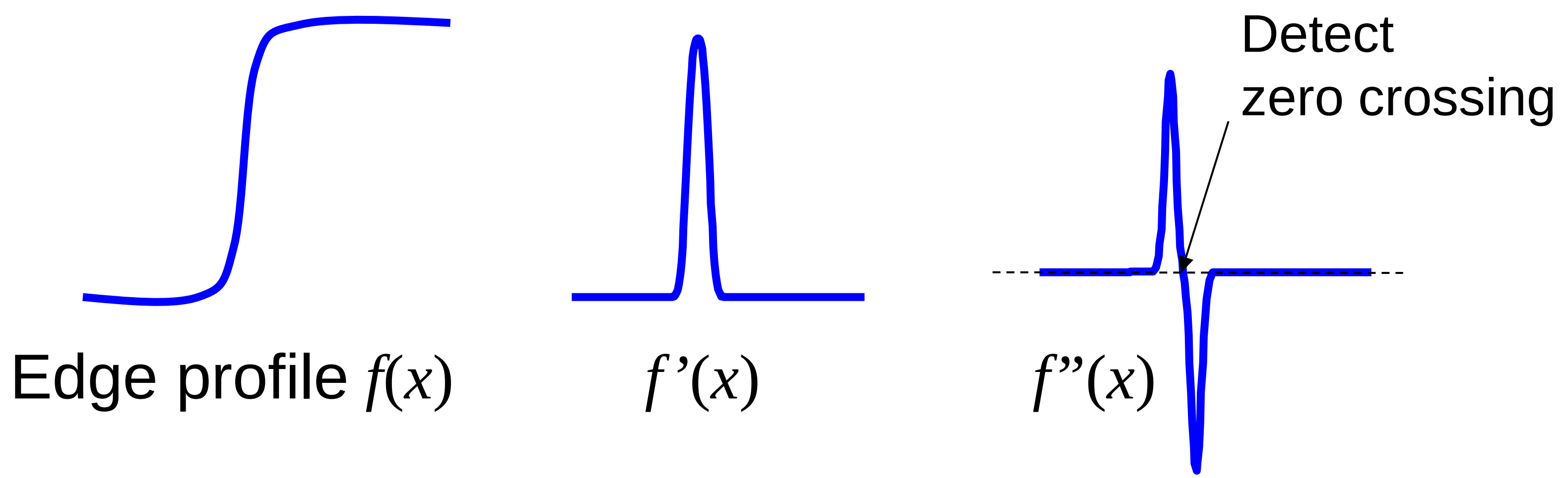
Laplacian operator

- Detect edges by considering second derivative

$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

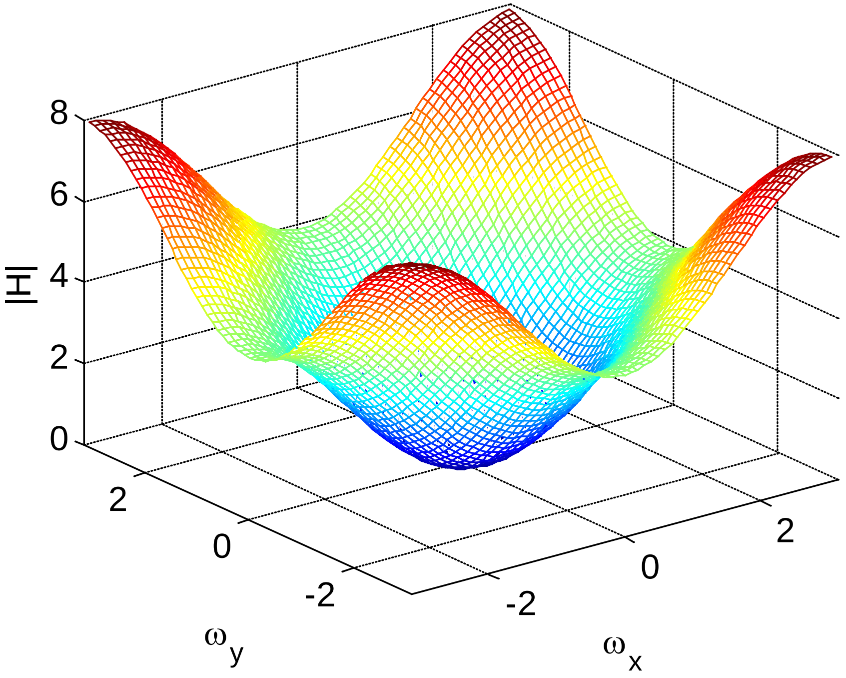
- Isotropic (rotationally invariant) operator
- Zero-crossings mark edge location

