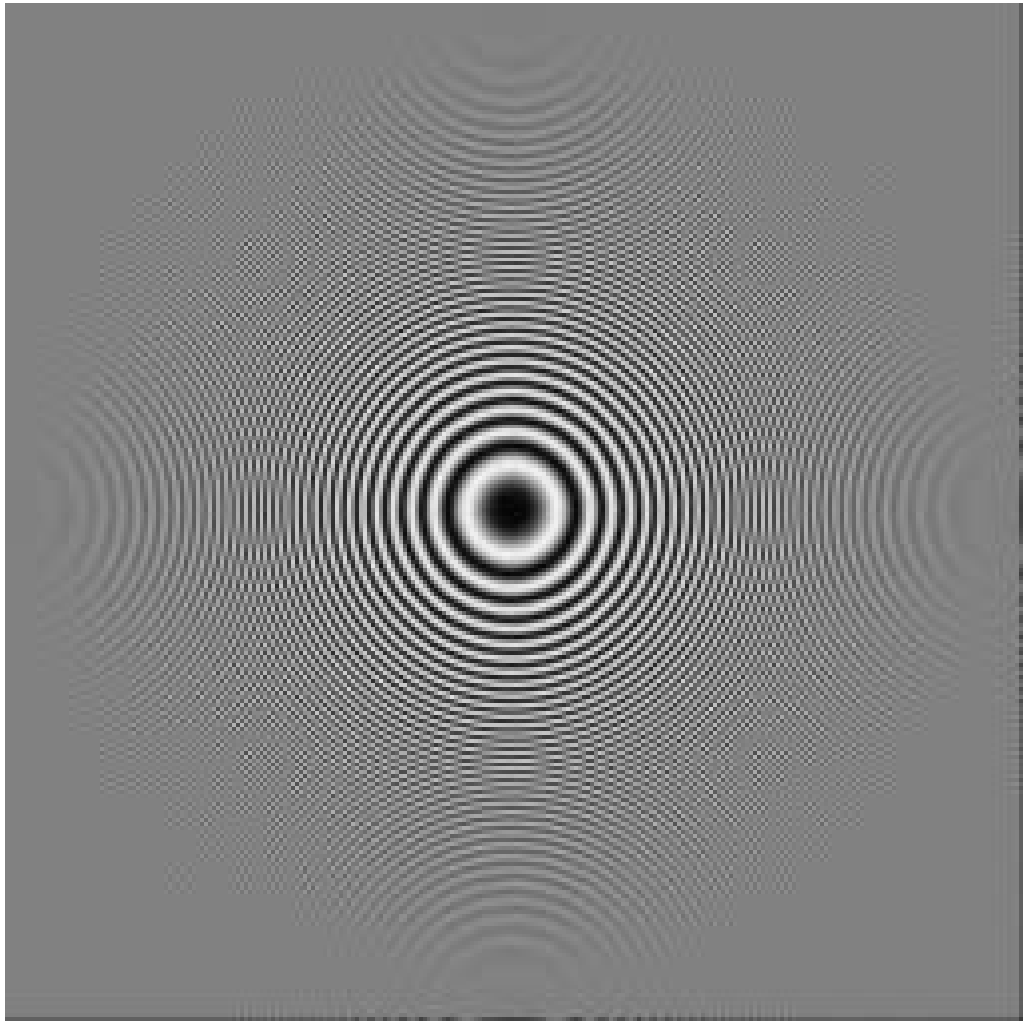
Horizontal/vertical 2:1 subsampling without prefiltering



Horizontal/vertical 2:1 subsampling with prefiltering

2d impulse response

$$\begin{pmatrix} \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{8} \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{16} \end{pmatrix}$$



