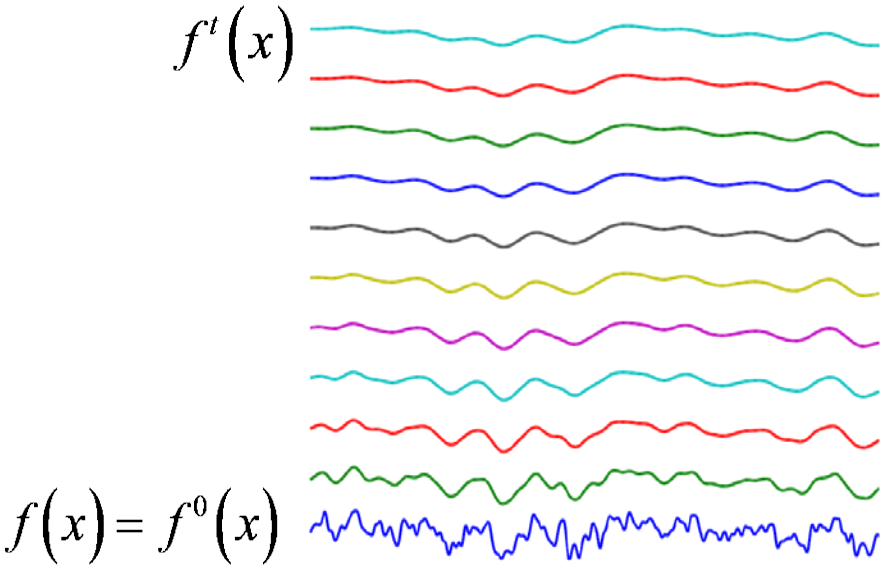
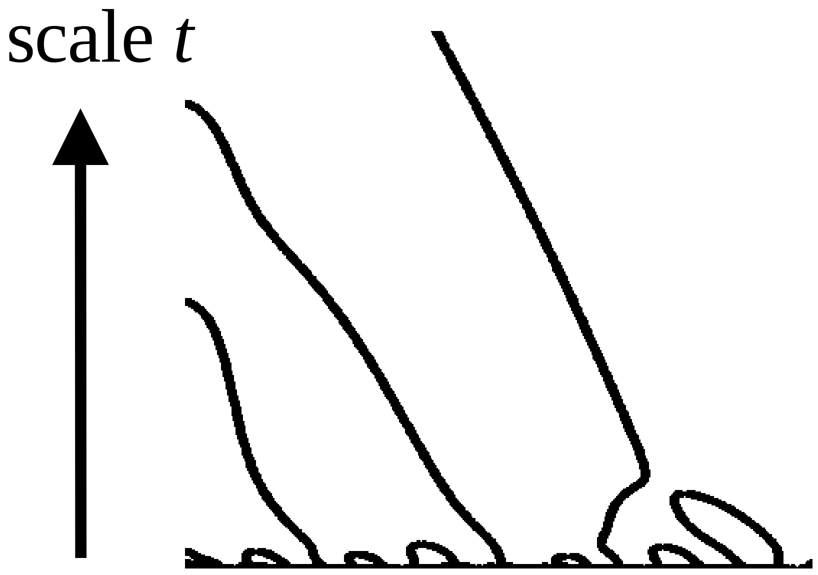
Scale-space representation of a signal

Parametric family of signals $f^t(x)$ where fine-scale information is successively attenuated



Successive smoothing
with a Gaussian filter



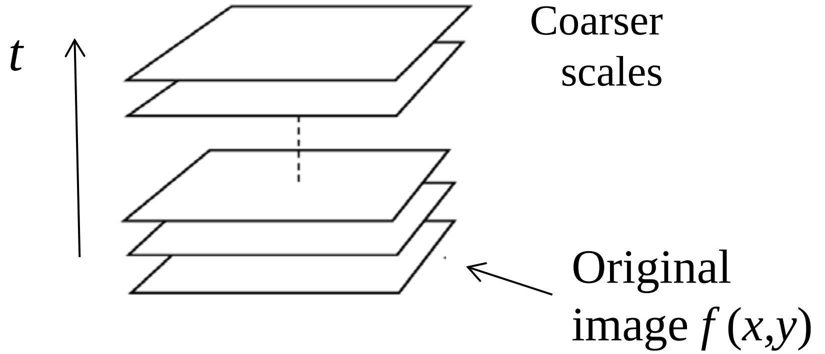
Zero-crossings of 2nd derivative $f^{t''}(x)$
Fewer edges at coarser scales

Scale-space representation of images

- Parametric family of images smoothed by Gaussian filter

$$f^t(x,y) = g^t(x,y) * f(x,y);\ t \geq 0 \quad \text{with} \ \ g^t(x,y) = \frac{1}{2\pi t}\exp\left(-\frac{x^2+y^2}{2t}\right)$$

$$F^t(\omega_x,\omega_y) = G^t(\omega_x,\omega_y)F(\omega_x,\omega_y) \ \ \text{with} \ \ G^t(\omega_x,\omega_y) = \exp\left(-\frac{t}{2}\left(\omega_x^2+\omega_y^2\right)\right)$$



- Shift-invariance

$$f^t(x-\Delta x, y-\Delta y) = g^t(x,y) * f(x-\Delta x, y-\Delta y)$$

- Rotation-invariance

$$f^t(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta) = g^t(x,y) * f(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$$

Scale-space representation of images (cont.)

- Commutative semigroup property

$$f^{t1+t2}(x,y) = g^{t1}(x,y) * f^{t2}(x,y) = g^{t2}(x,y) * f^{t1}(x,y)$$
$$= g^{t1}(x,y) * g^{t2}(x,y) * f(x,y)$$

- Separability

$$g^{t}(x,y) = \frac{1}{2\pi t}\exp\left(-\frac{x^2+y^2}{2t}\right) = \frac{1}{\sqrt{2\pi t}}\exp\left(-\frac{x^2}{2t}\right)\cdot\frac{1}{\sqrt{2\pi t}}\exp\left(-\frac{y^2}{2t}\right)$$
$$G^{t}(\omega_x,\omega_y) = \exp\left(-\frac{t}{2}\left(\omega_x^2+\omega_y^2\right)\right) = \exp\left(-\frac{t}{2}\omega_x^2\right)\exp\left(-\frac{t}{2}\omega_y^2\right)$$