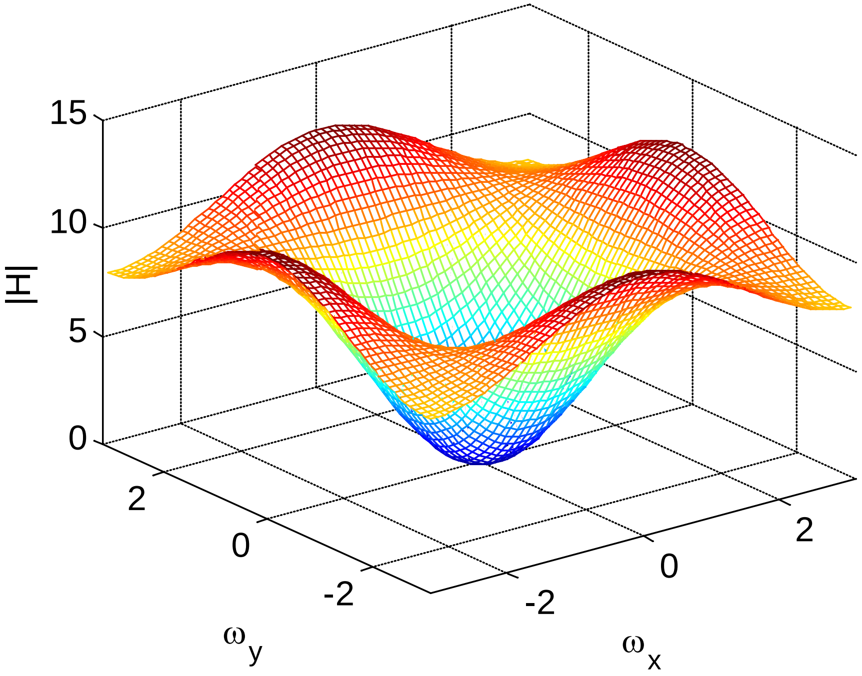


Approximations of Laplacian operator by 3x3 filter

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & [-4] & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & [-8] & 1 \\ 1 & 1 & 1 \end{pmatrix}$$