Frequency response

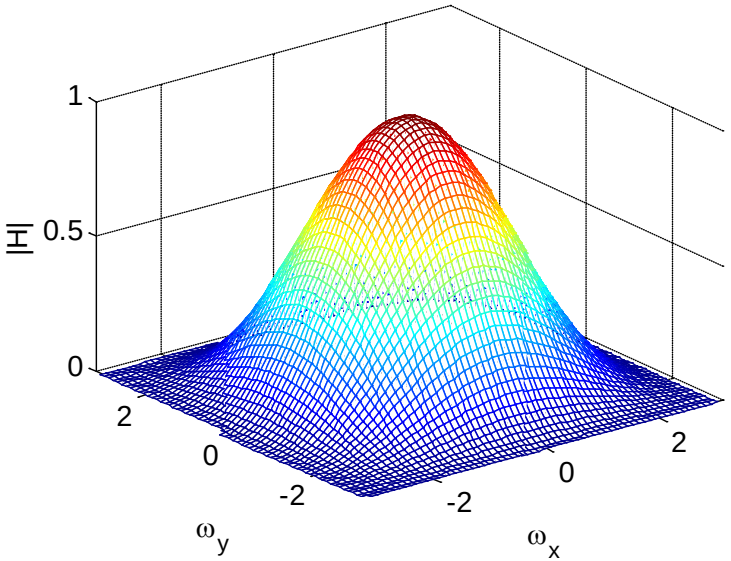
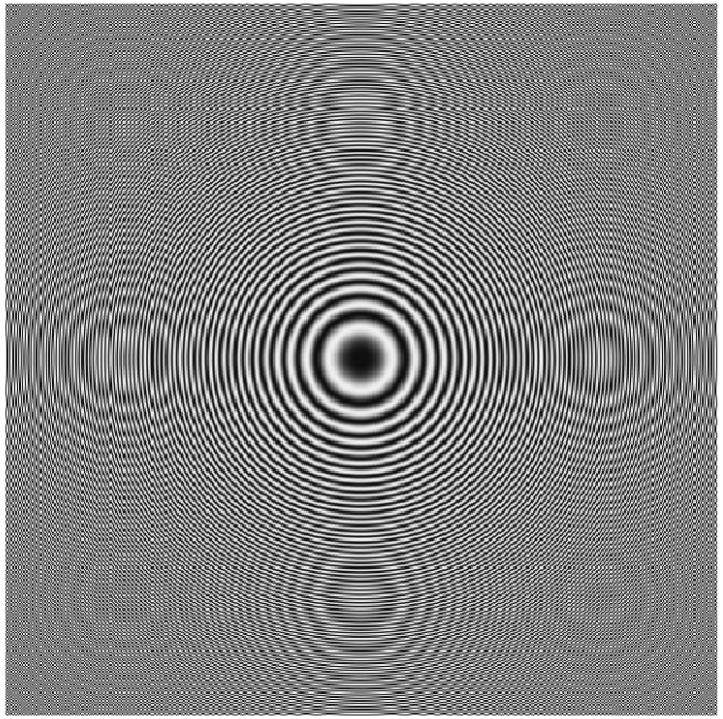
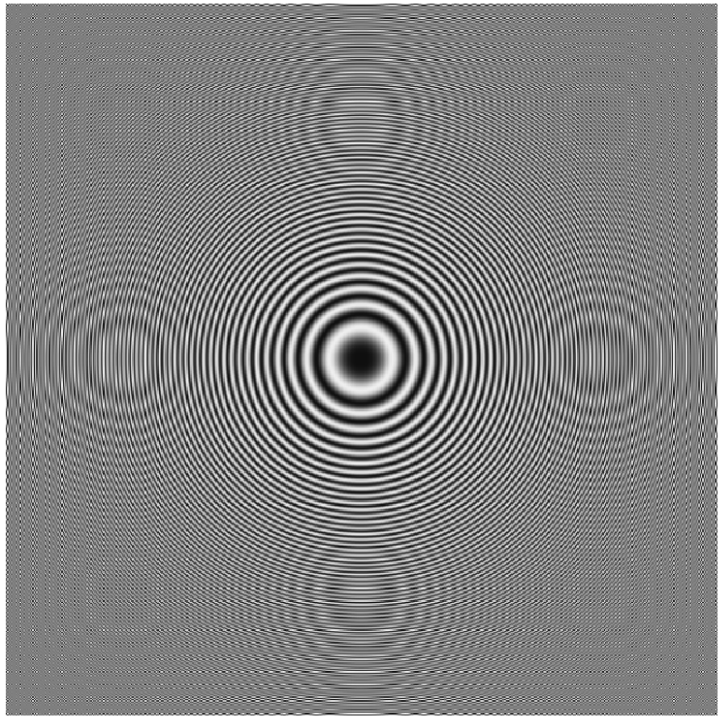


Image magnification (10%)



Nearest neighbor interpolation



Bilinear interpolation