Geometric mapping

- Notation:
  - Homogeneous coordinates; reference image $\underline{\mathbf{x}} = \begin{pmatrix} x & y & 1 \end{pmatrix}^T$
  - Inhomogeneous coordinates; target image $\mathbf{x}' = \begin{pmatrix} x' & y' \end{pmatrix}^T$
- Translation

$$\mathbf{x}' = \mathbf{x} + \mathbf{t} \quad \text{or} \quad \mathbf{x}' = \begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix} \underline{\mathbf{x}}$$

- Euclidean transformation (rotation and translation)

$$\mathbf{x}' = \begin{bmatrix} \cos\theta & -\sin\theta & t_x \\ \sin\theta & \cos\theta & t_y \end{bmatrix} \underline{\mathbf{x}}$$

- Scaled rotation (similarity transform)

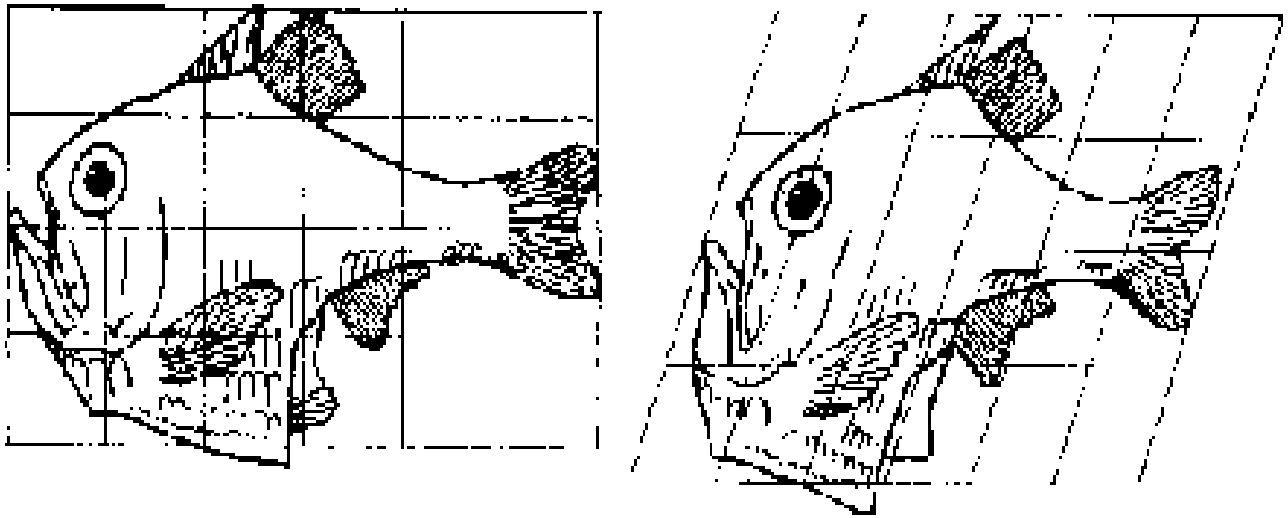
$$\mathbf{x}' = \begin{bmatrix} s\cdot\cos\theta & -s\cdot\sin\theta & t_x \\ s\cdot\sin\theta & s\cdot\cos\theta & t_y \end{bmatrix} \underline{\mathbf{x}}$$

Geometric mapping

- Affine transformation

$$\mathbf{x}' = \begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \end{bmatrix} \underline{\mathbf{x}}$$

- Motion of planar surface in 3d under orthographic projection
- Parallel lines are preserved

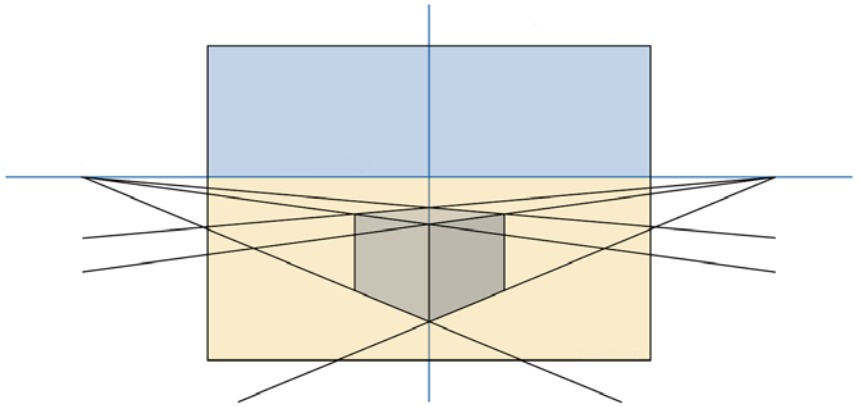


Argyropelecus olfersi. *Sternoptyx diaphana.*

Geometric mapping

- Motion of planar surface in 3d under perspective projection
- Homography

$$\underline{\mathbf{x}}' \sim \begin{pmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{pmatrix} \underline{\mathbf{x}}$$



- Inhomogeneous coordinates (after normalization)

$$x' = \frac{h_{00}x + h_{01}y + h_{02}}{h_{20}x + h_{21}y + h_{22}} \qquad y' = \frac{h_{10}x + h_{11}y + h_{12}}{h_{20}x + h_{21}y + h_{22}}$$

- Straight lines are preserved