

Set-theoretic interpretation

- Set of object pixels

$$F \equiv \left\{ (x, y) : f(x, y) = 1 \right\}$$

*Continuous (x,y).
Works for discrete [x,y]
in the same way.*

- Background: complement of foreground set

$$F^c \equiv \left\{ (x, y) : f(x, y) = 0 \right\}$$

- Dilation is Minkowski set addition

$$G = F \oplus \Pi_{xy}$$

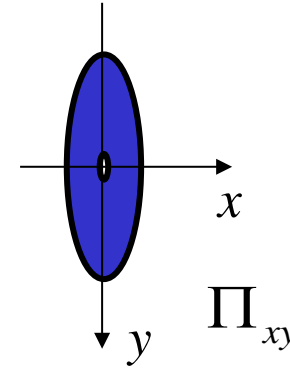
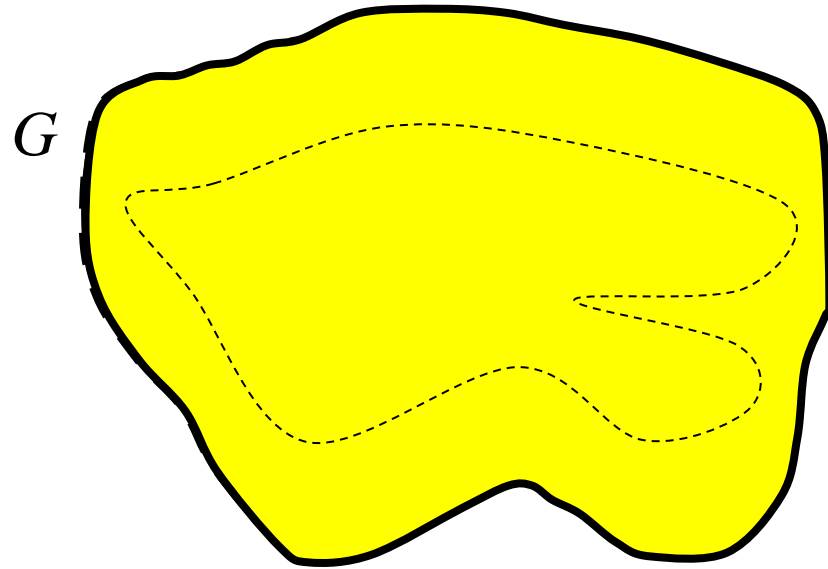
Commutative and associative!

$$= \left\{ (x + p_x, y + p_y) : (x, y) \in F, (p_x, p_y) \in \Pi_{xy} \right\}$$

$$= \bigcup_{(p_x, p_y) \in \Pi_{xy}} F^{+(p_x, p_y)}$$

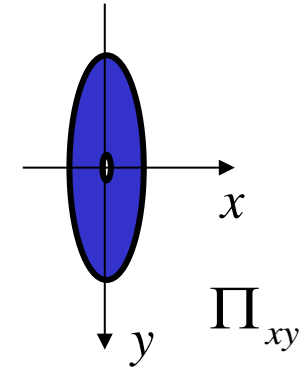
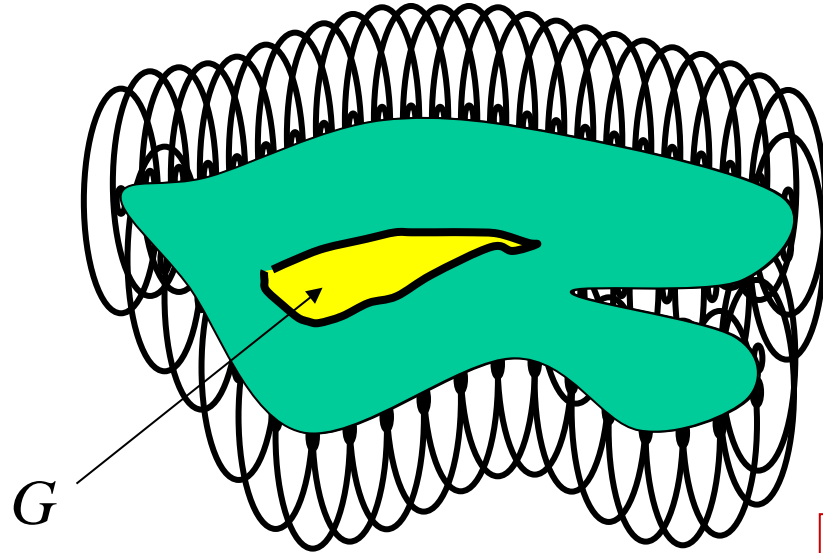
translation of F by vector (p_x, p_y)

Set-theoretic interpretation: dilation



$$\begin{aligned} G &= F \oplus \Pi_{xy} \\ &= \left\{ (x + p_x, y + p_y) : (x, y) \in F, (p_x, p_y) \in \Pi_{xy} \right\} \\ &= \bigcup_{(p_x, p_y) \in \Pi_{xy}} F_{+(p_x, p_y)} \end{aligned}$$

Set-theoretic interpretation: erosion



Not commutative!
Not associative!

Minkowski set subtraction

$$G = \bigcap_{(p_x, p_y) \in \Pi_{xy}} F_{+(p_x, p_y)} = F \ominus \Pi_{-xy}$$

Reversed structuring element