Unitary transforms

- Sort pixels $f[x,y]$ of an image into column vector $\vec{f}$ of length N
- Calculate N transform coefficients

$$\vec{c} = A\vec{f}$$

where $\boldsymbol{A}$ is a matrix of size N**x**N

- The transform A is unitary, iff

$$A^{-1} = \underbrace{A^{*T} \equiv A^{H}}_{\text{Hermitian conjugate}}$$

- If A is real-valued, i.e., $A=A^*$, transform is „orthonormal“

Energy conservation with unitary transforms

- For any unitary transform $\vec{c} = A\vec{f}$ we obtain

$$\|\vec{c}\|^2 = \vec{c}^{\,H}\vec{c} = \vec{f}^{\,H} A^H A\vec{f} = \|\vec{f}\|^2$$

- Interpretation: every unitary transform is simply a rotation of the coordinate system (and, possibly, sign flips)
- Vector length is conserved.
- Energy (mean squared vector length) is conserved.

Energy distribution for unitary transforms

- Energy is conserved, but, in general, unevenly distributed among coefficients.
- Autocorrelation matrix

$$R_{cc} = E\left[\vec{c}\vec{c}^{\,H}\right] = E\left[A\vec{f} \cdot \vec{f}^{\,H} A^H\right] = AR_{ff}A^H$$

- Diagonal of R_{cc} comprises mean squared values („energies“) of the coefficients c_i

$$E\left[c_i^2\right] = \left[R_{cc}\right]_{i,i} = \left[AR_{ff}A^H\right]_{i,i}$$