What patterns can be localized most accurately?

- Local displacement sensitivity (assuming continuous $f(x,y)$)

$$S(\Delta x,\Delta y)=\sum_{(x,y)\in window}\left[f(x,y)-f(x+\Delta x,y+\Delta y)\right]^2$$

- Linear approximation for small $\Delta x, \Delta y$

$$f(x+\Delta x,y+\Delta y)\approx f(x,y)+f_x(x,y)\Delta x+f_y(x,y)\Delta y$$

$f_x(x,y)$ – horizontal image gradient
$f_y(x,y)$ – vertical image gradient

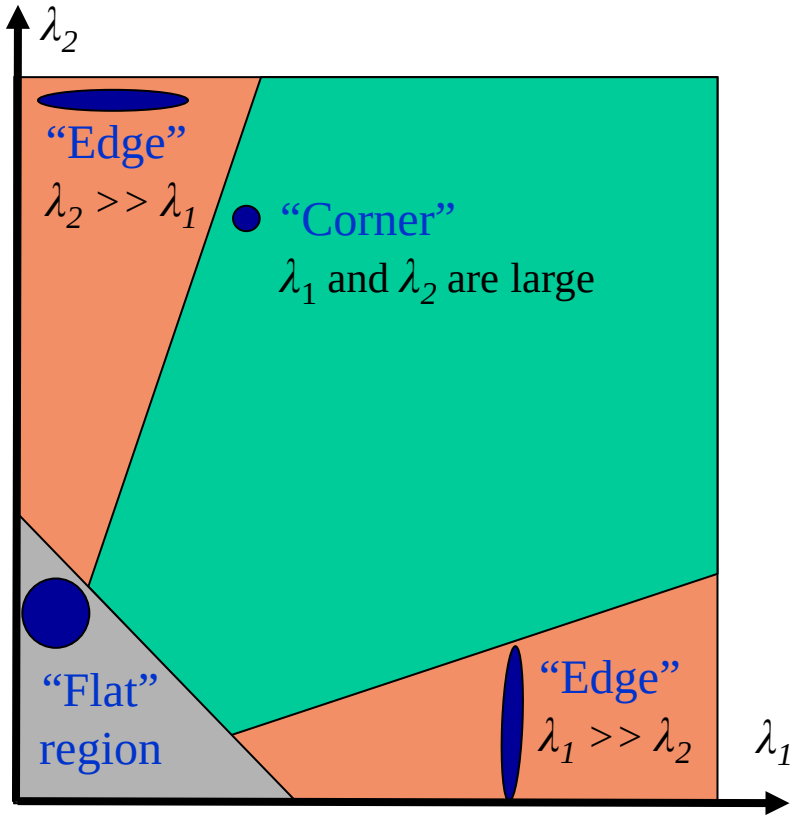
$$S(\Delta x,\Delta y)\approx\sum_{(x,y)\in window}\left[\begin{pmatrix} f_x(x,y) & f_y(x,y)\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}\right]^2$$

$$=\begin{pmatrix}\Delta x & \Delta y\end{pmatrix}\left(\sum_{(x,y)\in window}\begin{bmatrix} f_x^2(x,y) & f_x(x,y)f_y(x,y)\\ f_x(x,y)f_y(x,y) & f_y^2(x,y)\end{bmatrix}\right)\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}$$

$$=\begin{pmatrix}\Delta x & \Delta y\end{pmatrix}\mathbf{M}\begin{pmatrix}\Delta x\\ \Delta y\end{pmatrix}$$

- Iso-sensitivity curves are ellipses

Harris detector



Based on eigenvalues λ_1, λ_2 of "structure matrix" (aka "normal matrix" aka "second-moment matrix")