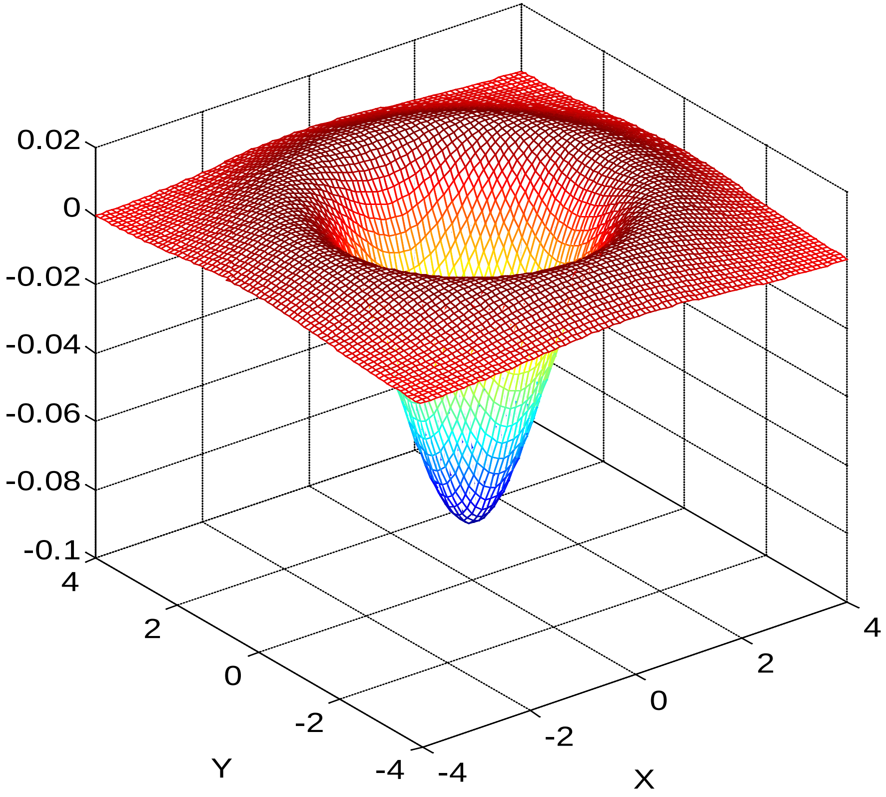
Zero crossings of Laplacian

- Sensitive to very fine detail and noise ➔ blur image first
- Responds equally to strong and weak edges
  ➔ suppress zero-crossings with low gradient magnitude

Laplacian of Gaussian

- Filtering of image with Gaussian and Laplacian operators can be combined into convolution with Laplacian of Gaussian (LoG) operator

$$LoG(x,y) = -\frac{1}{\pi\sigma^4}\left(1-\frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

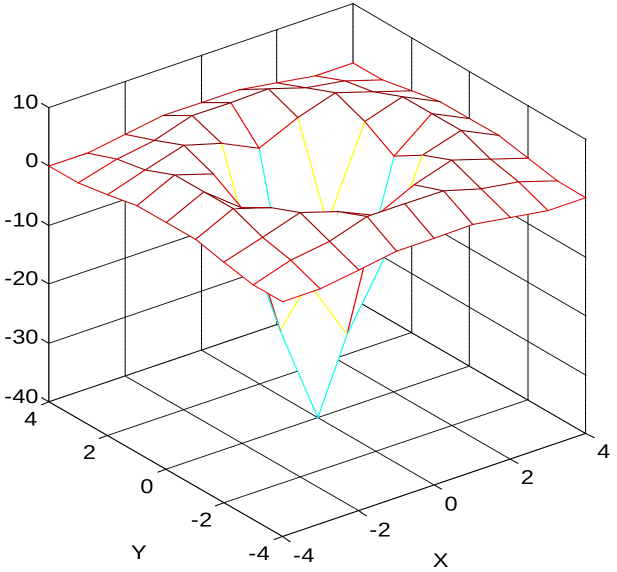


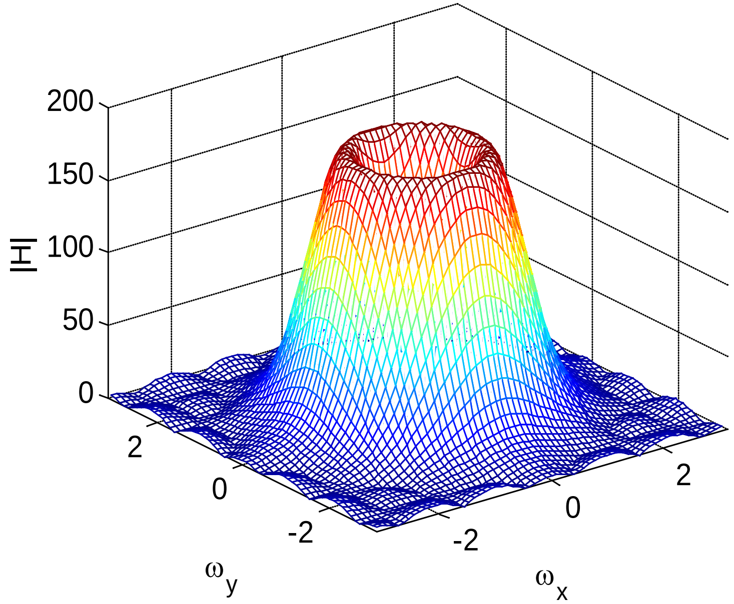
$\sigma = \sqrt{2}$

Discrete approximation of Laplacian of Gaussian

$$\sigma = \sqrt{2}$$

0	0	1	2	2	2	1	0	0
0	2	3	5	5	5	3	2	0
1	3	5	3	0	3	5	3	1
2	5	3	-12	-23	-12	3	5	2
2	5	0	-23	-40	-23	0	5	2
2	5	3	-12	-23	-12	3	5	2
1	3	5	3	0	3	5	3	1
0	2	3	5	5	5	3	2	0
0	0	1	2	2	2	1	0	0



