Histogram equalization

Idea:

Find a non-linear transformation

$$g = T(f)$$

that is applied to each pixel of the input image $f[x,y]$, such that a uniform distribution of gray levels results for the output image $g[x,y]$.

Histogram equalization

Analyse ideal, continuous case first ...

Assume

- Normalized input values $0 \leq f \leq 1$ and output values $0 \leq g \leq 1$
- $T(f)$ is differentiable, increasing, and invertible, i.e., there exists

$$f = T^{-1}(g) \qquad 0 \leq g \leq 1$$

Goal: pdf $p_g(g) = 1$ over the entire range $0 \leq g \leq 1$

Histogram equalization for continuous case

- From basic probability theory

$$p_f(f) \xrightarrow{f} \boxed{T(f)} \xrightarrow{g} \qquad p_g(g) = \left[p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)}$$

- Consider the transformation function

$$g = T(f) = \int_0^f p_f(\alpha)\, d\alpha \qquad 0 \le f \le 1$$

- Then . . .

$$\frac{dg}{df} = p_f(f)$$

$$p_g(g) = \left[p_f(f) \frac{df}{dg} \right]_{f=T^{-1}(g)} = \left[p_f(f) \frac{1}{p_f(f)} \right]_{f=T^{-1}(g)} = 1 \qquad 0 \le g \le 1$$