Linear image processing

- Image processing system $S(.)$ is linear, iff superposition principle holds:

$$S\left(\alpha \cdot f\left[x,y\right] + \beta \cdot g\left[x,y\right]\right) = \alpha \cdot S\left(f\left[x,y\right]\right) + \beta \cdot S\left(g\left[x,y\right]\right) \text{ for all } \alpha, \beta \in \mathbb{R}$$

- Any linear image processing system can be written as

$$\vec{g} = H\vec{f}$$

Note: matrix H need not be square.

by sorting pixels into a column vector

$$\vec{f} = \left(f\left[0,0\right] \;\; f\left[1,0\right] \;\; \cdots \;\; f\left[N-1,0\right] \;\; f\left[0,1\right] \;\; \cdots \;\; f\left[N-1,1\right] \;\; \cdots \;\; \cdots \;\; f\left[0,L-1\right] \;\; \cdots \;\; f\left[N-1,L-1\right] \right)^T$$

Impulse response

- Another way to represent any linear image processing scheme

$$g[\alpha,\beta]=\sum_{x=0}^{N-1}\sum_{y=0}^{L-1} f[x,y]\cdot h[x,\alpha,y,\beta]$$

impulse response

- Input: unit impulse at pixel $[a,b]$

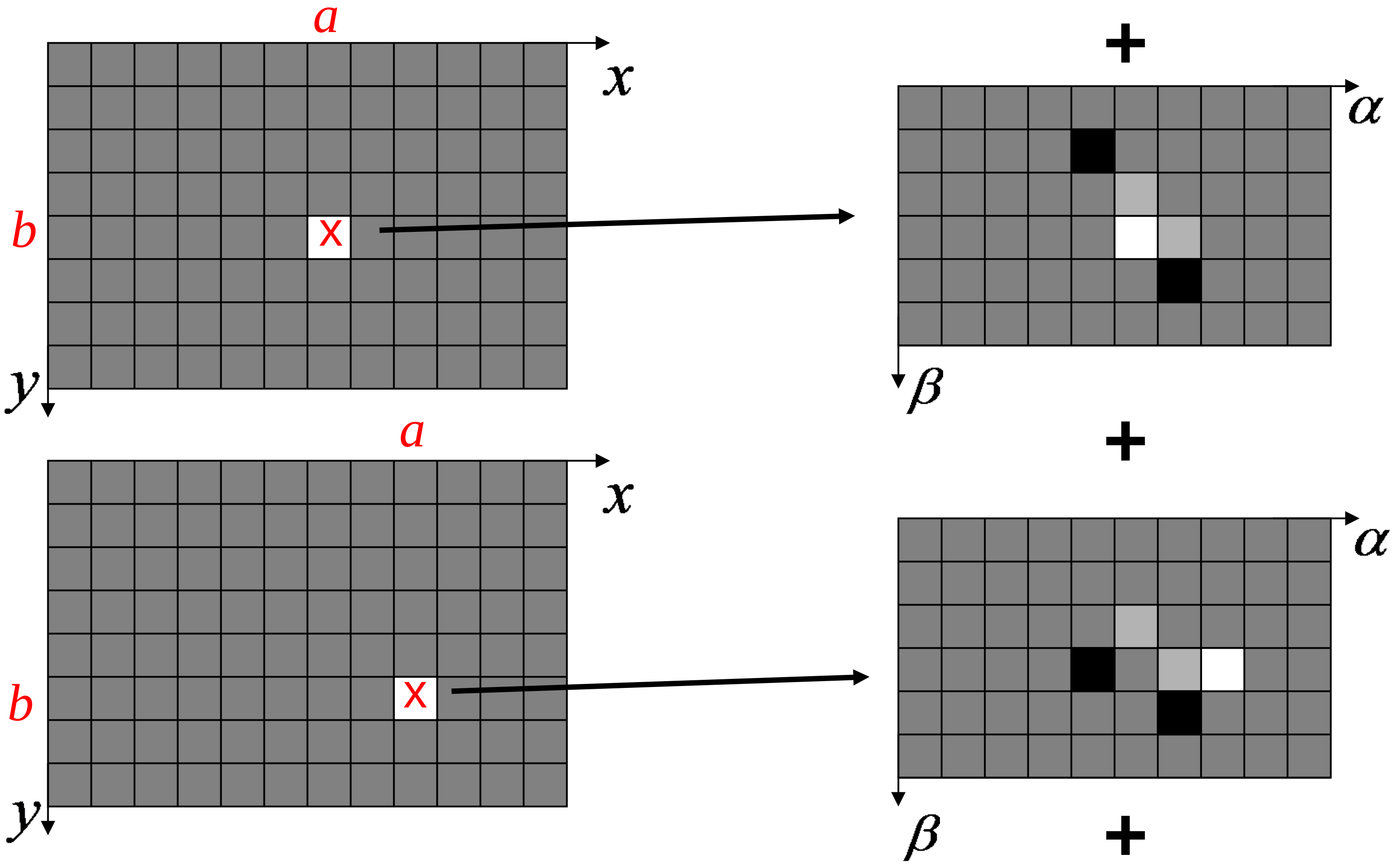
$$f[x,y]=\delta[x-a,y-b]=\begin{cases}1 & x=a\wedge y=b\\ 0 & else\end{cases}$$