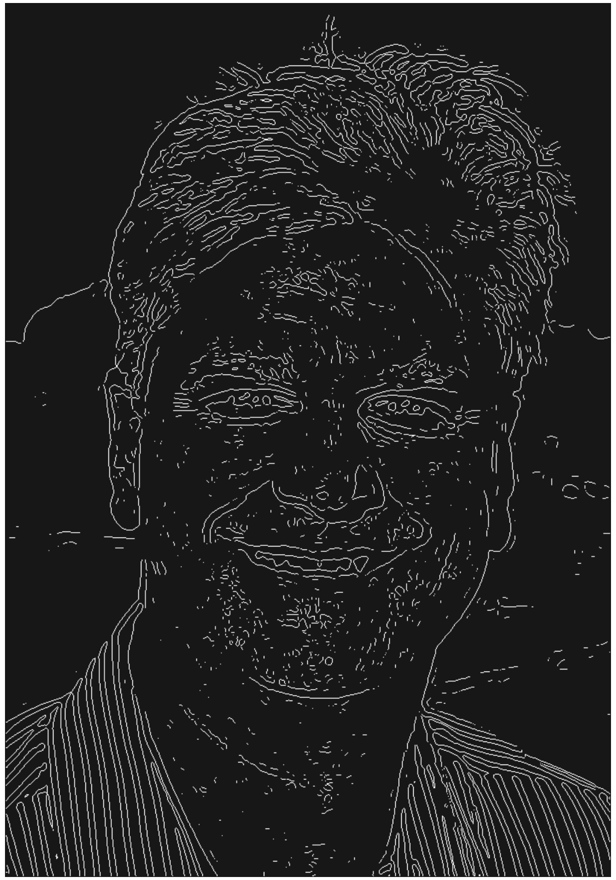


Zero crossings of LoG

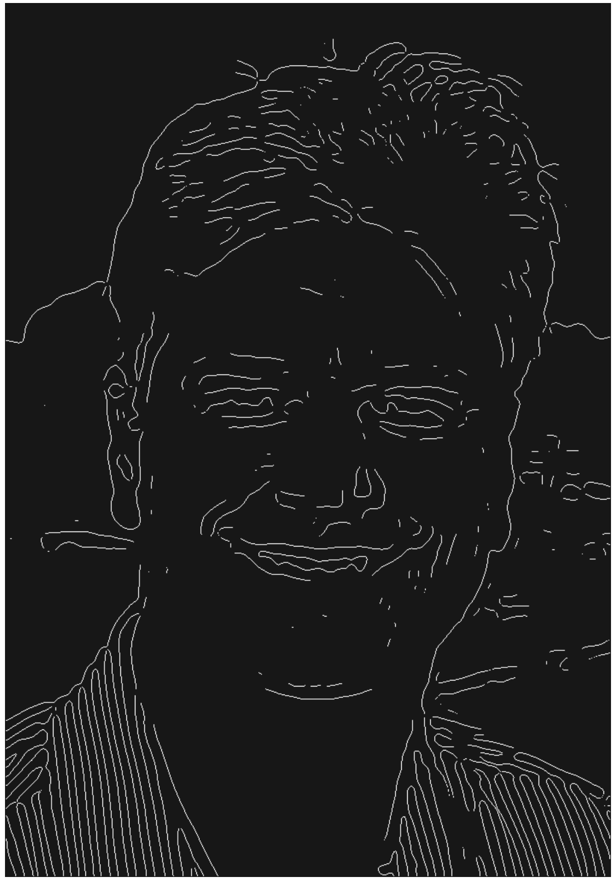
$\sigma = \sqrt{2}$ $\sigma = 2\sqrt{2}$ $\sigma = 4\sqrt{2}$ $\sigma = 8\sqrt{2}$

Zero crossings of LoG – gradient-based threshold

$\sigma = \sqrt{2}$



$\sigma = 2\sqrt{2}$



$\sigma = 4\sqrt{2}$

$\sigma = 8\sqrt{2}$