Beyond flat morphological operators

- General dilation operator

$$g[x,y]=\sup_{\alpha,\beta}\left\{f[x-\alpha,y-\beta]+w[\alpha,\beta]\right\}=\sup_{\alpha,\beta}\left\{w[x-\alpha,y-\beta]+f[\alpha,\beta]\right\}$$

- Like linear convolution, with sup replacing summation, addition replacing multiplication
- Dilation with “unit impulse”

$$d[\alpha,\beta]=\begin{cases}0 & \alpha=\beta=0\\ -\infty & \text{else}\end{cases}$$

does not change input signal:

$$f[x,y]=\sup_{\alpha,\beta}\left\{f[x-\alpha,y-\beta]+d[\alpha,\beta]\right\}$$

Flat dilation as a special case

- Find $w[\alpha,\beta]$ such that

$$f[x,y] = \sup_{\alpha,\beta}\left\{ f[x-\alpha, y-\beta] + w[\alpha,\beta] \right\} = dilate(f, W)$$

- Answer:

$$w[\alpha,\beta] = \begin{cases} 0 & [\alpha,\beta] \in \Pi_{xy} \\ -\infty & \text{else} \end{cases}$$

- Hence, write in general

$$g[x,y] = \sup_{\alpha,\beta}\left\{ f[x-\alpha, y-\beta] + w[\alpha,\beta] \right\}$$
$$= dilate(f, w) = dilate(w, f)$$

General erosion for gray-level images

- General erosion operator

$$g[x,y] = \inf_{\alpha,\beta}\left\{f[x-\alpha, y-\beta] - w[\alpha,\beta]\right\} = erode(f,w)$$

- Dual of dilation

$$g[x,y] = \inf_{\alpha,\beta}\left\{f[x-\alpha, y-\beta] - w[\alpha,\beta]\right\}$$

$$= -\sup_{\alpha,\beta}\left\{-f[x-\alpha, y-\beta] + w[\alpha,\beta]\right\} = -dilate(-f,w)$$

- Flat erosion contained as a special case