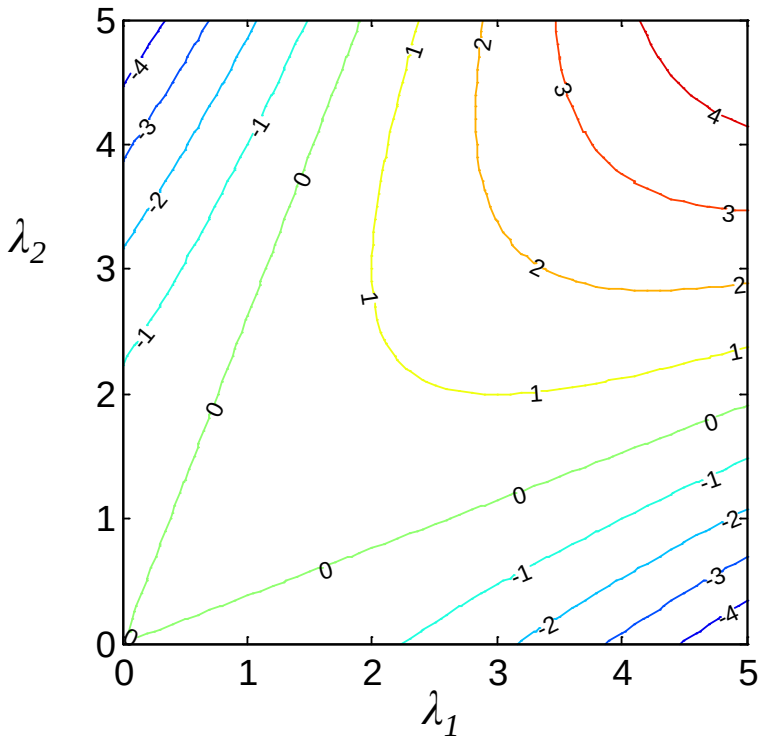
$$\mathbf{M} = \begin{bmatrix} \sum_{[x,y] \in window} f_x^2[x,y] & \sum_{[x,y] \in window} f_x[x,y] f_y[x,y] \\ \sum_{[x,y] \in window} f_x[x,y] f_y[x,y] & \sum_{[x,y] \in window} f_y^2[x,y] \end{bmatrix}$$

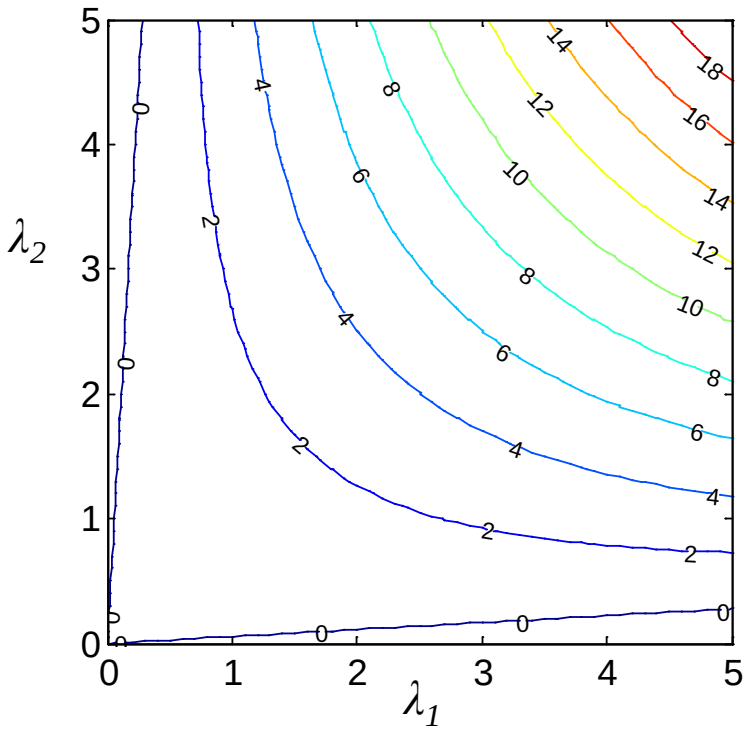
$f_x[x,y]$ – horizontal image gradient
$f_y[x,y]$ – vertical image gradient