- Output: impulse response

$$g[\alpha,\beta]=\sum_{x=0}^{N-1}\sum_{y=0}^{L-1}\delta[x-a,y-b]\cdot h[x,\alpha,y,\beta]=h[a,\alpha,b,\beta]$$

Interpretation #1: superposition of impulse responses

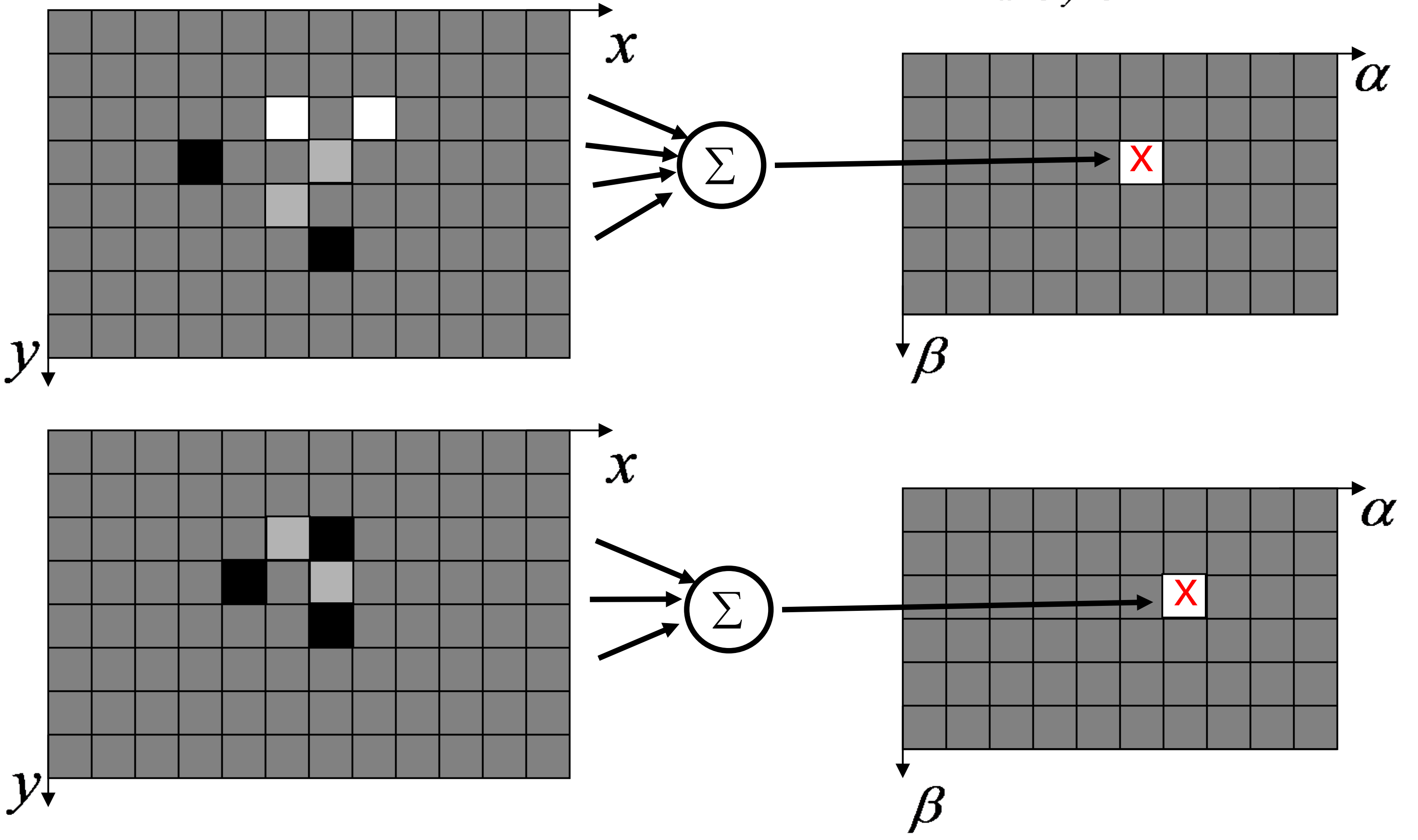
$$\delta[x-a, y-b]\, f[a,b] \qquad g[\alpha,\beta] = h[a,\alpha,b,\beta]\, f[a,b]$$



Interpretation #2: linear combination of input values

$$f[x,y]\cdot h[x,\alpha,y,\beta]$$

$$g[\alpha,\beta]=\sum_{x=0}^{N-1}\sum_{y=0}^{L-1} f[x,y]\cdot h[x,\alpha,y,\beta]$$



Relationship of H and $h[x,\alpha,y,\beta]$