Harris cornerness

$$C = \det(\mathbf{M}) - k \cdot \left(trace\left(\mathbf{M}\right)\right)^2 = \lambda_1\lambda_2 - k \cdot \left(\lambda_1 + \lambda_2\right)^2$$

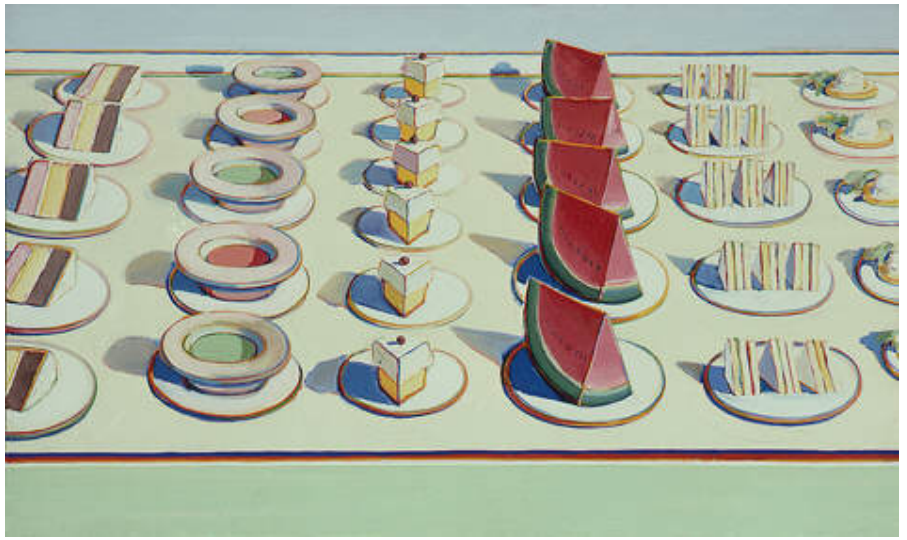
k = 0.2



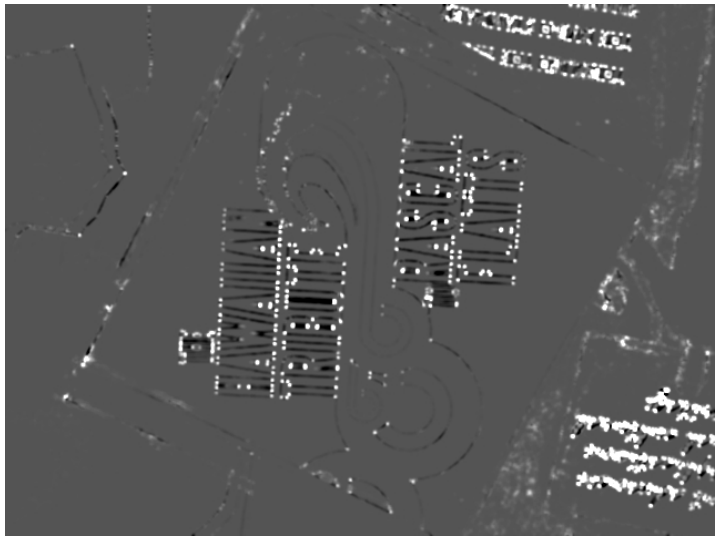


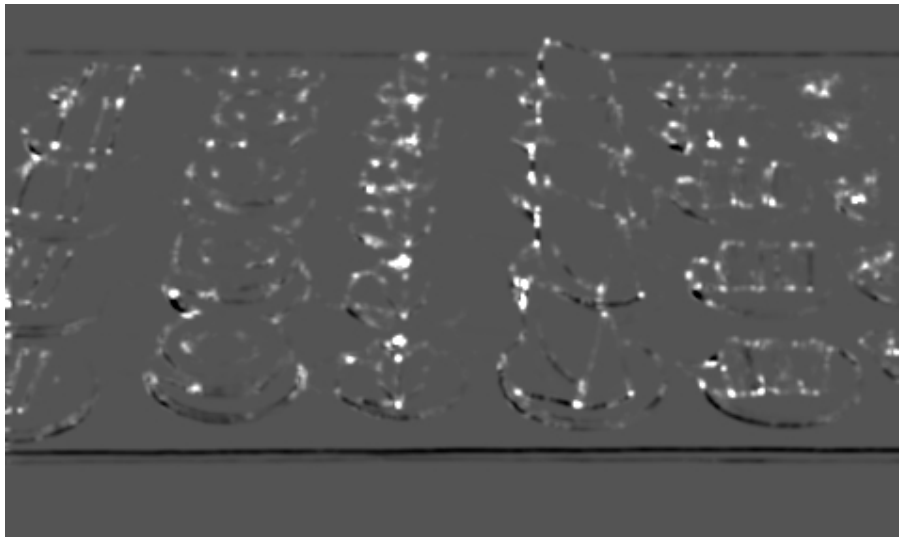
k = 0.05

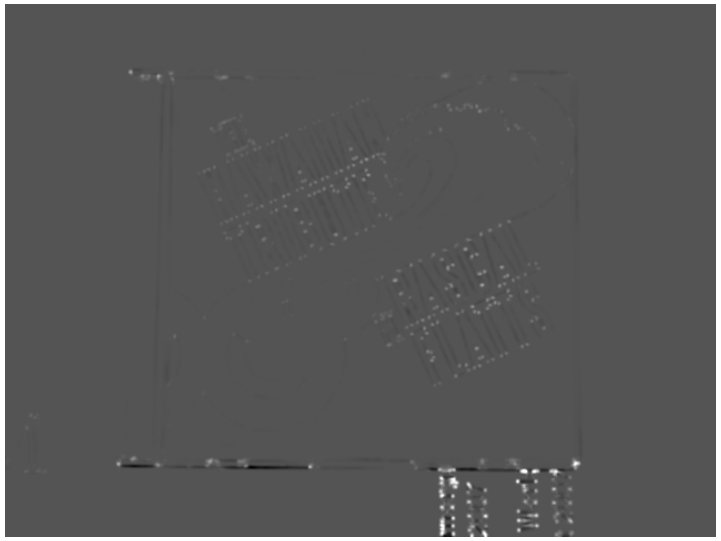
Input images

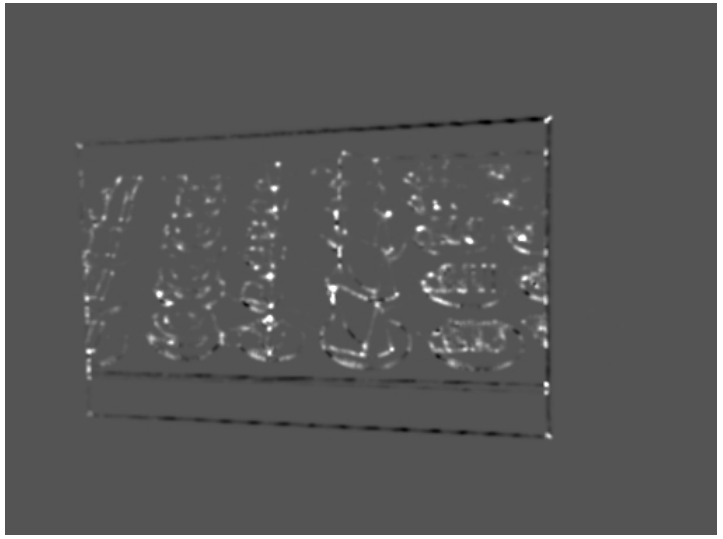


Harris cornerness

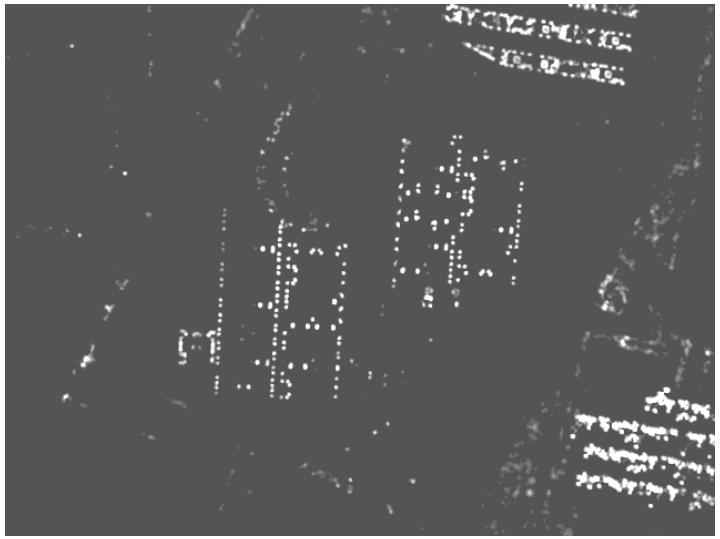


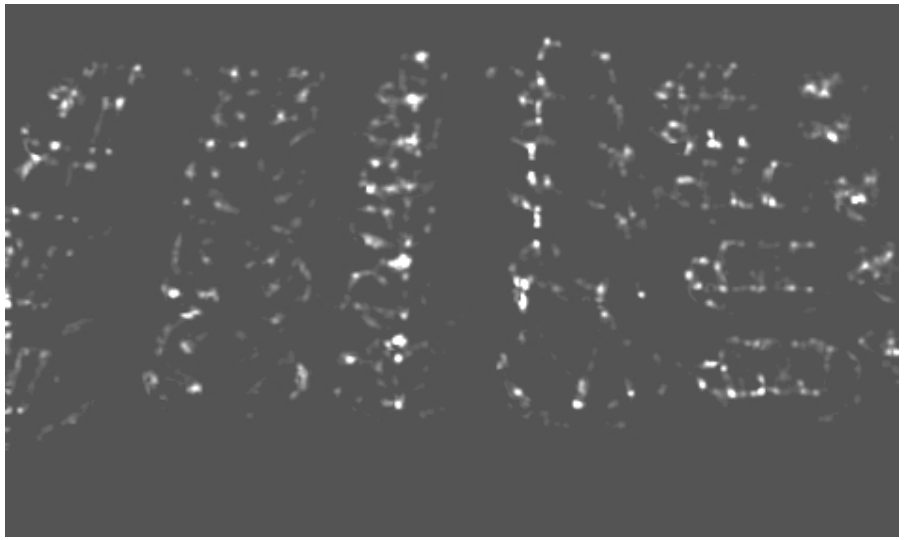


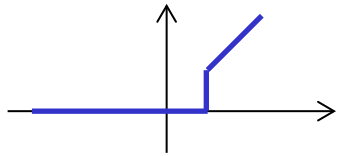


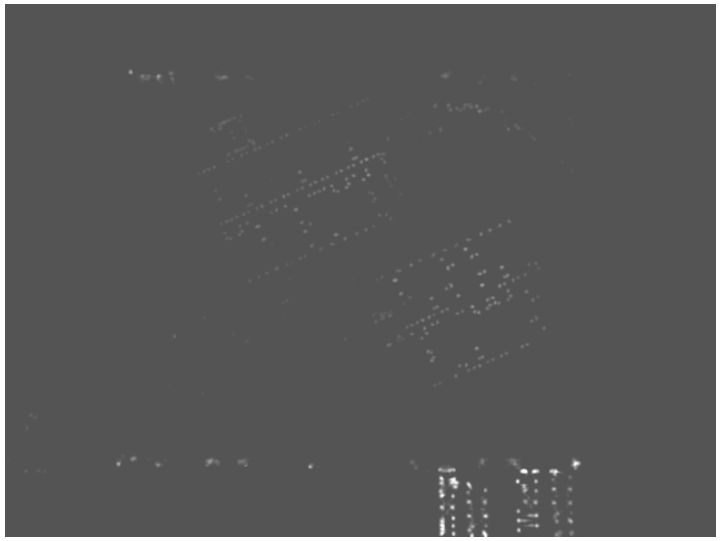


Thresholded cornerness

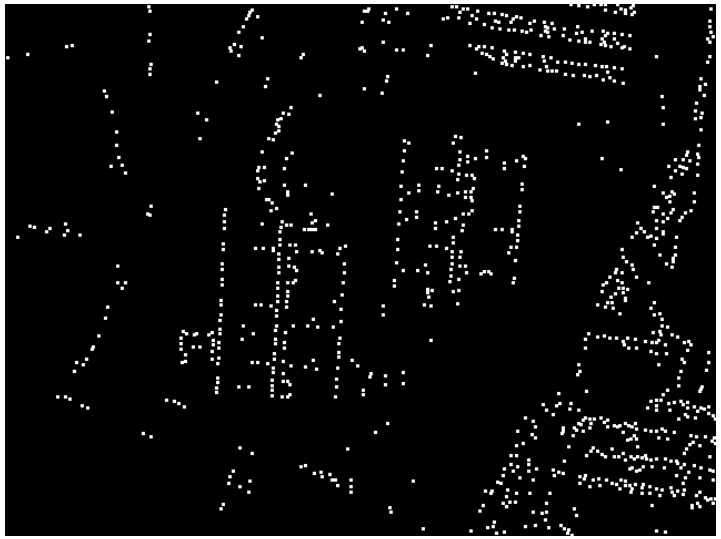


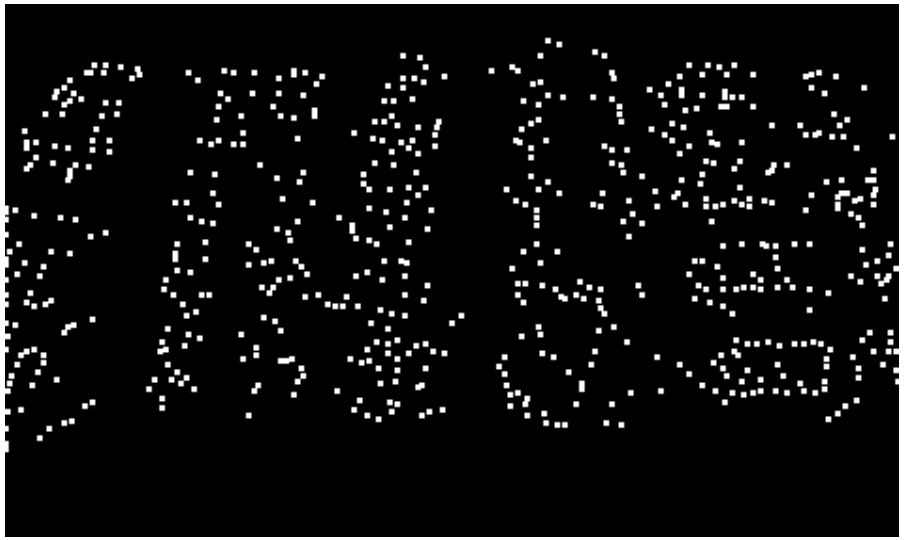




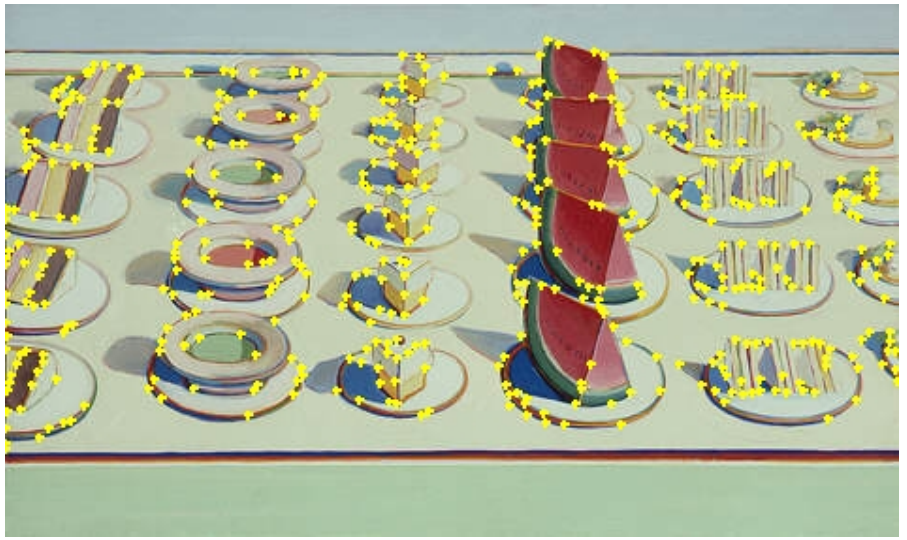


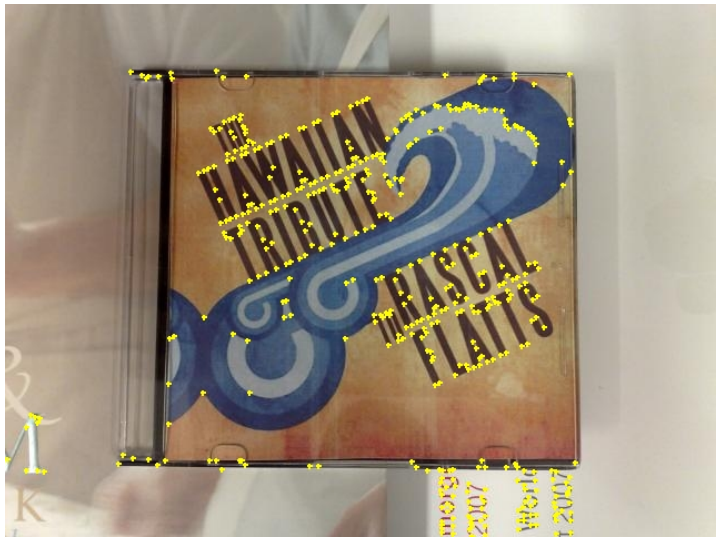
Local maxima of cornerness

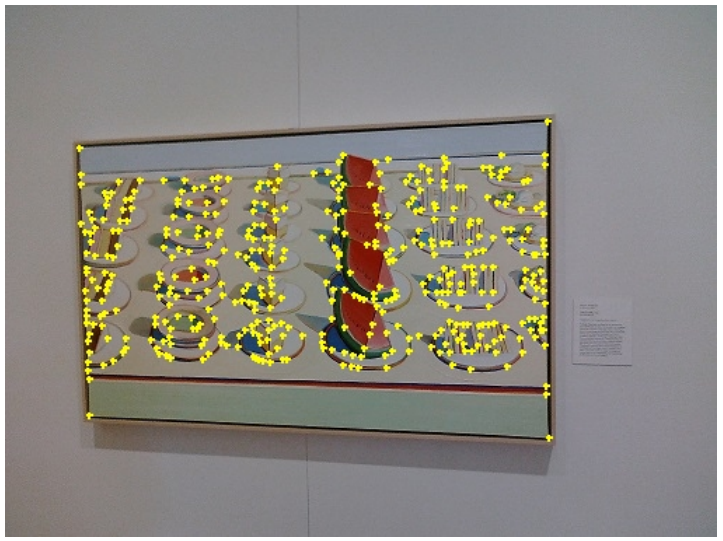




Superimposed Harris keypoints







500 strongest keypoints

Robustness of Harris detector

- Invariant to brightness offset: $f[x,y] \rightarrow f[x,y] + c$
- Invariant to shift and rotation

- Not invariant to scaling